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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 1, Due Mon Jan 22, 2018

Problem:	1	2	3	4	5	6	7	8	9	Total	Rescaled
Your Score:											
Possible:	10	10	10	10	10	10	10	20	10	100	20

- **Print this cover sheet and write your name on it.**
- **Except for Problem [9] don't write anything else on this sheet. Do your work on separate paper.**
- **You are encouraged to work together, but the words that you write should be your own.**
- **Assemble your pages in order and staple this cover sheet to the front.**
- **Turn in at the beginning of class on Monday, January 22, 2018. Late papers will not be accepted.**

[1] (similar to suggested problems 2.1#25, 36) Use DeMorgan's laws to find the following negations.
(a) Find the negation of statement *P*: *Bob is green and George is red.*
(b) Find the negation of statement *Q*: $5 \leq x < 6$.

[2] (similar to suggested problem 2.1#41)
(a) Make a truth table for the statement form $(\sim p \vee q) \vee (p \wedge \sim q)$
(b) Is the statement form in (a) a tautology, a contradiction, or neither? Explain.

[3] (similar to suggested problem 2.2#7) Make a truth table for the statement form $\sim p \wedge q \rightarrow r$

[4] (similar to suggested problems 2.2#15 and 2.1#14,16)
(a) Use a truth table to verify that $p \rightarrow q \equiv \sim p \vee q$
(b) Explain why $\sim(p \rightarrow q) \equiv p \wedge \sim q$

[5] Suppose that p and q are statements such that $p \rightarrow q$ is false. Find the truth values of the following:
(a) $\sim p \rightarrow q$ (b) $p \vee q$ (c) $q \rightarrow p$

[6] (similar to suggested problems 2.2#20, 22, 23) Consider statement *S*: If $x = 3$, then $x^2 = 9$.
(a) In words, write the *contrapositive of S.*
(b) In words, write the *converse of S.*
(c) In words, write the *inverse of S.*
(d) In words, write the *negation of S.*

[7] (a) Give an example of a conditional statement *A* such that *A* is true and *the converse of A* is false.
(b) Give an example of a conditional statement *B* such that *B* is true and the *converse of B* is also true.

[8] (similar to suggested problem 2.2 # 20) Write the negation of each statement:
(a) If the car is red then the house is blue.
(b) If today is February 28, then tomorrow is March 1.
(c) If *n* is divisible by 6, then *n* is divisible by 2 and *n* is divisible by 3.

[9] (sim to sugg prob 2.2 # 20,22,23) Let *S* be the conditional statement: "If Alex is a Shark then Betty is a Jet." Some of the following statements below are the converse, inverse, contrapositive, or negation of *S*. Indicate which by writing the appropriate word in the blanks to the right. If none, then write "none".
(a) If Alex is not a Shark, then Betty is not a Jet. ____________________
(b) If Alex is a Shark, then Betty is not a Jet. ____________________
(c) If Betty is a Jet, then Alex is a Shark. ____________________
(d) If Betty is not a Jet, then Alex is not a Shark. ____________________
(e) Alex is a Shark and Betty is not a Jet. ____________________
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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 2, Due Mon Jan 29, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1](a)(b) (Similar to suggested problems 2.3 #6, 7, 8, 12)
Two argument forms are shown at right. Use truth tables to determine whether they are valid or invalid. Be sure to indicate which columns represent the premises and which represent the conclusion, and include an explanation to support your answers.

form [1] (a)	form [1] (b)
$a \to b$	$a \to c$
$\sim a$	$b \to c$
$\therefore \sim b$	$\therefore a \vee b \to c$

[2] (Similar to suggested problems 2.3 #24, 27, 29)
An argument is shown below. The argument might be valid, or it may exhibit the converse error or the inverse error. Use symbols to write the logical form of the argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse error or the inverse error was made.

If $x^2 < 25$, then $x < 5$.
$x < 5$
Therefore, $x^2 < 25$.

[3] (Similar to suggested problem 2.3 #41) A set of premises and a conclusion are given at right. Use the valid argument forms listed in table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in example 2.3.8. Assume all variables are statement variables.

premise 1:	$a \vee b$
premise 2:	$b \to c$
premise 3:	$a \wedge d \to e$
premise 4:	$\sim c$
premise 5:	$\sim b \to f \wedge d$
conclusion:	e

[4] Let x and y be variables with domain the set **R** of real numbers.
Let $P(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$."

(a) Explain why $P(x, y)$ is false for $(x, y) = (-3,2)$.
(b) Give (x, y) values different from those in part (a) for which $P(x, y)$ is false.
(c) Explain why $P(x, y)$ is true for $(x, y) = (2,3)$.
(d) Explain why $P(x, y)$ is true for $(x, y) = (2, -3)$.
(e) Give values different from those in parts (c) and (d) for which $P(x, y)$ is true.

[5] Find the truth set of each predicate.

(a) The domain of the variable d is the set **Z**. The predicate $P(d)$ is the sentence "$6/d$ is an integer."
(b) The domain of the variable d is the set $\mathbf{Z}^+$. The predicate $P(d)$ is the sentence "$6/d$ is an integer."
(c) The domain of the variable x is the set **R**. The predicate $S(x)$ is the sentence "$1 \leq x^2 \leq 4$."
(d) The domain of the variable x is the set **Z**. The predicate $S(x)$ is the sentence "$1 \leq x^2 \leq 4$."

[6] The statement "The square of any rational number is rational" can be written formally in at least two ways:

1. "For all rational numbers x, x^2 is rational."
2. "For all real numbers x, if x is rational, then x^2 is rational."

Notice that in Version 1, the domain (the set of rational numbers) is small, and the predicate ("x^2 is rational") is just a simple statement about the numbers in the domain. And notice that in Version 2, the domain is larger, and so the predicate needs to be in the form of a conditional statement, with the hypothsis ("x is rational") serving to further narrow down the numbers being talked about. The conclusion of the second version ("x^2 is rational") is the simple statement that was the entire predicate of the first version.

The assignment continues on back. ➔

The two versions presented above could be generalized to the following two general forms:

1. For all items in some small domain, BLAH.
2. For all items in some larger domain, if the item is in the smaller subset then BLAH.

By this notation, I mean that the statement BLAH that is the predicate in the first form is the same as the statement BLAH that is the conclusion of the conditional statement in the predicate of the second form.

Rewrite each of the following statements in the two general forms described above.

(a) The derivative of any polynomial function is a polynomial function.
(b) The negative of any irrational number is irrational.
(c) The product of any two rational numbers is rational.

[7] (Based on suggested problem 3.1 #30)

Let n be a variable with domain the set **Z** of integers.
Let $Odd(n)$ be the statement "n is odd".
Let $Prime(n)$ be the statement "n is prime".
Let $Square(n)$ be the statement "n is a perfect square".

(An integer n is said to be a perfect square if it equals the square of some integer. For example, 9 is a perfect square because $9 = 3^2$.)

Consider the following two statements

Statement A: $\exists n \in \mathbf{Z}\big(Prime(n) \wedge \sim Odd(n)\big)$".
Statement B: $\exists n \in \mathbf{Z}\big(Odd(n) \wedge Square(n)\big)$".

Rewrite each statement without using mathematical notation (quantifiers, variables, functions, logic symbols). Determine whether the statements are true or false, and justify your answers as best as you can.

[8] (Based on suggested problem 3.1 #30) Let u, v, x, y be variables with domain the set **R** of real numbers.

Consider the following two statements

Statement A: $xy = 0 \ \Rightarrow \ x = 0 \ \vee \ y = 0$.
Statement B: $u < x \ \wedge \ v < y \ \ \Rightarrow \ uv < xy$.

Are the statements true or false? Give counterexamples for the statements that are false.

[9] (Similar to suggested problem 3.2 #5) Consider the following two statements

statement A: Some exercises have answers.
statement B: Every rational number is a real number.

Write a formal and informal negation for each statement. (Hint: start by rewriting the original statements formally, using variables and quantifiers. Then find the formal negation of the formal original, then finally find the rewrite the formal negation as an informal negation.

[10] (Similar to sugg exercises 3.2 #19, 29) Consider Statement A.

Statement A: For all integers n, if $12/n$ is an integer, then $n = 4$.

(a) Write the following four statements: $converse(A), contrapositive(A), inverse(A), \sim A$

(b) Consider the five statements $A, converse(A), contrapositve(A), inverse(A), \sim A$.
Which of those five statements is true? Explain.

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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 3, Due Fri Feb 9, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] (Similar to suggested problem 4.1 # 20) Show how a direct proof of Statement *A* would begin and end. That is, show the proof structure. You do not have to fill in the proof.

Statement *A*: For all real numbers x, if $x > 1$ then $x^2 > x$.

[2] (Similar to suggested problems 4.1 # 25) Consider the following statement.

Statement *B*: The sum of any two odd integers is even.

(a) Rewrite Statement *B* using variables and quantifiers.

(b) Prove or disprove Statement *B*.

[3] (Similar to suggested problems 4.1 # 25, 39, 41, 43) Prove or disprove Statement *C*.

Statement *C*: For all integers n and m, if $n - m$ is even then $n^3 - m^3$ even.

Hint: To start, use long division to see if $n - m$ is a factor of $n^3 - m^3$. (This will require that you review long division of polynomials. That's why I assigned the problem.)

[4] (Similar to suggested problems 4.1 # 35, 53, 54) Prove or disprove Statement *D*.

Statement *D*: There exists an integer n such that $6n^2 + 27$ is prime.

[5] (Suggested exercise 4.1 # 58) Consider the following statement.

Statement *E*: The difference of the squares of any two consecutive integers is odd.

(a) Rewrite Statement *E* using variables and quantifiers.

(b) Prove or disprove Statement *E*.

[6] The Zero Product Property says that if a product of two real numbers is 0, then at least one of the numbers must be 0.

(a) Write this property formally using quantifiers and variables.

(b) Write the contrapositive of your answer to (a).

(c) Write an informal version (without quantifier symbols or variables) of the contrapositive.

[7] (Similar to suggested exercise 4.2 # 9) Suppose that a and b are both integers and that $a \neq 0$ and $b \neq 0$. Explain why $(5a + 12b)/(7a^2b)$ must be a rational number. Hint: You will need to use [6c].

[8] (Suggested exercise 4.2 # 14) Consider the following statement.

Statement *F*: The square of any rational number is a rational number.

(a) Rewrite Statement *F* using variables and quantifiers.

(b) Prove or disprove Statement *F*.

[9] (Similar to suggested exercise 4.2 # 15) Consider the following statement.

Statement *G*: The difference of any two rational numbers is a rational number.

(a) Rewrite Statement *G* using variables and quantifiers.

(b) Prove or disprove Statement *G*.

[10] (Suggested exercise 4.2 # 20) Given two rational numbers r and s, with $r < s$, prove that there is a rational number between r and s. Hint: Use the results of 4.2 # 18, 19 (even if you were not able to do those exercises).

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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 4, Due Wed Feb 14, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] (Similar to suggested problem 4.3 #15) Prove that for all integers a, b, c, if $a|b$ and $a|c$, then $a|(b-c)$.

[2] (Similar to suggested problems 4.3 # 26, 27, 30) Prove or disprove Statement *D*.

Statement *D*: for all integers a, b, c, if $a|bc$, then $a|b$ or $a|c$.

[3] (Similar to suggested problem 4.3 # 41) How many zeros are at the end of $45^6 \cdot 44^{13}$? Explain how you can answer this question without actually computing the number. (Hint: $10 = 2 \cdot 5$)

[4] (Similar to suggested problem 4.3 #47) Prove that for any nonnegative integer n, if the sum of the digits of n is divisible by 3, then n is divisible by 3.

[5] (Sim. to sugg. probs 4.4 # 2,6) For each given n, d
(i) Write the corrsponding $n = dq + r$ equation with $q, r \in \boldsymbol{Z}$ and $0 \le r < d$.
(ii) Write the corresponding *mod* and *div* expressions.

(a) $n = 61, d = 8$ (b) $n = -27, d = 5$ (c) $n = 32, d = 4$

[6] (Sim. to sugg. probs 4.4 # 9) For the given *mod* and *div* expressions,
(i) Write the corrsponding $n = dq + r$ equation with $q, r \in \boldsymbol{Z}$ and $0 \le r < d$.
(ii) Find the value of the given *mod* and *div* expression. (Present answers as $32\ div\ 5 = BLAH$.)

(a) $32\ div\ 5$ and $32\ mod\ 5$
(b) $29\ div\ 2$ and $29\ mod\ 2$

[7] (Similar to suggested problem 4.4 # 21)
Suppose that c is an integer such that $c\ mod\ 15 = 3$. What is $7c\ mod\ 15$? In other words if division of c by 15 gives a remainder of 3, what is the remainder when $7c$ is divided by 15?

[8] (Suggested problem 4.4 # 30)
Use the quotient-remainder theorem with $d = 3$ to prove that the product of any two consecutive integers has the form $3k$ or the form $3k + 2$, for some integer k.

[9] (Suggested problem 4.4 # 44) Prove this property of the absolute value. (Use book's hint.)

$$\forall x, y \in \boldsymbol{R}(\ |x| \cdot |y| = |xy|\ )$$

[10] (Suggested problem 4.4 # 46) Prove this property of the absolute value.

$$\forall c \in \boldsymbol{R}^+\big(\forall r \in \boldsymbol{R}(\ If\ |r| \le c\ \ then\ \ -c \le r \le c\ )\big)$$

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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 5, Due Fri March 2, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] (a) Let $a_k = 2k + 1$ for $k \geq 0$. Write the first four terms of the sequence. ($k = 0,1,2,3$) Label your terms.

(b) Let $b_k = (k-1)^3 + k + 2$ for $k \geq 0$. Write the four terms of the sequence. Label your terms.

[2] Find an explicit formula for the sequence that begins $0, -\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \frac{6}{7}, \ldots$ (Remember that an explicit formula consists of an expression for computing the terms, along with a starting value for the index.)

[3] Compute the sums & products. (Exact, simplified answers, not decimal approximations.) Show calculations.

$$(a)\ \sum_{k=0}^{4} \frac{1}{3^k} \qquad (b)\ \prod_{k=3}^{5} (-1)^k k^3 \qquad (c)\ \sum_{k=1}^{998} (-1)^k \qquad (d)\ \prod_{k=1}^{998} (-1)^k \qquad (d)\ \sum_{k=1}^{1000} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

[4] Compute the following quantities. Your solution must contain a clear presentation of the calculation.

$$(a)\ \frac{5!}{7!} \qquad (b)\ \frac{5!}{0!} \qquad (c)\ \frac{n!}{(n-2)!} \qquad (d)\ \binom{7}{4} \qquad (e)\ \binom{7}{7} \qquad (f)\ \binom{n+1}{n}$$

[5] Rewrite the following using summation notation. (Do not compute the numbers!!)

$(a)\ 2 + 4 + 6 + \cdots + 738$
$(b)\ 2 + 4 + 6 + \cdots + 2n$
$(c)\ 1^3 + 2^3 + 3^3 + \cdots + 597^3$
$(d)\ 1^3 + 2^3 + 3^3 + \cdots + n^3$
$(e)\ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 615 \cdot 616$
$(f)\ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n$

[6] What does it mean to say that $205_{10} = 11001101_2$? Explain by expressing 205 as a sum of terms that is each a product of either 1 or 0 and some power of the number 2.

[7] (Similar to book example on p.244-247, Proposition 5.2.1, and Suggested Exercise 5.2 # 1)

The goal is to use the Principle of Mathematical Induction to prove statement *S*:

Statement *S*: Any postage of at least 12 cents can be obtained by using 3 cent and 7 cent stamps.

Let $P(n)$ be the predicate "A postage of *n* cents can be obtained using 3 cent and 7 cent stamps."

Using this definition of $P(n)$ statement *S* can be abbreviated:

$$Statement\ S\colon \forall n \in \mathbf{Z}, n \geq 12(P(n))$$

Prove statement *S* using Mathematical Induction.

(Hint: Use as your model the book's proof of Prop 5.2.1 or the book's solution of Suggested Exercise 5.2 # 1.)

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[8] The goal of this problem is to use the Method of Induction to prove statement *S*:

$$Statement\ S\text{: } \forall n \in \mathbf{Z}, n \geq 1\left(1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n-3)}{2}\right)$$

$$Predicate\ P(n)\ is\ the\ following\ :\ \ 1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n-3)}{2}$$

Questions (a),(b),(c),(d) are similar to Suggested Exercises 5.2 # 3,4 and are about identifying the parts. This is what goes on in the portion of the *Handout on Induction* titled *"Preliminary Work".*

(a) Write $P(1)$.

(b) Write $P(k)$.

(c) Write $P(k + 1)$.

(d) In a proof by mathematical induction that the predicate $P(n)$ is true for all $n \geq 1$, what must be shown in the Inductive Step?

Question (e) is similar to Suggested Exercise 5.2 # 6, 8, 10, 13 and is about actually doing the proof. This is what goes on in the portion of the *Handout on Induction* titled *"Build a proof of Statement S using the following structure".*

(e) Prove statement *S* using Mathematical Induction.

[9] Same questions (a),(b),(c),(d),(e) as in [8], but this time with the following Statement and Predicate.

$$Statement\ S\ is\ the\ following\text{: } \forall n \in \mathbf{Z}, n \geq 1\left(1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2\right)$$

$$Predicate\ P(n)\ \ is\ the\ following\text{: } 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

[10] (Conceptual Questions)

(a) Why is 0! defined to be 1 instead of 0? Explain.

(b) In your first job after college, you are working at the *Induction Hotline*. Somebody calls wanting help with the Inductive Step. They want to know how they are supposed to prove that $P(k)$ is true if they don't know the value of k. What do you tell them?

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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 6, Due Fri March 9, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] The goal of this problem is to use Induction to prove Statement *S*:

$$\textit{Statement } S\text{: } \forall n \in \mathbf{Z}, n \geq 0(7^n - 2^n \textit{ is divisible by } 5)$$

Questions (a),(b),(c),(d),(e) are about identifying the parts. This is what goes on in the portion of the *Handout on Induction* titled *"Preliminary Work"*.

(a) Write $P(n)$. **(b)** Write $P(0)$. **(c)** Write $P(k)$. **(d)** Write $P(k+1)$.

(e) In a proof of Statement S using the Method of Induction, what must be shown in the Inductive Step?

Question (f) is about actually doing the proof. This is what goes on in the portion of the *Handout on Induction* titled *"Build a proof of Statement S using the following structure"*.

(f) Prove Statement *S* usingthe Method of Induction.

[2] Using a web site or Excel, make a table (with column headings) that compares the values of the three functions n^3 and 3^n and $n!$ for $n = 0,1,2, \dots, 12$. Figure out how to print your table, with your name included in the computer printout. (I don't want your name to be hand-written.) Print the table and include it here.

[3] (a) From the three functions n^3 and 3^n and $n!$, pick one of the functions to play the role of $f(n)$ and another function to play the role of $g(n)$, and pick an integer value of "a" (the lowest possible value) to build a true statement of this form: $\textit{Statement } S\text{: } \forall n \in \mathbf{Z}, n \geq a(f(n) < g(n))$. (Write down your true Statement S.)

(b) Use the Method of Induction to prove your Statement S.

[4] (a) From the three functions n^3 and 3^n and $n!$, pick another pair of functions to play the role of $f(n)$ and $g(n)$, and again pick an integer value of "a" (the lowest possible value) to build another true statement of the form $\textit{Statement } S\text{: } \forall n \in \mathbf{Z}, n \geq a(f(n) < g(n))$. (Write down your true Statement S.)

(b) Use the Method of Induction to prove your Statement S.

In [5] – [10], be sure to present your solutions as equations, with left sides that name the quantity.

[5] 6.1 #12. Present each answer three ways: **(i)** Set Notation **(ii)** Interval Notation **(iii)** Number Line.

[6] 6.1#14 **[7]** 6.1#16 **[8]** 6.1#17 **[9]** 6.1#23

[10] 6.1#35 but using $A = \{c, d\}$ and $B = \{4,6\}$ and $C = \{6,7\}$.

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2017 - 2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 7, Due Fri March 30, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] A *bit string* is a string of zeros and ones. A *4-bit string* is bit string with four digits. We need an organized way of listing all the 4-bit strings. The standard way is to count from 0 to 15 in binary. Fill in the table:

number in base 10	representation by a 4-bit string
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

[2] Fill in the following table by rotating your list of 4-bit strings 90° counterclockwise. The finished table shows the outputs of all possible two-place Boolean functions.

		output of f_0	output of f_1	output of f_2	output of f_3	output of f_4	output of f_5	output of f_6	output of f_7	output of f_8	output of f_9	output of f_{10}	output of f_{11}	output of f_{12}	output of f_{13}	output of f_{14}	output of f_{15}
Input	(0,0)																
	(0,1)																
	(1,0)																
	(1,1)																

[3] (a) What is $f_{11}(0,1)$?
(b) What is $f_{11}(1,0)$?
(c) For what input (a,b) is $f_4(a,b) = 1$?
(d) What is is $f_3^{-1}(1)$?

[4] The function g shown at right is a two-place Boolean function. $g(x_1, x_2) = (4x_1 + 3x_2)\ mod\ 2$
(a) What is $g(0,0)$?
(b) What is $g(0,1)$?
(c) What is $g(1,0)$?
(d) What is $g(1,1)$?
(e) Which of the sixteen two-place Boolean functions in your table from exercise [2] is g?

[5] Let $A = \{-3, -2, -1, 0\}, B = \{0, 1\}$, and let $f: \boldsymbol{R} \to \boldsymbol{R}$ be defined by the formula $f(x) = x^2$.

(a) What is $A \cap B$?

(b) What is $f(A \cap B)$?

(c) What is $f(A)$?

(d) What is $f(B)$?

(e) What is $f(A) \cap f(B)$?

(f) Prove or disprove: ∀ sets A and B and for all functions f, $f(A \cap B) = f(A) \cap f(B)$.

[6] Define $f: \boldsymbol{Z} \to \boldsymbol{Z}$ by $f(n) = 2n + 7$.

(a) Is f one-to-one? Prove or give a counterexample.

(b) Is f onto? Prove or give a counterexample.

(c) Is f a one-to-one correspondence? If it is, find the inverse function.

[7] Define $g: \boldsymbol{R} \to \boldsymbol{R}$ by $g(x) = 2x + 7$.

(a) Is g one-to-one? Prove or give a counterexample.

(b) Is g onto? Prove or give a counterexample.

(c) Is g a one-to-one correspondence? If it is, find the inverse function

[8] (a) Circle the statements that mean " f is one-to-one"

Statement S: $\forall x_1, x_2 \left(If\ x_1 = x_2\ then\ f(x_1) = f(x_2)\right)$.

Converse of S: $\forall x_1, x_2 (If\ f(x_1) = f(x_2)\ then\ x_1 = x_2)$.

Inverse of S: $\forall x_1, x_2 \left(If\ x_1 \neq x_2\ then\ f(x_1) \neq f(x_2)\right)$.

Contrapositive of S: $\forall x_1, x_2 (If\ f(x_1) \neq f(x_2)\ then\ x_1 \neq x_2)$.

(b) Circle the statements that mean "f is onto"

$\forall x\left(\exists y(f(x) = y)\right)$.

$\exists x\left(\forall y(f(x) = y)\right)$.

$\forall y\left(\exists x(f(x) = y)\right)$.

$\exists y\left(\forall x(f(x) = y)\right)$.

[9] $Let\ \ f(x) = \dfrac{x + a}{x + b}\ \ for\ all\ real\ numbers\ x \neq -b.$

(a) Find f^{-1}.

(b) What is the domain of f^{-1}?

[10] (a) Give an example of a function $f: \boldsymbol{R} \to \boldsymbol{R}$ that is one-to-one but not onto. **To get credit for this, your function cannot be the same as the function of anybody else in the class.**

(b) Give an example of a function $g: \boldsymbol{R} \to \boldsymbol{R}$ that is onto but not one-to-one. **To get credit for this, your function cannot be the same as the function of anybody else in the class.**

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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 8, Due Fri April 6, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

Remark: In [1] and [2], if you give a counterexample to demonstrate that a statement is false, your counterexample has to be unique: It can't be the same counterexample given by another student or by the book.

[1] (related to suggested exercises 7.3 # 16,18) Suppose that $f: A \to B$ and $g: B \to C$.
(a) If $g \circ f$ is one-to-one, must f be one-to-one? Prove or give a counterexample.
(b) If $g \circ f$ is one-to-one, must g be one-to-one? Prove or give a counterexample.

[2] (related to suggested exercises 7.3 # 17,19) Suppose that $f: A \to B$ and $g: B \to C$.
(a) If $g \circ f$ is onto, must f be onto? Prove or give a counterexample.
(b) If $g \circ f$ is onto, must g be onto? Prove or give a counterexample.

[3] (Suggested exercise 7.3 # 26) Prove that if if $f: A \to B$ and $g: B \to C$ are both one-to-one and onto, then $(g \circ f)^{-1}$ exists and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$
(Hint: The book gives a hint, but there is a nicer solution than the one that the book suggests. Investigate what happens when you compose the expression $g \circ f$ with the expression $f^{-1} \circ g^{-1}$, and investigate what happens when you compose $f^{-1} \circ g^{-1}$ with $g \circ f$. Then use the result of 7.3#25, which I presented in class on Friday as the "missing Theorem B".)

[4] Let $A = \{2,3,4,5\}$ and $B = \{3,4\}$. Define a *binary relation* R from A to B by $R = \{(x, y) \in A \times B | x \geq y\}$

(a) Is $2\ R\ 4$? (b) Is $4\ R\ 3$? (c) Is $(4,4) \in R$? (d) Is $(3,2) \in R$? (e) Write R as ordered pairs.

[5] (a) List all the *binary relations* from $A = \{1,2\}$ to $B = \{x, y\}$
(b) How many of the *binary relations* from part (a) are *functions*?
(c) Now suppose that C is a set with m elements and D is a set with n elements. How many *binary relations* are there from C to D?
(d) How many of the *binary relations* from part (c) are *functions*?

[6] Let $B = \{a, b, c, d\}$. Define a binary relation S on B by $S = \{(a, b), (a, c), (b, c), (d, d)\}$
Draw the directed graph of S.

[7] Let $A = \{2,3,4,5,6,7,8\}$. Define a binary relation T on A by $T = \{(x, y) \in A \times A \text{ such that } 3|(x - y)\}$
Draw the directed graph of T.

[8] Let $A = \{0,1,2,3\}$. Define a binary relation R on A by $R = \{(0,0), (0,1), (1,1), (1,2), (2,2), (2,3)\}$
(a) Draw the directed graph for R.
(b) Is R *reflexive*? (c) Is R *symmetric*? (d) Is R *transitive*? (If not, give counterexamples to demonstrate.)

[9] Let C be the unit circle relation on the set of real numbers $\boldsymbol{R}$.
That is $C = \{(x, y) \in \boldsymbol{R}^2 \text{ such that } x^2 + y^2 = 1\}$
(a) Draw a picture of the relation C showing it as a subset of $\boldsymbol{R}^2$.
(b) Is C *reflexive*? (c) Is C *symmetric*? (d) Is C *transitive*? (If not, give counterexamples to demonstrate.)

[10] Define the binary relation P on the set $\boldsymbol{Z}$ by

$$P = \{(m, n) \in \boldsymbol{Z}^2 \text{ such that } m \text{ and } n \text{ have a common prime factor}\}$$

(a) Is P *reflexive*? (b) Is P *symmetric*? (c) Is P *transitive*? (If not, give counterexamples to demonstrate.)

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2017 - 2018 Spring Semester MATH 3050 Section 101 (Barsamian) Revised H9, Due Wed Apr 18, 2018

Problem:	1	2	3	4	5	6	7	Total	Rescaled
Your Score:									
Possible:	10	10	10	20	20	10	20	100	20

[1] (Similar to book Example 9.1.1 and suggested exercises 9.1 #3,5)

A card is drawn at random from a standard 52-card deck.

(A) Draw the sample space S (note that this is not done in the book.) and find $N(S)$.

(B) Define E to be the event that the drawn card is red and has an odd number on it. Illustrate E on your drawing of the sample space, find $N(E)$, and find $P(E)$.

[2] (Similar to book Example 9.1.2 and suggested exercises 9.1 #7,9)

A pair of standard dice is rolled. One die is red; the other, white

(A) Draw the sample space S using a table (easier than the drawing done in the book example) and find $N(S)$.

(B) Define E to be the event that the sum of the numbers is at least 8. Illustrate E on your drawing of the sample space, find $N(E)$, and find $P(E)$.

[3] (Similar to book suggested exercise 9.1 #18)

An urn contains three green balls (denoted G_1, G_2, G_3) and two red balls (denoted R_1, R_2).

One ball is drawn and its color is recorded, and it is replaced in the urn.

Then another ball is drawn and its color is recorded.

(A) Illustrate the experiment two ways: Draw the sample space S using a table, and also construct a possibility tree showing all the possible outcomes. Find $N(S)$.

(B) Define E to be the event that first ball drawn is green. Illustrate E on your table, find $N(E)$ and $P(E)$.

(C) Define F to be the event that both balls drawn are green. Illustrate F on your tree, find $N(F)$ and $P(F)$.

[4] (Similar to previous problem)

Same urns, same balls as in previous problem, but this time the first ball is not put back into the urn before the second ball is drawn. Answer questions (A),(B),(C) from problem [3] in this new situation. (Make new illustrations.)

(Hint: the key is to realize that the new illustrations can be made by making small changes to the old ones.)

[5] (Similar to book Example 9.1.3 and suggested exercise 9.1 #20, but using Section 9.2 tree technique)

Analyze the *Monty Hall Problem* but with n doors, instead of 3 doors. That is, there are n doors, one that has a prize and the remaining doors empty. (I'll tell you which value of n to use below.) The contestant chooses a door. The host does not open that door, but opens one of the remaining doors. The host always chooses to open one of the remaining doors that is empty. At this point, the contestant has the option of staying with their original choice of door, or switching to a new choice. (Of course, the contestant would not choose the door that the host has opened, because the contestant now knows that that door is empty.)

Define two strategies that the contestant can use:

- The *Stay Strategy* is to stay with the original choice of door.
- The *Switch Strategy* is to switch doors.

Here are your assignments

- Ryan, Ashley, Noah, Matthew: Use $n = 6$.
- Devon, Ben, Alyssa, Blue: Use $n = 5$.
- Kristen, Peter, Jalen, Alexandria, Allison: Use $n = 4$.

(A) Using your assigned value of n, make a possibility tree for the *Stay Strategy* and find $N(Stay)$.

(B) Define $WinStay$ to be the event that the contestant wins using the *Stay Strategy*. Illustrate $WinStay$ on your tree, find $N(WinStay)$ and $P(WinStay)$.

(C) Start over: Using your assigned value of n, make a possibility tree for the *Switch Strategy* and find $N(Switch)$.

(D) Define $WinSwitch$ to be the event that the contestant wins using the *Switch Strategy*. Illustrate $WinSwitch$ on your tree, find $N(WinSwitch)$ and $P(WinSwitch)$.

[6] (Similar to book suggested exercise 9.2#6,7)

An urn contains three green balls (denoted G_1, G_2, G_3) and one red ball (denoted R_1).

A second urn contains two green balls (denoted G_4, G_5) and two red balls (denoted R_2, R_3).

An experiment is performed in which one of the two urns is chosen at random and then two balls are randomly chosen from it, one after the other, without replacement.

(A) Construct a possibility tree showing all the possible outcomes. Find $N(S)$.

(B) Define E to be the event that two green balls are chosen. Illustrate E on your table, find $N(E)$ and $P(E)$.

[7] (Similar to book Example 9.2.1 and suggested exercises 9.2#1,3)

(A) How many ways can a Best 3 of 5 Contest be played? Draw a possibility tree to illustrate.

(B) In a Best 3 of 5 Contest, what is the probability that the team that loses the first game will go on to win the contest? (Explain using your possibility tree.)

(C) How many ways can a Best 3 of 5 Contest be played if no team wins three games in a row? Draw a new possibility tree to illustrate.

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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 10, Due Wed April 25, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

On all problems, give answers in sentences, with explanations. Don't just give a number as an answer.

[1] (similar to suggested exercises 9.2#32,39)
(a) How many ways can the letters of the word *EDUCATION* be arranged in a row?
(b) How many ways can the letters of the word *EDUCATION* be arranged in a row if the letters *CAT* must remain together (in that order)?
(c) How many ways can four letters of the word *EDUCATION* be selected and arranged in a row?
(d) How many ways can four letters of the word *EDUCATION* be selected and arranged in a row if the first two letters must be *DT* (in that order)?

[2] (similar to suggested exercises 9.2#41,42) Prove that $\forall n \geq 4\big(P(n+1,4) - P(n,4) = 4P(n,3)\big)$.

[3] (sim to Example 9.3.3 and sugg ex 9.3# 3,16) Your credit card needs a PIN, which can be any four digits.
(a) How many PINs are possible?
(b) How many PINs do not have any repeated digits? ("Repeated" means digits used more than once. It does not mean that the repeated digits have to be adjacent in the PIN.)
(c) How many PINs do have repeated digits?
(d) What is the probability that a randomly chosen 4-digit PIN does have repeated digits?

[4] (similar to Examples 9.1.4 and 9.3.6 and suggested exercise 9.3#23)
(a) How many integers from 1 through 1000 are multiples of 5 or multiples of 7?
(b) Suppose that an integer from 1 through 1000 is chosen at random. What is the probability that the integer is a multiple of 5 or a multiple of 7?
(c) How many integers from 1 through 1000 are neither multiples of 5 nor multiples of 7?

[5] (similar to suggested exercise 9.3#31) **The Birthday Problem Part 1: Group of 5 people**
Assume that all years have 365 days, and all birthdays have equal probability.
Given a group of five people, *A,B,C,D,E*,
(a) What is the total number of ways in which the birthdays of *A,B,C,D,E* could occur?
(b) What is the total number of ways in which the birthdays of *A,B,C,D,E* could occur so that no two people share the same birthday?
(c) What is the total number of ways in which the birthdays of *A,B,C,D,E* could occur so that at least two people share a birthday?
(d) What is the probability that at least two people in the group *A,B,C,D,E* share a birthday?

[6] (similar to suggested exercise 9.3#32) **The Birthday Problem Part 2: Group of k people**
We are interested in the probability that at least two people in a group of k people share a birthday. We will define $P(k)$ to be the probability that at least two people in a group of k people share a birthday.
Our goal is to find a formula for $P(k)$, make a table of values and a graph for it, and use that table of values to answer some questions. But before doing that, we can observe the following:

- In problem [5](d), you found the value of $P(5)$.
- If $k = 1$, then there is only one person in the group, so there is no way that two people in the group can share a birthday. That is, $P(1) = 0$.
- If $k = 366$ or more, then at least two people must share a birthday, because there are more people then birthdays. Since a shared birthday is certain, we can say that $If\ k \geq 366\ \ then\ P(k) = 1$.
- In general, as k increases in the interval $1 \leq k \leq 366$, the value of $P(k)$ should increase from 0 to 1.

We will now work towards getting a formula for $P(k)$.

(a) What is the total number of ways in which the birthdays of the k people in the group could occur?

(b) What is the total number of ways in which the birthdays of the k people in the group could so that no two people share the same birthday?

(c) What is the total number of ways in which the birthdays of the k people in the group could occur so that at least two people share a birthday?

(d) What is the probability that at least two people in the group share a birthday? That is, what is $P(k)$?

(e) Using Desmos, make a table of values for $P(k)$, for $n = 1,2, \ldots, 50$, along with a graph of that data. (Integer domain, not real numbers: the graph should be dots.) Make your graph large and clear, so that it the curve spans the whole screen and the axes and their labels are clearly visible. Print your graph.

(f) Using your data, how many large does k need to be for $P(k)$ to be at least 0.5? Illustrate on your graph.

[7] (similar to suggested problem 9.5#5) Use Theorem 9.5.1 to compute each of the following.
(Show how the expressions can be simplified. Do not use a calculator!)

(a) $\binom{7}{0}$ (b) $\binom{7}{1}$ (c) $\binom{7}{2}$ (d) $\binom{7}{3}$ (e) $\binom{7}{4}$ (f) $\binom{7}{5}$ (g) $\binom{7}{6}$ (h) $\binom{7}{7}$

[8] (similar to Examples 9.5#4,5,6,7 and sugg ex 9.5#5) A computer programming department has 15 members.

(a) How many ways can a group of six be chosen to work on a project?

(b) Suppose eight department members are women and seven are men.

(i) How many groups of six can be chosen that contain four women and two men?

(ii) How many groups of six can be chosen that contain at least one man?

(iii) How many groups of six can be chosen that contain at most three women?

(c) Suppose two department members refuse to work together on projects. How many groups of six can be chosen to work on a project?

(d) Suppose two department members insist on either working together or not at all on projects. How many groups of six can be chosen to work on a project?

[9] (similar to suggested exercise 9.5# 17) (similar to suggested problem 9.5#17) Eight points labeled *A,B,C,D,E,F,G,H* are arranged in a plane in a way that no three of them lie on the same line.

(a) How many straight lines are determined by the eight points?

(b) How many of those straight lines do not pass through point A?

(c) How many triangles have three of the eight points as vertices?

(d) How many of these triangles do not have A as a vertex?

[10] (similar to suggested problem 9.5#16) Suppose that three computer boards in a production run of fifty are defective. A sample of five is to be selected to be checked for defects.

(a) How many different samples can be chosen?

(b) How many samples will contain at least one defective board?

(c) What is the probability that a randomly chosen sample of five contains at least one defective board?

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