

BMA

Basic Matrix Algebra (V1.0)

User's Guide &

Lesson Plan

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BMA

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Introduction

Basic Matrix Algebra (BMA) performs basic matrix algebra routines to assist more advanced students.

It is important for advanced students in applied statistics courses to understand matrix algebra at least at a basic level. Unfortunately, it is often difficult to practice these procedures without learning SPSS and SAS matrix programming languages. BMA provides students a visual, user-friendly interface to learn basic matrix algebra. The BMA program performs basic matrix algebra and helps to illustrate the mathematical procedures of multiple regression and multivariate analysis. The primary purpose of the program is to provide a convenient learning tool with a user-friendly interface as well as to promote the effective teaching of multiple regression and multivariate analyses.

Matrix operations can be used to obtain important quantities in statistical work, such as the matrix of variances and covariances for a set of variables, the determinant of a matrix involved in several multivariate test statistics, and eigenvalues which are fundamental to many multivariate analyses. Using matrix algebra, for example, the statistics can be acquired for multiple regression. The BMA program includes simple, calculator-like buttons for matrix addition, matrix subtraction, matrix multiplication, and division by a scalar. Buttons are also included for transposing a matrix, calculating a determinant, matrix inversion, and finding eigenvalues. Other special features include quick buttons for calculations necessary for regression using a matrix algebra approach. A Review of Basic Matrix Algebra Using BMA^M Program

1. Outlook of BMA

BMA program provides two windows: **Operation Window** and **Results Window**.

In the **Operation Window**, all the calculating buttons are in grey if cell values are not registered in Matrices. Once calculating buttons are activated, or in bold, click on any one and a Results Window will pop up.

The **Results Window** is designed to present calculating results and it will stay until the whole program ends. If you choose to close it in a complex calculation procedure, the data are still stored in the program unless you shut down the Operation Window. For each calculation, the Results Window presents its results only once and you need to either save them temporarily or copy them back to the Operation Window and save them in *.txt files. The details will be illustrated later.

😽 BMA: Basic Matrix Alg	yebra						
File Function Manage Help	ē.						
# Rows	# Columns				# Rows	# Columns	
Matrix A: I	by	Set Dimensions		Matrix B:	l t	by .	Set Dimensions
Register C	ell Values in Mattix A						
			A+B				
			A-B				
			A*B				
			A/B				
			Clear A				
			Clear B				
Transpose Matrix A		n of 1's				Create M	latric of A Means
Determinant of Malrix A		_		Dete		B	
Invert Matin A		s of A				Find Ei	genvalues of B

👯 Result	s Window				
Res	ults Matr	# Rov	Transpose R	esuits Matrix	
			-/ .	Invert Res	ults Matrix
Туре о	f Result	S:		Results Matrix	Eigenvalues
Matrix A	+ Matrix B				
4.0000	8.0000	9.0000	3.0000	View Eige	nvectors
4.0000	1.0000	9.0000	10.0000	Create Diagon	al 1/SD Matrix
0.0000	7.0000	6.0000	11.0000	Coloridate Distort	Win First Caluma
				Save as Matrix 1 Save as Matrix 2 Save as Matrix 3	Load Matrix 1 Load Matrix 2 Load Matrix 3
					Lopy to Matrix B

Figure 1: BMA Operation Window (Left) and Results Window (Right)



2. Matrix Addition, Subtraction, Multiplication and Division by a Scalar

The four functions are shown between two Matrix Boxes in the Operation Window. For each function, the major four steps should be conducted:

Step1: Set numbers of rows and columns and confirm them by clicking on the button

Step2: Input desired data in Matrix	boxes;	
Step3: Click on the button	Register Cell Values in Matrix A 🛛 🗟 🖉	or
Register Cell Values in Matrix B	to confirm your entries;	
Step4: Choose a desired function	outton to calculate results:	

Example: As the operation procedures for the four functions are similar, we only illustrate the **Division Operation** herein:



1) Operations in the Operation Window

Figure 2: Operations in the Operation Window

2) Operations in the Results Window

			1
	10	1.1	
A	/ H	1	

Once you click on **Line Results Window** pops up and the calculation results will be presented. Based on individual needs, the results could be saved temporarily for further calculations or permanently as *.txt file for use next time.

Temporary Save Mode: The maximum number of matrices saved in this mode is THREE and they could be reloaded in the further calculation.

Permanent Save Mode: Copy results into either Matrix A or B box in the Operation Window and save them by choosing ave Matrix A or ave matrix B in the File Menu.

In this **Division** example, the results saved in the two modes are illustrated in the following:

📉 Result	s Window			
Res	ults Matr	# Rov	vs # Columns by 4	Transpose Results Matrix
			100. -	Invert Results Matrix
Туре о	f Result	S :		Results Matrix Eigenvalues
Matrix A	/ Scalar B			
1.0000	1.6667	2.3333	0.6667	View Eigenvectors
0.6667	0.0000	1.3333	1.0000	Create Diagonal 1/SD Matrix
0.0000	0.3333	0.6667	1.0000	Calculate B2 for DV in First Column
Resu by a	lts fron Scalar	n Divisi	on ^T	Save as Matrix 1
			Save Results Temporarily	Save as Matrix 2 Load Matrix 2
	Lo: Fui	ad save ther Ca	Copy to Matrix A Copy to Matrix B	
(Copy Ro Save or	esults f Furthe		



🍇 BMA: Basic Mat	trix Algebr	a							
File Function Mana	ige Help								
New	Ctrl+N	# Columns				# Rov	NS .	# Columns	
Open into Matrix A	Ctrl+O	14	Set Dimensions		Matri	× B: 3	by	4	Set Dimensions
Cours Mobile A	chi c	ilues in Matrix A				Reg	ister Cell Value	es in Matrix B	
Save Matrix B	Ctri+5	2			1.0000	1.6667	2.3333	0.6667	
Exit 🥄	Ctrl+F4	3		A+B	0.6667	0.0000	1.3333	1.0000	
0 1	2	3			0.0000	0.3333	0.6667	1.0000	
-		L		A-B	-				-
				A*B					
1	Perma	nent Save	e						
				A/B					
				Clear A					
				Clear B					
]				
Transpose Matrix	xA	Insert Initial Colum	in of 1's			Transpose	Matrix B	Create M	latrix of A Means
Determinant of Ma	trix A					Determinant	of Matrix B		
Invert Matrix A		Find Eigenvalue	s of A			Invert M	atrix B	Find Ei	genvalues of B

Figure 3: Temporary and permanent save modes

Note: Other function buttons designed specifically for Statistical courses in the Results Window will be introduced in the later Multiple Regression lesson plan.



3. The Transpose, Inverse, Determinant and Eigenvalues

In the **Operation Window**, the function buttons for **Transpose**, **Inverse**, **Determinant and Eigenvalues** are provided for Matrix A and B separately. Once you choose a desired function, the results for the matrix will be displayed in the Results Window as illustrated above.

If there is any inappropriate operation, a dialog box will pop up and give you a specific instruction shown as follows:

🐜 BMA: Basic Matrix Algebra	
File Function Manage Help	
# Rows # Columns	# Rows # Columns
Matrix A: 3 by 2 Set Dimensi	ions Matrix B: 2 by 3 Set Dimensions
Register Cell Values in Matrix A	Register Cell Values in Matrix B
1 0	6 2 3
2 1 Warning	A+B 2 3 4
-3 2 Warning	
-	
Barran Barran	
You can only find the determinant of	a square matrix (number of rows must equal number of columns).
	RO
Separate Functions for	
Matrix A and B: Transpose,	
Figenvalues	ClearB
Transpose Matrix A Insert Initial Column of 1's	Transpose Matrix B Create Matrix of A Means
Determinant of Matrix A	Determinant of Matrix B
Invert Matrix A Find Eigenvalues of A	Upen Results Invert Matrix B Find Eigenvalues of B

Figure 4: Dialog box popping up for inappropriate operation

4. Special Functions

1. To move, copy, swap matrices in Matrix A and B boxes, go directly to Manage Menu in the Operation Window and find the function you want.



Figure 5: Manage menu for matrix swap, copy and move

2. Under the **Function Menu**, **special reset**, **set and save functions** are provided, such as *Reset Matrix* A(B) *to empty*, *Set Matrix* A(B) *as Identity Matrix*, and *Save Matrix* A(B) *in Memory as Data Matrix* X(Y), etc.





Figure 6: Function menu for special reset, set and save functions

II. BMA Lesson Plan: Multiple Regression Case Study

In this section, students are required to learn about the calculation of important elements in Multiple Regression Analysis using BMA^{IIII}, such as Matrix of Raw (Standardized) Regression Coefficients, Sum of Squares about Regression and Residuals, Mean Squares of Regression and Residuals, F-statistic, Matrix of Standard Deviation, Variance-Covariance Matrix, and Zero-order correlation Matrix, etc.

Case Study: Use the data provided to answer the multiple regression questions below:

Person	Y	X1	X2	X3
1	63	11	69	32
2	71	17	75	50
3	85	16	88	33
4	60	25	60	25
5	65	15	65	30
6	63	08	87	29
7	95	20	71	49
8	50	06	65	21
9	75	12	80	24
10	91	16	77	42

Question01 Observed Scores

What is the observed dependent variable score (Y) for Person_4?

(Answer: 60)

Question02 Raw Regression Coefficients

What is the raw regression coefficient for predictor variable X2? (Hint: You need to run BMA to answer this question; $\beta = (x'x)^{-1}(x'y)$; create column vector of 1's and combine with x matrix to calculate intercept)



Answer:

Step1: Input X1, X2 and X3 into Matrix A box;

Step2: Click on the button of "Insert Initial Column of 1's" to calculate intercept;

🐘 BMA: Basic Matrix Algebra										
File Funct	ion Manage	Help								
	# Row	IS	# Columns				# Rows		# Columns	
Matrix	A: 10	by	1	Set Dimensions		Matrix	K B: 10	by 1	1	Set Dimensions
Register Cell Values in Matrix A					Register (Cell Value:	s in Matrix B			
1.0000	11.0000	69.0000	32.0000			63.0000				
1.0000	17.0000	75.0000	50.0000	-	A+B	71.0000				
1.0000	16.0000	88.0000	33.0000			85.0000				
1.0000	25.0000	60.0000	25.0000		A-B	60.0000				
1.0000	15.0000	65.0000	30.0000			65.0000				
1.0000	8.0000	87.0000	29.0000		∆*B	63.0000				
1.0000	20.0000	71.0000	49.0000			95.0000				
1.0000	6.0000	65.0000	21.0000		A/D	50.0000				
1.0000	12.0000	80.0000	24.0000		AVB	75.0000				
1.0000	16.0000	77.0000	42.0000		Clear A	91.0000				
		Inse	rt Colu	mn	Clear B					
		of 1'	S							
Trans	spose Matrix A	Inse	rt Initial Colum	n of 1's			Transpose Matr	ix B	Create M	latrix of A Means
Determ	inant of Matrix	A					Determinant of Ma	atrix B		
Inv	vert Matrix A	Fir	nd Eigenvalue	s of A	Upen Results		Invert Matrix I	в	Find Ei	genvalues of B

Step3: Calculate $(x'x)^{-1}$ using Transpose, Swap, Inverse, Temporary Save Functions provided in either Operation Window or Results Window;

👯 BMA: Basic Matrix Algebra						
File Function Manage Help						
Open Results Window	Ctrl+W		# Rov	vs	# Columns	
Matrix A: Swap Matrix A and Matrix B		Matri	x B: 10	by	4	Set Dimensions
Copy Matrix A to Matrix B (overwrite B) Copy Matrix B to Matrix A (overwrite A)			Regi	ster Cell Value	es in Matrix B	
1.0000 1 Mayo Matrix A to Matrix P (oursurite P, close	- A)	1 1.0000	11.0000	69.0000	32.0000	
11.0000 1 Move Matrix B to Matrix A (overwrite B, deal	(A) (FB) B	1.0000	17.0000	75.0000	50.0000	
69.0000 7 Copy Matrix A to Results Window	Ctrl+C	1.0000	16.0000	88.0000	33.0000	
32.0000 50.0000 33.0000 25.0000 30.0000	25 A-B	1.0000	25.0000	60.0000	25.0000	_
		1.0000	15.0000	65.0000	30.0000	-
	A*B	1.0000	8.0000	87.0000	29.0000	
		1.0000	20.0000	71.0000	49.0000	_
Coloulate VIV	A/D	1.0000	6.0000	65.0000	21.0000	_
Calculate X X	AVD	1.0000	12.0000	80.0000	24.0000	
	Clear A	1.0000	16.0000	77.0000	42.0000	
	Clear B					
	>					
Transpose Matrix A Insert Initial Column of 1's			Transpose	Matrix B	Create M	atrix of A Means
Determinant of Matrix A			Determinant	of Matrix B		
Invert Matrix A Find Eigenvalues of A	Upen Hesult	\$	Invert M	ətrix B	Find Eig	genvalues of B

Step 4: Calculate (x'y) in the similar manner as illustrated in Step3;

Step 5: Calulate $(x'x)^{-1}(x'y)$.

👯 Results Window				
Results Matrix: 4 by 1	Transpose Results Matrix Invert Results Matrix			
Type of Results:	Results Matrix Eigenvalues			
-19.0693 0.9973 0.7429	View Eigenvectors Create Diagonal 1/SD Matrix Calculate R2 for DV in First Column			
Raw Regression Coefficient for Predictor X2	Save as Matrix 1 Save as Matrix 2 Save as Matrix 3 Load Matrix 2 Load Matrix 3 Copy to Matrix A Copy to Matrix A			
	<u>I</u> <u>C</u> lose			

Question03 Regression Model

Write the regression model using raw regression coefficients (include intercept).(Hint: Refer to the results generated for Question 02)

Answer: Y = -19.069 + .997X1 + .743X2 + .644X3

Question04 Predicted Values

What is the predicted Y value for Person_4? (Hint: Yhat = Xb or Yhat = $x^*(x'x)^{-1}(x'y)$; refer back to the results produced for Question 02 and continue your calculation using BMA)

🛚 BMA: Basic Matrix Algebra 📰 🗖 🔀								
File Funct	ion Manage	Help						
	# Row	s	# Columns				# Rows	# Columns
Matrix	A: 10	by 4	1	Set Dimensions		Matrix	B: 4 by	1 Set Dimensions
	Regi	ster Cell Value	s in Matrix A				Register Cell Valu	es in Matrix B
1.0000	11.0000	69.0000	32.0000			-19.0693		
1.0000	17.0000	75.0000	50.0000		A+B	0.9973		
1.0000	16.0000	88.0000	33.0000			0.7429	Paw P	Pegression
1.0000	25.0000	60.0000	25.0000	1	A-B	0.6435	Coeff	icient
1.0000	15.0000	65.0000	30.0000					
1.0000	8.0000	87.0000	29.0000	1	A \$B			
1.0000	20.0000	71.0000	49.0000		0-			
1.0000	6.0000	65.0000	21.0000	1	A/B			
1.0000	12.0000	80.0000	24.0000		RUD			
1.0000	16.0000	77.0000	42.0000		Clear A			
		×	M	atrix X	Clear B			
Transpose Matrix A Insert Initial Column of 1's							Transpose Matrix B	Create Matrix of A Means
Determi	inant of Matrix	A					Determinant of Matrix B	
Inv	ert Matrix A	Fin	id Eigenvalue:	of A			Invert Matrix B	Find Eigenvalues of B

🍇 Results Window	🔳 🗖 🔀
#Rows #Columns Results Matrix: 10 by 1	Transpose Results Matrix
	Invert Results Matrix
Type of Results:	Results Matrix Eigenvalues
Matrix A * Vector B	
63.7529	View Eigenvectors
85.7778	Create Diagonal 1/SD Matrix
83.4976	Calculate B2 for DV in First Column
66.5244	
63.4835	Save as Matrix 1 Load Matrix 1
72.2021 Predicted Y	Cause as Matrix 2
85.1547 Value for Person 4	
48.7158	Save as Matrix 3 Load Matrix 3
67.7734	
81.1179	Copy to Matrix A Copy to Matrix B
	<u>I</u> Close

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Question05 Squared Residuals

What is the squared residual (Y-Yhat) for Person_4?

(Hint: Be careful!!! This is to calculate squared errors of prediction. You first have to calculate Y-Yhat using BMA. Use the results for Yhat in Question04)







Question06 Degrees of Freedom for Residual

How many degrees of freedom are there for the residual? (Hint: $df_{residual} = n - k - 1$)

Answer: 10 - 3 - 1 = 6

Question07 Degrees of Freedom for Regression

How many degrees of freedom are there for the regression? (Hint: $df_{reg} = k$)

Answer: k = 3.

Question08 Mean Squares for Regression

What is the Mean Squares for the regression? (Hint: $MS_{reg} = SS_{reg}/number$ of predictors = [(Yhat - Ybar)' (Yhat - Ybar)]/number of predictors; Ybar = (Sum of Y)/ number of cases)

🍇 BMA: Basic Matrix Al	gebra			
File Function Manage He # Rows Matrix A: 10	p # Columns by 1 Set Dimensio	ns Matri	×B: 10 by	# Columns 1 Set Dimensions
Register C	Cell Values in Matrix A		Register Cell Value	es in Matrix B
63.7529 85.7778 83.4976 66.5244 63.4835 72.2021 85.1547 48.7158 67.7734 81.1179	Yhat	A+B 71.8 71.8 71.8 71.8 71.8 71.8 71.8 71.8		Ybar
		Clear B	C	alculate Ybar
Transpose Matrix A	Insert Initial Column of 1's		Transpose Matrix B	Create Matrix of A Means
Determinant of Matrix A			Determinant of Matrix B	
Invert Matrix A	Find Eigenvalues of A		Invert Matrix B	Find Figenvalues of B



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🙀 Results Window				
Results Matrix: 1 by 1	Transpose F	Transpose Results Matrix		
	Invert Re:	sults Matrix		
Type of Results:	Results Matri	x Eigenvalues		
436.1305 Mean Square of Regression	View Eigenvectors Create Diagonal 1/SD Matrix Calculate R2 for DV in First Column			
	Save as Matrix 2 Save as Matrix 2 Save as Matrix 3	Load Matrix 2 Load Matrix 3		
	Copy to Matrix A	Copy to Matrix B		
	ı.	lose		

Question09 Means

What is the mean of predictor variable X2? (Hint: Use the function button under Matrix B box)

A	nsw	er:								
8	BMA: B	asic Matri	c Algebra							
F	ile Functio	on Manage	Help							
		# Row	s #	Columns			# Rov	is	# Colur	nns
1	Matrix	A: 10	by 3	Set Dimensions		Matr	ix B: 10	by	3	Set Dimensions
		Regis	ster Cell Values	in Matrix A			Regi	ster Cell Value	s in Mat	rix B
ľ	11.0000	69.0000	32.0000			14.600) 73.7000	33.5000		
	17.0000	75.0000	50.0000		A+B	14.6000) <mark>73.7000</mark>	33.5000		
	16.0000	88.0000	33.0000			14.6000) <mark>73.7000</mark>	33.5000		
	25.0000	60.0000	25.0000		A-B	14.6000) <mark>73.7000</mark>	33.5000		Means
	15.0000	65.0000	30.0000	Matrix X		14.6000) <mark>73.7000</mark>	33.5000	-	of
	3.0000	87.0000	29.0000		A*B	14.6000) <mark>73.7000</mark>	33.5000		Matrix X
	20.0000	71.0000	49.0000			14.6000) <mark>73.7000</mark>	33.5000		
	5.0000	65.0000	21.0000		A/B	14.6000) <mark>73.7000</mark>	33.5000		
	12.0000	80.0000	24.0000		AVD	14.6000) <mark>73.7000</mark>	33.5000		
	16.0000	77.0000	42.0000		Clear A	14.6000) <mark>73.7000</mark>	33.5000		
					Clear B	Cli	ck on thi	s Butto	n	
						to	get mear	IS		
Ĩ	Transp	ose Matrix A	Insert	Initial Column of 1's			Transpose	Matrix B	Crea	ate Matrix of A Means
	Determin	ant of Matrix	A			1	Determinant o	of Matrix B		
	Inve	ert Matrix A	Find	Eigenvalues of A	Upen Results		Invert Ma	ətrix B	Fir	nd Eigenvalues of B

Question10 Mean Square of Residuals

What is the Mean Square of residuals?

(Hint: $MS_{resid} = SS_{resid}$ / (number of cases – number of predictors - 1) = [(Y - Yhat)'(Y - Yhat)] / df_{resid}).

Answer: Step 1: Calculate SS_{residual}

👯 BMA: Basic Matrix Algebra				
File Function Manage Help				
Matrix A: 1 by 10 Set Dimensions Register Cell Values in Matrix A		Matrix	H Rows B: 10 by Register Cell Value	# Columns 1 Set Dimensions es in Matrix B
-0.7529 -14.7778 1.5024 -6.5244 1.5185 -4	A+B	-0.7529 -14.7778 1.5024		
(Y- Yhat)'	А-В А*В	-6.5244 1.5165 -9.2021	• (Y- Y	hat)
	A/B	9.8453 1.2842 7.2266	_	
	Clear A Clear B	9.8821		
Transpose Matrix A Insert Initial Column of 1's		1	Transpose Matrix B	Create Matrix of A Means
Determinant of Matrix A			Determinant of Matrix B	
Invert Matrix A Find Eigenvalues of A	Open Results		Invert Matrix B	Find Eigenvalues of B

Nesults Window			
Results Matrix: 1 by 1	Transpose Results Matrix		
	Invert Results Matrix		
Type of Results:	Results Matrix Eigenvalues		
Vector A* Vector B			
599.2124	View Eigenvectors		
Sum of Squares	Create Diagonal 1/SD Matrix		
about residuals	Calculate R2 for DV in First Column		
	Save as Matrix 1Load Matrix 1Save as Matrix 2Load Matrix 2Save as Matrix 3Load Matrix 3		
	Copy to Matrix A Copy to Matrix B		
	<u>I</u> Close		

```
🍇 Results Window
                           # Rows
                                            # Columns
                                                                     Transpose Results Matrix
   Results Matrix: 1
                                     by 1
                                                                       Invert Results Matrix
Type of Results:
                                                                    Results Matrix Eigenvalues
Scalar A / Scalar B
 99.8687
                                                                       View Eigenvectors
                                                                   Create Diagonal 1/SD Matrix
             Mean Square of
                                                                 Calculate R2 for DV in First Column
             Residuals
                                                              Save as Matrix 1
                                                                                   Load Matrix 1
                                                              Save as Matrix 2
                                                                                   Load Matrix 2
                                                              Save as Matrix 3
                                                                                   Load Matrix 3
                                                                Copy to Matrix A
                                                                                Copy to Matrix B
                                                                         <u>I</u>Close
```

Step 2: Calculate $MS_{residual} = SS_{residual}/(10-3-1)$

Question11 R-squared

Calculate R-squared (be sure to show your work). Hint: $R^2 = SS_{reg} / SS_{total} = [(Yhat - Ybar)' (Yhat - Ybar)] / (SS_{residual} + SS_{reg})$; Refer to Question 8 and 10 to get SS_{reg} and $SS_{residual}$)

👯 Results Window			
Results Matrix: 1 by 1	Transpose Results Matrix		
	Invert Re:	sults Matrix	
Type of Results:	Results Matri	x Eigenvalues	
0.6859	View Eige	nVALUES	
	Create Diagor	al 1/SD Matrix	
R-Squared	Calculate R2 for DV in First Column		
	Save as Matrix 1	Load Matrix 1	
	Save as Matrix 2	Load Matrix 2	
	Save as Matrix 3	Load Matrix 3	
	Copy to Matrix A	Copy to Matrix B	
	<u>i</u> c	lose	

Question11 F-statistic

What is the F-statistic?

(Hint: F-statistic= $MS_{reg}/MS_{residual}$; refer to results for Question 8 and 10)

😽 Results Window			
Results Matrix: 1 by 1	Transpose Results Matrix		
	Invert Results Matrix		
Type of Results:	Results Matrix Eigenvalues		
A.3670 F-statistic	View EigerVALUES Create Diagonal 1/SD Matrix Calculate R2 for DV in First Column Save as Matrix 1 Save as Matrix 2 Save as Matrix 3 Copy to Matrix A Copy to Matrix A		



Question13 Leverage Value

What is the leverage value for Person_4?

(Hint: Leverage value = $x(x'x)^{-1}x'$; be sure to insert initial column 1 in Matrix A box when calculating leverage values.)

👯 Results	s Window			26: Rida	2.5373				
Besu	ults Matr	# Row	s	# Columns 10		Transpose F	Results Matrix		
			27			Invert Re	Invert Results Matrix		
Туре о	f Results	5:				Results Matr	ix Eigenvalues		
Matrix A*	* Matrix B								
0.2214	0.1157	-0.0860	-0.0226	0.1670	0.	View Eige	enVALUES		
0.1157	0.4052	0.0295	-0.1078	0.0567	0.	Create Diagor	nal 1/SD Matrix		
-0.0860	0.0295	0.4820	0.1078	-0.0649	0.	Calculate B2 for	DV in First Column		
-0.0226	-0.1078	0.1078	0.8341	0.2291	-0		Dy In hist Column		
0.1670	0.0567	-0.0649	0.2291	0.2008	-0	Save as Matrix 1	Load Matrix 1		
0.0794	0.0186	0.3016	-0.1978	-0.0214	0.				
0.0880	0.3744	0.0165	0.0763	0.0927	-0	Save as Matrix 2	Load Matrix 2		
0.3187	-0.0492	-0.1972	-0.0335	0.2426	0.	Save as Matrix 3	Load Matrix 3		
0.0421	-0.0899	0.2799	0.1263	0.0498	0.				
0.0764	0.2468	0.1308	-0.0119	0.0476	0.	Copy to Matrix A	Copy to Matrix B		
<	Lever Perso	age Va on 4	lue for		>	<u>i</u> .	Zlose		



Question14 Deviation Score

What is the deviation score (X-Xbar) for Person_4 on dependent variable X1?

👯 BMA: B	lasic Matrix	c Algebra						
File Functi	ion Manage	Help						
	# Row	s #	Columns			# Row	IS	# Columns
Matrix	A: 10	by 3	Set Dimen	sions	Matrix	CB: 10	by	3 Set Dimensions
	Regis	ster Cell Values i	n Matrix A			Regi	ster Cell Value	es in Matrix B
11.0000	69.0000	32.0000			14.6000	73.7000	33.5000	
17.0000	75.0000	50.0000		A+B	14.6000	73.7000	33.5000	
16.0000	88.0000	33.0000			14.6000	73.7000	33.5000	
25.0000	60.0000	25.0000	(2014))	A-B	14.6000	73.7000	33.5000	
15.0000	65.0000	30.0000	▲ X		14.6000	73.7000	33.5000	♦ Xhar
8.0000	87.0000	29.0000		A*B	14.6000	73.7000	33.5000	Abda
20.0000	71.0000	49.0000			14.6000	73.7000	33.5000	
6.0000	65.0000	21.0000		A/D	14.6000	73.7000	33.5000	
12.0000	80.0000	24.0000		AVD	14.6000	73.7000	33.5000	
16.0000	77.0000	42.0000		Clear A	14.6000	73.7000	33.5000	Calculate
		i i i i i i i i i i i i i i i i i i i		Clear P			K	Means of
								Matrix X
		L. Lorent	LN101			T	4.1.5. P	Contraction of the
Trans	pose Matrix A	Insert	Initial Column of 1's		_	I ranspose	Matrix B	Lreate Matrix of A Means
Determi	nant of Matrix	A				Determinant o	of Matrix B	
Inv	ert Matrix A	Find	Eigenvalues of A	Upen Hesults		Invert Ma	atrix B	Find Eigenvalues of B

🖌 Results	: Window				
Resi	ilts Matri	# Rows	# Columns	Transpose F	Results Matrix
11000		A .] ¹⁰	Invert Re:	sults Matrix	
Гуре о	f Results	11		Results Matri	x Eigenvalues
Matrix A*	* Matrix B				l
-3.6000	-4.7000	-1.5000		View Eige	INVALUES
2.4000	1.3000	16.5000	-	Create Diagor	al 1/SD Matrix
1.4000	14.3000	-0.5000	-	Coloulate P2 for l	D) (in First Column
10.4000	-13.7000	-8.5000			
0.4000	-8.7089	-3.5000	-	Save as Matrix 1	Load Matrix 1
-6.6000	13.3000	-4.5080	-		
5.4000	-2.7000	15.5000	Deviation	Save as Matrix 2	Load Matrix 2
-8.6000	-8.7000	-12.5000	Score for	Save as Matrix 3	Load Matrix 3
-2.6000	6.3000	-9.5000	Person 4 on		1
1.4000	3.3000	8.5000	Variable X1	Copy to Matrix A	Copy to Matrix B
				<u>n</u> c	lose



Question15 Z-scores

What is the z-score for Person_4 on dependent variable Y? Hint: $Z = (Y - Ybar)/SD_v$; Get variance of variable Y first (Y - Ybar)' (Y-Ybar)/(N-1).

Answer:

🖏 BMA: Basic Matrix Algebra		26	×
File Function Manage Help			
# Rows # Columns Matrix A: by 10 Set Dimensions	Mati	trix B: 10 by 1 Set Dimensions	s
Register Lell Values in Matrix A		Register Cell Values in Matrix B	
-8.8000 -0.8000 13.2000 -11.8000 -6.8000 -8 (Y - Ybar)'	A+B A-B A-B A+B A+B A+B A+B 23.000 23.000 A/B 21.800 3.000 Clear A 19.200 Clear A	Deviation Score for Person 4 Deviation Score for Person 4 (Y- Ybar)	
2			
Transpose Matrix A Insert Initial Column of 1's		Transpose Matrix B Create Matrix of A Means	
Determinant of Matrix A		Determinant of Matrix B	
Invert Matrix A Find Eigenvalues of A	Open Results	Invert Matrix B Find Eigenvalues of B	

👯 Results Window	
Besults Matrix: 1 by 1	Transpose Results Matrix
	Invert Results Matrix
Type of Results:	Results Matrix Eigenvalues
Scalar A / Scalar B	
211.9556	View EigenVALUES
	Create Diagonal 1/SD Matrix
Variance for	Calculate R2 for D1/ in First Column
Variable Y	
	Save as Matrix 1 Load Matrix 1
	Save as Matrix 2 Load Matrix 2
	Save as Matrix 3
	Copy to Matrix A Copy to Matrix B
	<u>i</u> <u>C</u> lose

SD = Square Root of Variance = 14.587. Z = (-11.8)/14.587 = -0.8105.

```
вма 25
```

Question 16 Variances

What are the variances for Y, X1, X2 and X3 respectively? Hint: Create variance-covariance matrix for Y, X1, X2 and X3.

Answer:

Step1: Create a Matrix (A) for Y, X1, X2 and X3;

Step2: Calculate means for the four variables;

🍇 BMA: E	lasic Matrix	c Algebra										
File Functi	on Manage	Help										
	# Row	s	# Columns				# Rows		# Columns			
Matrix	A: 10	by 4		Set Dimensions		Matrix	B: 10	by 4		Set Dimensions		
Register Cell Values in Matrix A							Regist	er Cell Values	in Matrix B			
63.0000	11.0000	69.0000	32.0000			71.8000	14.6000	73.7000	33.5000			
71.0000	17.0000	75.0000	50.0000		A+B	71.8000	14.6000	73.7000	33.5000			
85.0000	16.0000	88.0000	33.0000	Calculate		71.8000	14.6000	73.7000	33.5000			
60.0000	25.0000	60.0000	25.0000	(A-Abar)	A-B	71.8000	14.6000	73.7000	33.5000			
65.0000	15.0000	65.0000	30.0000	1 3	1 7			71.8000	14.6000	73.7000	33.5000	
63.0000	8.0000	87.0000	29.0000		A*B	71.8000	14.6000	73.7000	33.5000			
95.0000	20.0000	71.0000	49.0000	-		71.8000	14.6000	73.7000	33.5000			
50.0000	6.0000	65.0000	21.0000	← A	← A	A/D	71.8000	14.6000	73.7000	33.5000	- Abar	
75.0000	12.0000	80.0000	24.0000		AVD	71.8000	14.6000	73.7000	33.5000			
91.0000	16.0000	77.0000	42.0000			Clear A	71.8000	14.6000	73.7000	33.5000		
Y	X1	X2	ХЗ	-	Clear B	Ybar	X1bar	X2bar	X3bar			
							C	alculat	te Mea	ns		
Trans	pose Matrix A	Inser	t Initial Colum	in of 1's			Transpose M	atrix B	Create Ma	trix of A Means		
Determi	nant of Matrix	A					Determinant of	Matrix B				
Inv	ert Matrix A	Fin	d Eigenvalue	s of A	Open Results		Invert Mat	rix B	Find Eig	envalues of B		

Step3: Calculate (A - Abar)'(A - Abar);

BMA: B	Basic Matri:	x Algebra Help									
Matrix	A: 4	by [# Columns 1 0	Set Dimens	ions		Matrix	# Rov 8: 10	by F	# Columns 4	Set Dimensions
	Regi	ster Cell Value	s in Matrix A					Regi	ster Cell Value	s in Matrix B	
-8.8000	-0.8000	13.2000	-11.8000	-6.8000	-8		-8.8000	-3.6000	-4.7000	-1.5000	
-3.6000	2.4000	1.4000	10.4000	0.4000	-6	A+B	-0.8000	2.4000	1.3000	16.5000	
-4.7000	1.3000	14.3000	-13.7000	-8.7000	1:		13.2000	1.4000	14.3000	-0.5000	
-1.5000	16.5000	-0.5000	-8.5000	-3.5000	-4	A-B	-11.8000	10.4000	-13,7000	-8.5000	
							-6.8000	0.4000	-8.7000	-3.5000	
						∆ *B	-8.8000	-6.6000	13.3000	-4.5000	
	(A -	Abar)'					23.2000	5.4000	-2.7000	15.5000	
						A/D	·21.8000	-8.6000	-8.7000	-12.5000	
						AD	3.2000	-2.6000	6.3000	-9.5000	
						Clear A	19.2000	1.4000	3.3000	8.5000	
						Class	-				
2								Ų	A - Abar)	
Trans	spose Matrix A	Inse	rt Initial Colum	in of 1's			-	Transpose	Matrix B	Create M	atrix of A Means
Determ	inant of Matrix						-	Determinant	of Matrix B		
Determ	Inductor Matrix					Open Results	-	Determinant			
Inv	ert Matrix A	Fir	nd Eigenvalue	sofA				Invert Ma	atrix B	Find Eig	envalues of B

Ж ВМА: В	asic Matrix	Algebra			-12. 					
File Functio	on Manage	Help								کا (۲۰) کا
Matrix	# Rows A: 4	by 4	# Columns	Set Dimensions	ļ.	Matrix	# Rows B : 1	by 🛙	# Columns	Set Dimensions
	Regis	ter Cell Values	: in Matrix A	-10			Register	Cell Value:	s in Matrix B	
1907.6000 312.2000	312.2000 284.4000	543.4000 -145.2000	922.0000 212.0000		A+B	9				
543.4000	-145.2000	802.1000	143.5000		-					
922.0000	212.0000	143.5000	938.5000		A-B					
(A - Abar)'(A - Abar)				A*B		N - 1 =	10 - 1	= 9		
					A/B	÷				
					Clear A Clear B					
Transp	oose Matrix A	Inser	t Initial Colum	n of 1's			Transpose Mal	trix B	Create Ma	atrix of A Means
Determin	nant of Matrix A	۹ (T					Determinant of M	fatrix B		
Inve	ert Matrix A	Fin	d Eigenvalue	s of A	Open Results		Invert Matrix	В	Find Eig	envalues of B

Step4: Use (A - Abar)'(A - Abar)/ (N-1) to calculate variance;



Question 17 Zero-order Correlation

What is the zero-order correlation between variable X2 and variable Y? What is the correlation between predictor variable X1 and predictor variable X3? (Hint: zero-order correlation = 1/SD * Variance * 1/SD; refer back to the results for Question 16)

Answer:

Step1: Once you obtain the variance matrix, click on calculate 1/SD (you can use the Inverse function to get SD);

👯 Results	Window						
Resu	ilts Matri	# Rows X: 4	Transpose Results Matrix				
1		-	/ 1		Invert Results Ma	atrix	
Type of	Results	E.			Results Matrix Eiger	ivalues	
Matrix A /	Scalar B						
211.9556	34.6889	60.3778	View EigenVALL	JES			
34.6889 🕇	31.6000	-16.1333	23.5556	Calculate	Create Diagonal 1/SI	D Matrix	
60.3778	-16.1333 🛉	89.1222	15.9444	HSD	Calculate B2 for DV in E	iret Column	
102.4444	23.5556	15.9444 🔶	104.2778				
Variai Y	nce Variar X1	nce Variar X2	Save as Matrix 1 Li Save as Matrix 2 Li Save as Matrix 3 Li Copy to Matrix A Copy	oad Matrix 1 oad Matrix 2 oad Matrix 3			
			<u>jî</u> <u>C</u> lose	•			

👯 Result	s Window					
Res	ults Matr	# Rov ix: 4	Transpose Results Matrix			
12.01					Invert Resu	ults Matrix
Туре о	f Result	S:			Results Matrix	Eigenvalues
1/SQRT	(Diagonal)	on Diagon	al			
0.0687	0.0000	0.0000	0.0000		View Eiger	WALUES
0.0000	0.1779	0.0000	0.0000		Create Diagona	al 1/SD Matrix
0.0000	0.0000	0.1059	0.0000		Calculate B2 for D	V in First Column
0.0000	0.0000	0.0000	0.0979			Y III III III IIII
	1/9	SD		-	Save as Matrix 1 Save as Matrix 2 Save as Matrix 3	Load Matrix 1 Load Matrix 2 Load Matrix 3
					Copy to Matrix A	Copy to Matrix B

Step2: Capitalize the temporary save mode to calculate Correlation Matrix (1/SD \ast Variance \ast 1/SD)

⁸ ₩ Result	s Window					
Besi	ults Matr	# Rov	Transpose R	Transpose Results Matrix		
1 1001		.	111		Invert Res	ults Matrix
Туре о	f Results	50			Results Matrix	Eigenvalues
Matrix A	* Matrix B					
1.0000	0.4239	0.4393	0.6891	Y	View Eige	nVALUES
0.4239	1.0000	-0.3040	0.4103	X1	Create Diagon	al 1/SD Matrix
0.4393	-0.3040	1.0000	0.1654	X2	Calculate B2 for [) V in First Column
0.6891 🔺	0.4103	0.1654	1.0000	ХЗ		
Y	X1	X2	ХЗ		Save as Matrix 1	Load Matrix 1
			Capitali	ze the _	Save as Matrix 2	Load Matrix 2
Correlation between Y			use of t tempor	he arv	Save as Matrix 3	Load Matrix 3
and X	2		save m	ode	Copy to Matrix A	Copy to Matrix B
	Corr	elation	between			
	X1 a	nd X3	<u>i</u> c	lose		
<u></u>						

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Question 18 Standardized Regression Coefficients

What is the standardized regression coefficient for predictor variable X3? Hint: Standardized regression coefficient = (correlation matrix for predictors)⁻¹×correlation matrix for Y and predictors; refer to results for Question17 to get correlation matrix for predictors and that for Y and predictors.

Answer:

Step1: Enter Correlation Matrix for Predictors (Rxx) and Correlations between Y and Predictors (Rxy) respectively;





Step2: Calcualte the inverse of Predictor Correlation Matrix (Rxx)⁻¹;

Step3: Calculate $B = (Rxx)^{-1}(Rxy)$

🍇 Results Window					
Results Matrix: 3 by 1	Transpose Results Matrix				
	Invert Results Matrix				
Type of Results:	Results Matrix Eigenvalues				
Matrix A * Vector B 0.3851	View EigenVALUES				
0.4817	Create Diagonal 1/SD Matrix				
0.4514	Calculate R2 for DV in First Column				
	Save as Matrix 1 Load Matrix 1				
Stardardized Regression	Save as Matrix 2 Load Matrix 2				
Coefficients					
	Copy to Matrix A Copy to Matrix B				
	<u>i</u> <u>C</u> lose				

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Question 19 Intercept

What is the intercept for the regression model using standardized regression coefficients? (Hint: Be careful!)

Answer: 0.