Topic for this video: Given specified limit behavior, sketch graph and discuss continuity behavior

Reading:

- **General:** Section 2.3 Continuity
- **More specifically:** Pages 118 – 121, about a graphical approach to continuity

Homework:

H17: Given specified limit behavior, sketch graph and discuss continuity behavior (2.3#11,13)

Recall some background concepts:

The terminology of *one-sided limits*, introduced in Section 2.1 and discussed in the Video for Homework H02.

The *limit from the left*

- **Symbol:** $\lim_{x \to c^-} f(x) = L$
- **Spoken:** "The limit, as x approaches c from the left, of $f(x)$ is L."
- **Graphical Significance:** The graph of f appears to be heading for location $(x, y) = (c, L)$ from the left.

The *limit from the right*

- **Symbol:** $\lim_{x \to c^+} f(x) = L$
- **Spoken:** "The limit, as x approaches c from the right, of $f(x)$ is L."
- **Graphical Significance:** The graph of f appears to be heading for location $(x, y) = (c, L)$ from the right.

Another background concept:

The definition of *continuity*, introduced in Section 2.3 and discussed in the the previous video, which accompanied Homework H16.

Definition of *Continuity at a particular x value*

Words: *The function f is continuous at $x = c$.*

Meaning: the function f passes these three tests:

Test 1: $\lim_{x \to c} f(x)$ exists

Test 1a: $\lim_{x \to c^-} f(x)$ exists

Test 1b: $\lim_{x \to c^+} f(x)$ exists

Test 1c: The numbers in test 1a and 1b agree.

Test 2: $f(c)$ exists

Test 3: The numbers in *Test 1* and *Test 2* agree.

IMPORTANT EXAMPLE OF A TYPE NOT DISCUSSED IN BOOK:

Given a description of limit behavior of f,

sketch a possible graph of f and discuss the continuity

[Example]

(a) Sketch a graph that satisfies all these conditions:

$$f(1) = 3$$

$$\lim_{x \to 1^-} f(x) = 2$$

$$\lim_{x \to 1^+} f(x) = -4$$

(b) Discuss the continuity of f at $x = 1$.

Solution to (a) This part of the question was actually presented in the video for Homework H03, and represented a kind of problem was on that homework set:

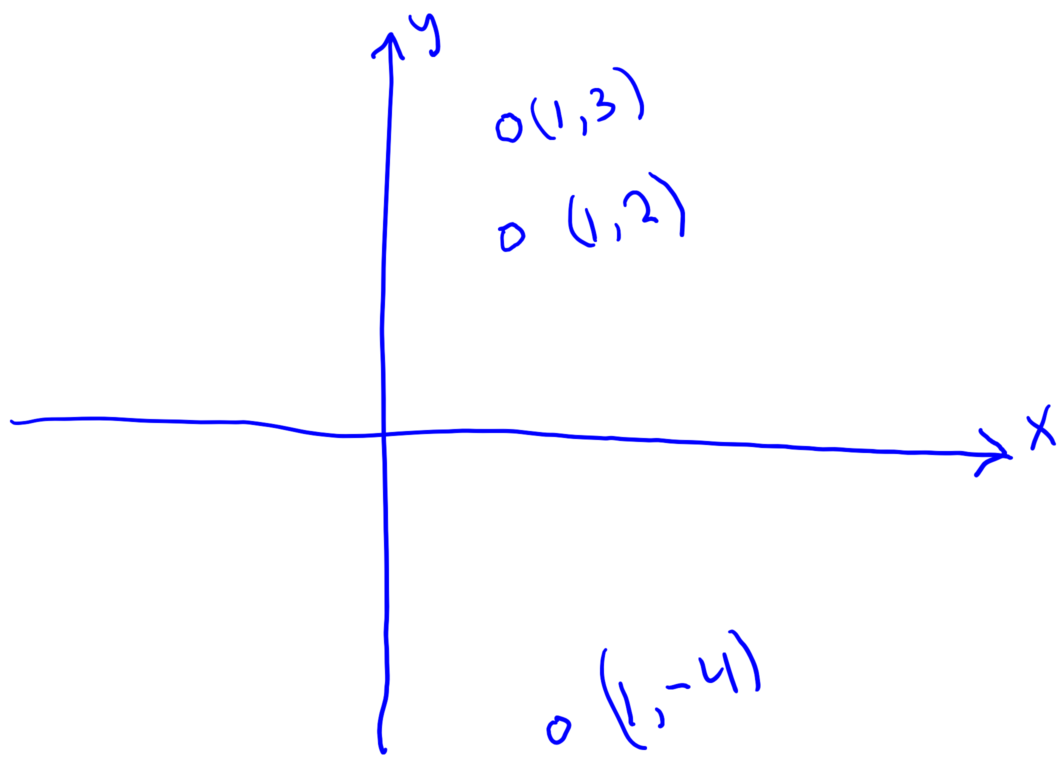
Given a description of limit behavior of f, sketch a possible graph of f.

For simplicity, I will just repeat the solution presented in that video.

Start by noting that in the given information, three (x, y) locations are mentioned.

- The symbol $f(1) = 3$ is about the location $(x, y) =$ (1,3)
- The symbol $\lim_{x\to 1^-} f(x) = 2$ is about the location $(x, y) =$ (1,2)
- The symbol $\lim_{x\to 1^+} f(x) = -4$ is about the location $(x, y) =$ (1,-4)

On one set of axes, plot these three locations with open circles.

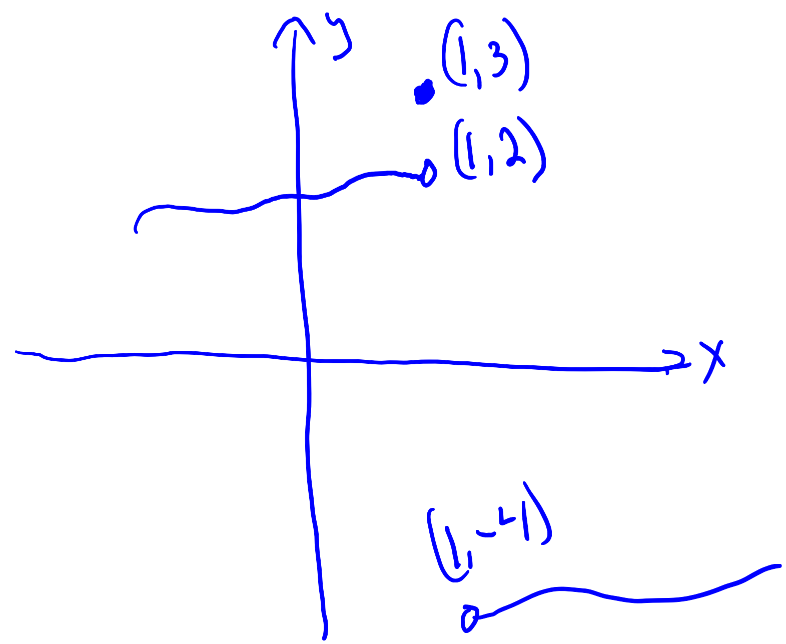


Then add features to the figure that convey what the given information tells us about those locations.

- The symbol $f(1) = 3$ tells us that there is a point on the graph at $(x,y) = (1,3)$

- The symbol $\lim_{x \to 1^-} f(x) = 2$ tells us that the graph is heading for the location $(x,y) = (1,2)$ from the left.

- The symbol $\lim_{x \to 1^+} f(x) = -4$ tells us that the graph is heading for the location $(x,y) = (1,-4)$ from the right.



Solution to (b): This is the only part of this problem that is new. To discuss the continuity of f at $x = 1$, we should see how f does on the *three-part test* presented in the *definition of continuity*.

Test 1: Does $\lim_{x \to 1} f(x)$ exist?

Test 1a: Does $\lim_{x \to 1^-} f(x)$ exist? yes. we were told that $\lim_{x \to 1^-} f(x) = 2$

Test 1b: Does $\lim_{x \to 1^+} f(x)$ exist? yes we were told that $\lim_{x \to 1^+} f(x) = -4$

Test 1c: Do the numbers in test 1a and 1b agree? no $2 \neq -4$

f is not continuous at $x=1$. It fails test 1c.

Test 2: Does $f(c)$ exist? yes. we are told that $f(1)=3$

Test 3: Do the numbers in *Test 1* and *Test 2* agree? the function failed test 1, so there is no way that it can pass test 3.

Conclusion:

f is not continuous at $x=1$.

[End of Example] and End of Video