BMA

Basic Matrix Algebra (V1.0)

User's Guide
&
Lesson Plan

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Contents

Introduction

Basic Matrix Algebra (BMA) performs basic matrix algebra routines to assist more advanced students.

It is important for advanced students in applied statistics courses to understand matrix algebra at least at a basic level. Unfortunately, it is often difficult to practice these procedures without learning SPSS and SAS matrix programming languages. BMA provides students a visual, user-friendly interface to learn basic matrix algebra. The BMA program performs basic matrix algebra and helps to illustrate the mathematical procedures of multiple regression and multivariate analysis. The primary purpose of the program is to provide a convenient learning tool with a user-friendly interface as well as to promote the effective teaching of multiple regression and multivariate analyses.

Matrix operations can be used to obtain important quantities in statistical work, such as the matrix of variances and covariances for a set of variables, the determinant of a matrix involved in several multivariate test statistics, and eigenvalues which are fundamental to many multivariate analyses. Using matrix algebra, for example, the statistics can be acquired for multiple regression. The BMA program includes simple, calculator-like buttons for matrix addition, matrix subtraction, matrix multiplication, and division by a scalar. Buttons are also included for transposing a matrix, calculating a determinant, matrix inversion, and finding eigenvalues. Other special features include quick buttons for calculations necessary for regression using a matrix algebra approach.

I.

A Review of Basic Matrix Algebra Using BMA Program

1. Outlook of BMA

BMA program provides two windows: **Operation Window** and **Results Window**.

In the **Operation Window**, all the calculating buttons are in grey if cell values are not registered in Matrices. Once calculating buttons are activated, or in bold, click on any one and a Results Window will pop up.

The **Results Window** is designed to present calculating results and it will stay until the whole program ends. If you choose to close it in a complex calculation procedure, the data are still stored in the program unless you shut down the Operation Window. For each calculation, the Results Window presents its results only once and you need to either save them temporarily or copy them back to the Operation Window and save them in *.txt files. The details will be illustrated later.

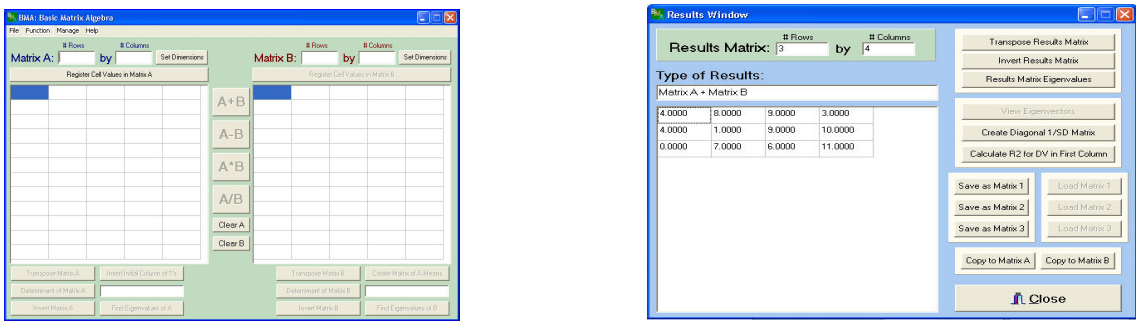


Figure 1: BMA Operation Window (Left) and Results Window (Right)

2. Matrix Addition, Subtraction, Multiplication and Division by a Scalar

The four functions are shown between two Matrix Boxes in the Operation Window. For each function, the major four steps should be conducted:

Step1: Set numbers of rows and columns and confirm them by clicking on the button 

;
Step2: Input desired data in Matrix boxes;
Step3: Click on the button 

or 

to confirm your entries;
Step4: Choose a desired function button to calculate results;

Example: As the operation procedures for the four functions are similar, we only illustrate the **Division Operation** herein:

1) Operations in the Operation Window

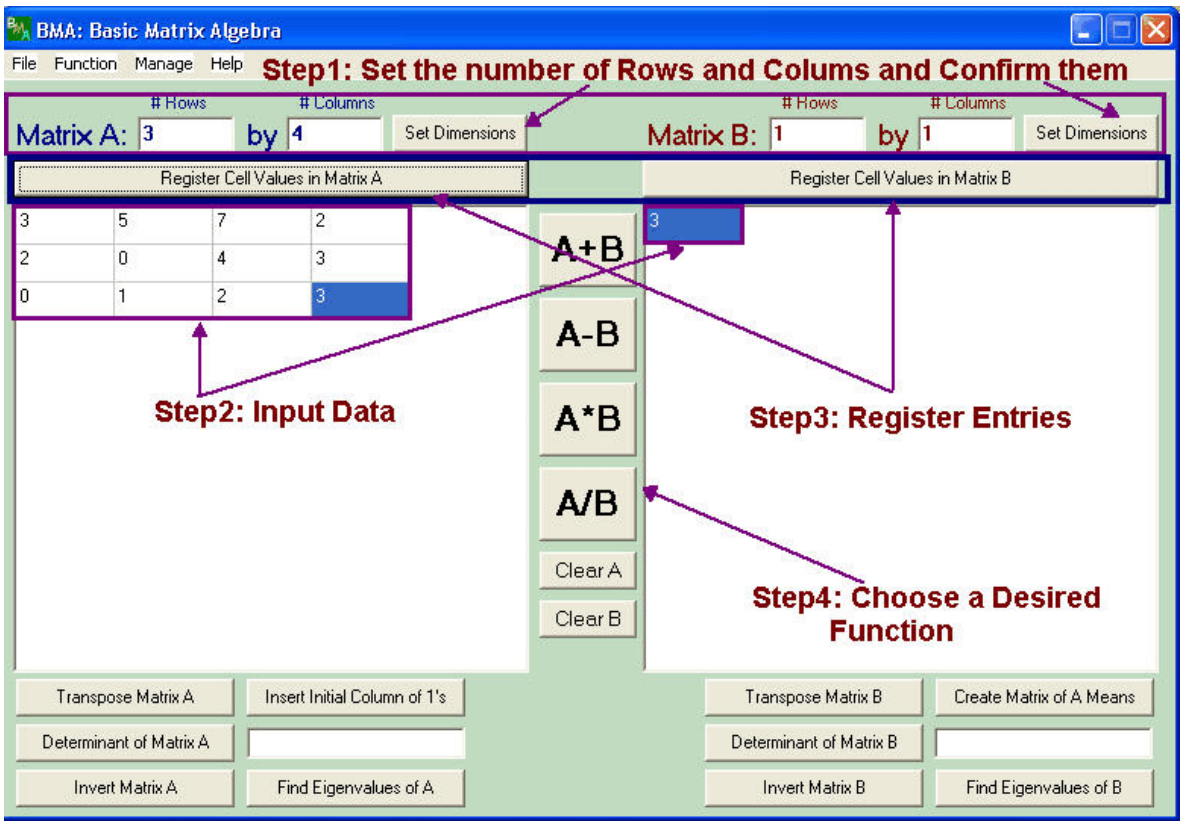


Figure 2: Operations in the Operation Window

2) Operations in the Results Window

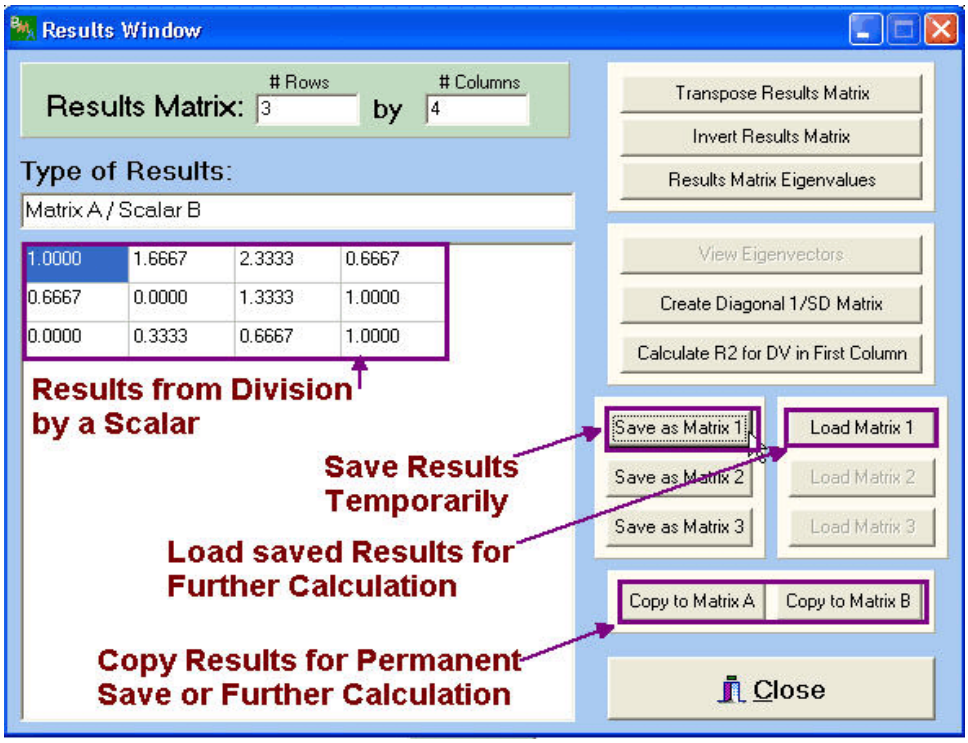
Once you click on 

, the **Results Window** pops up and the calculation results will be presented. Based on individual needs, the results could be saved temporarily for further calculations or permanently as *.txt file for use next time.

Temporary Save Mode: The maximum number of matrices saved in this mode is THREE and they could be reloaded in the further calculation.

Permanent Save Mode: Copy results into either Matrix A or B box in the Operation Window and save them by choosing ave Matrix A or ave matrix B in the File Menu.

In this **Division** example, the results saved in the two modes are illustrated in the following:



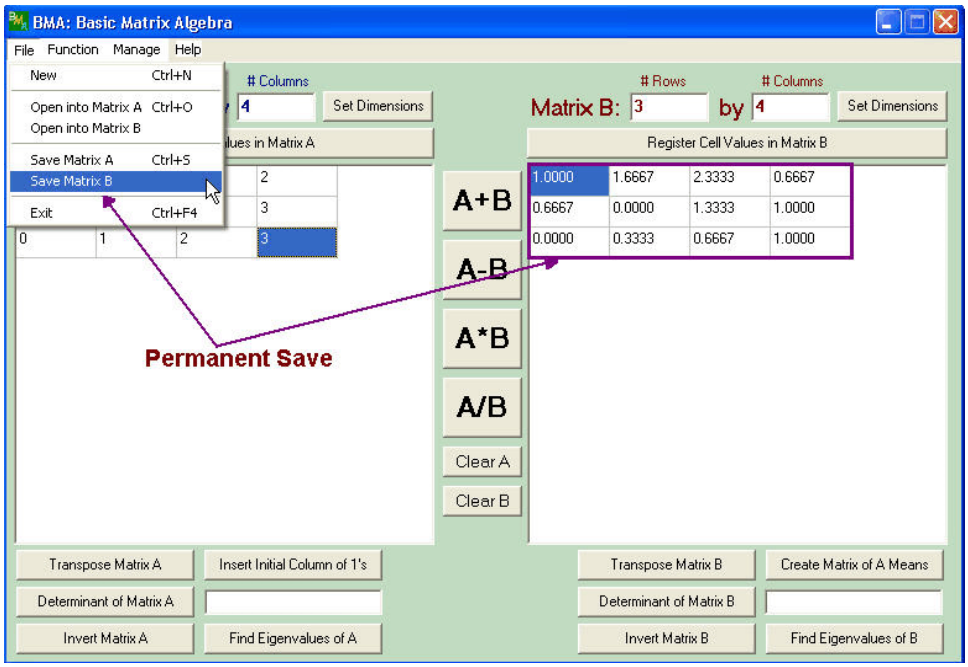


Figure 3: Temporary and permanent save modes

Note: Other function buttons designed specifically for Statistical courses in the Results Window will be introduced in the later Multiple Regression lesson plan.

3. The Transpose, Inverse, Determinant and Eigenvalues

In the **Operation Window**, the function buttons for **Transpose, Inverse, Determinant and Eigenvalues** are provided for Matrix A and B separately. Once you choose a desired function, the results for the matrix will be displayed in the Results Window as illustrated above.

If there is any inappropriate operation, a dialog box will pop up and give you a specific instruction shown as folows:

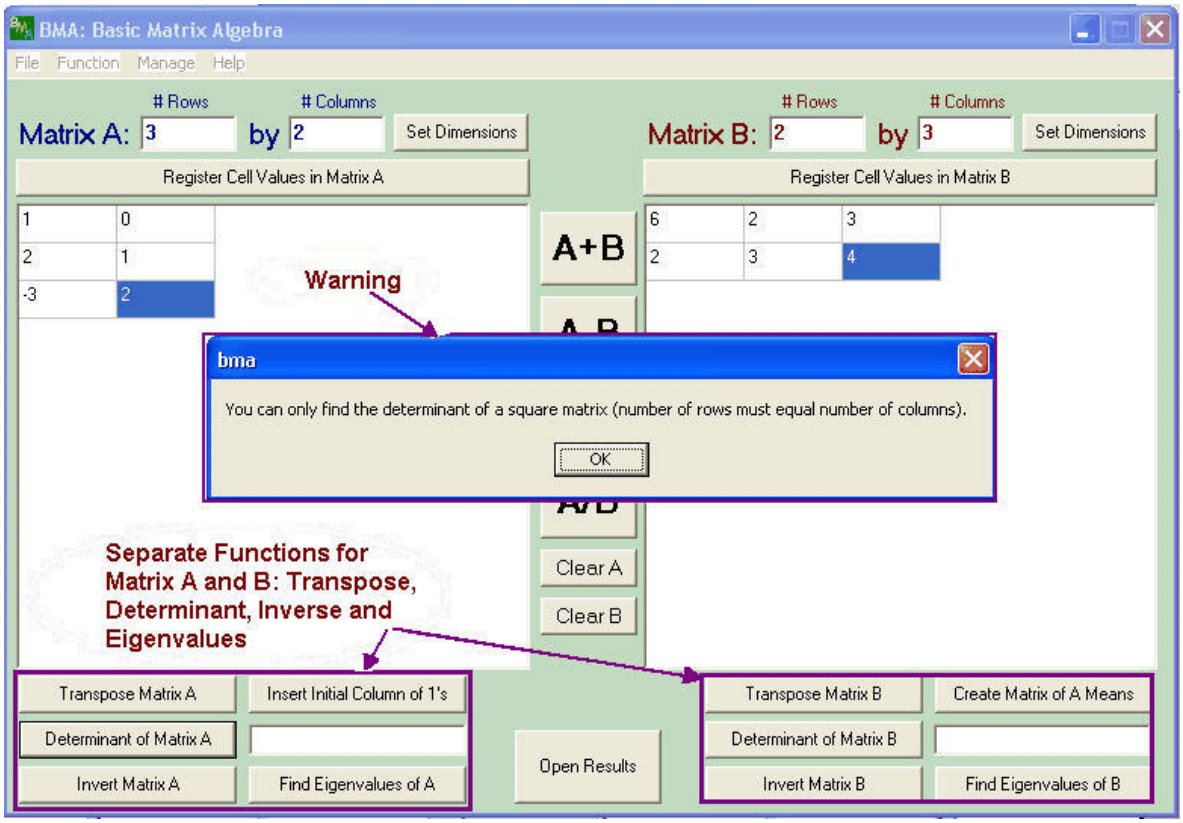


Figure 4: Dialog box popping up for inappropriate operation

4. Special Functions

1. To **move, copy, swap matrices** in Matrix A and B boxes, go directly to **Manage Menu** in the Operation Window and find the function you want.

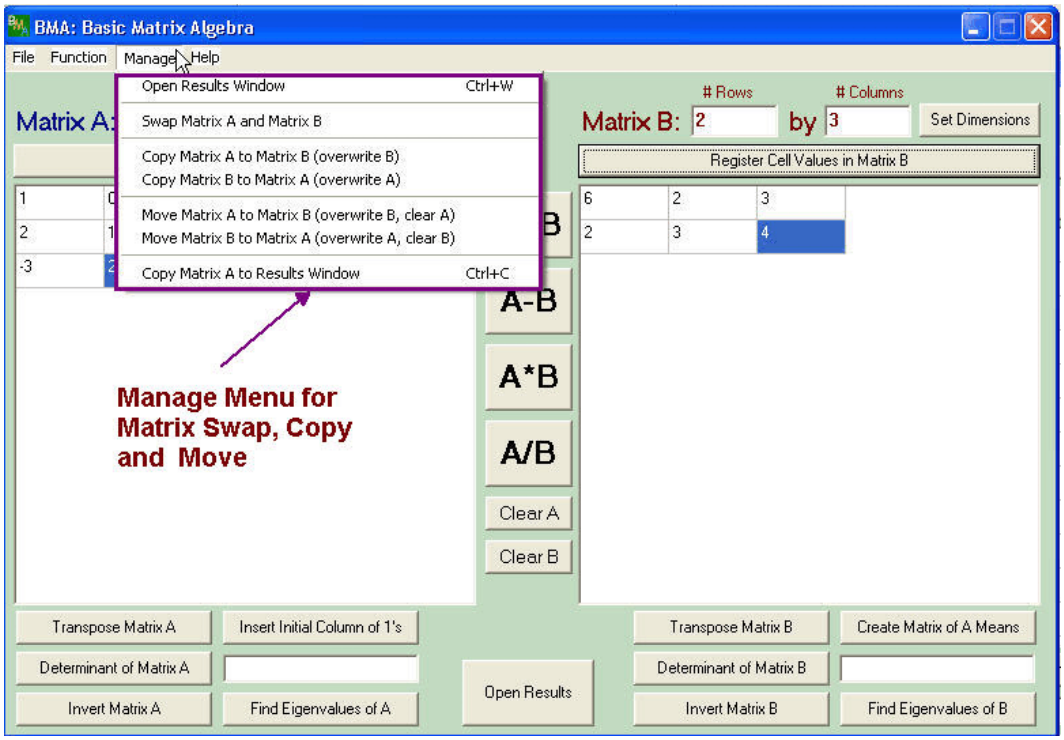


Figure 5: Manage menu for matrix swap, copy and move

2. Under the **Function Menu**, **special reset, set and save functions** are provided, such as *Reset Matrix A(B) to empty*, *Set Matrix A(B) as Identity Matrix*, and *Save Matrix A(B) in Memory as Data Matrix X(Y)*, etc.

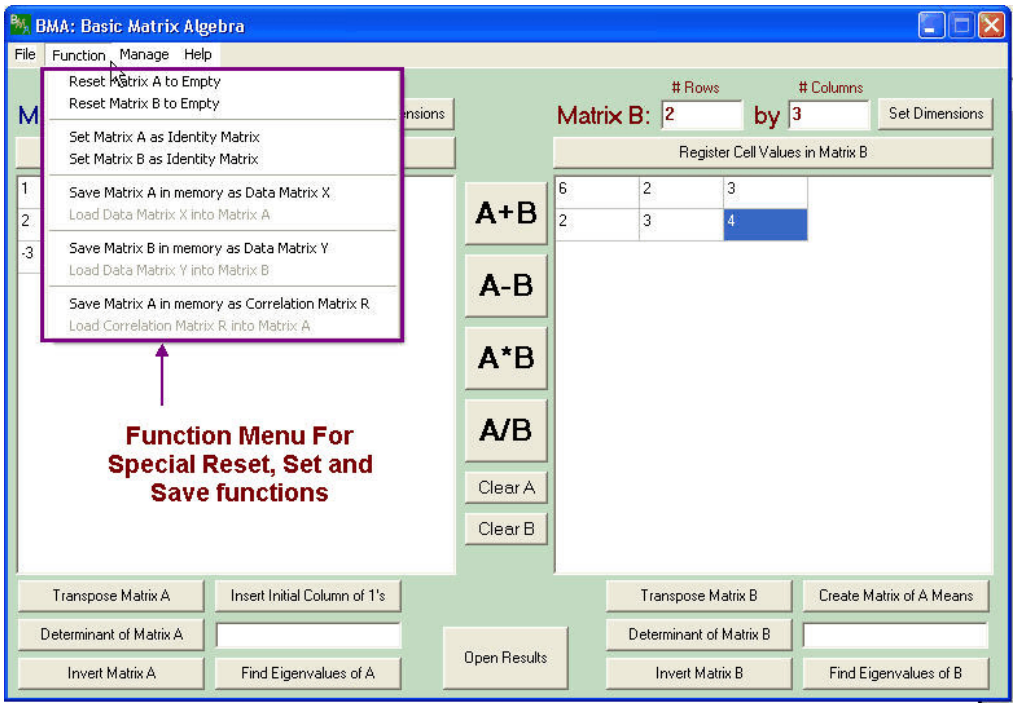


Figure 6: Function menu for special reset, set and save functions

II. BMA Lesson Plan: Multiple Regression Case Study

In this section, students are required to learn about the calculation of important elements in Multiple Regression Analysis using BMA, such as Matrix of Raw (Standardized) Regression Coefficients, Sum of Squares about Regression and Residuals, Mean Squares of Regression and Residuals, F-statistic, Matrix of Standard Deviation, Variance-Covariance Matrix, and Zero-order correlation Matrix, etc.

Case Study: Use the data provided to answer the multiple regression questions below:

Person	Y	X1	X2	X3
1	63	11	69	32
2	71	17	75	50
3	85	16	88	33
4	60	25	60	25
5	65	15	65	30
6	63	08	87	29
7	95	20	71	49
8	50	06	65	21
9	75	12	80	24
10	91	16	77	42

Question01 Observed Scores

What is the observed dependent variable score (Y) for Person_4?

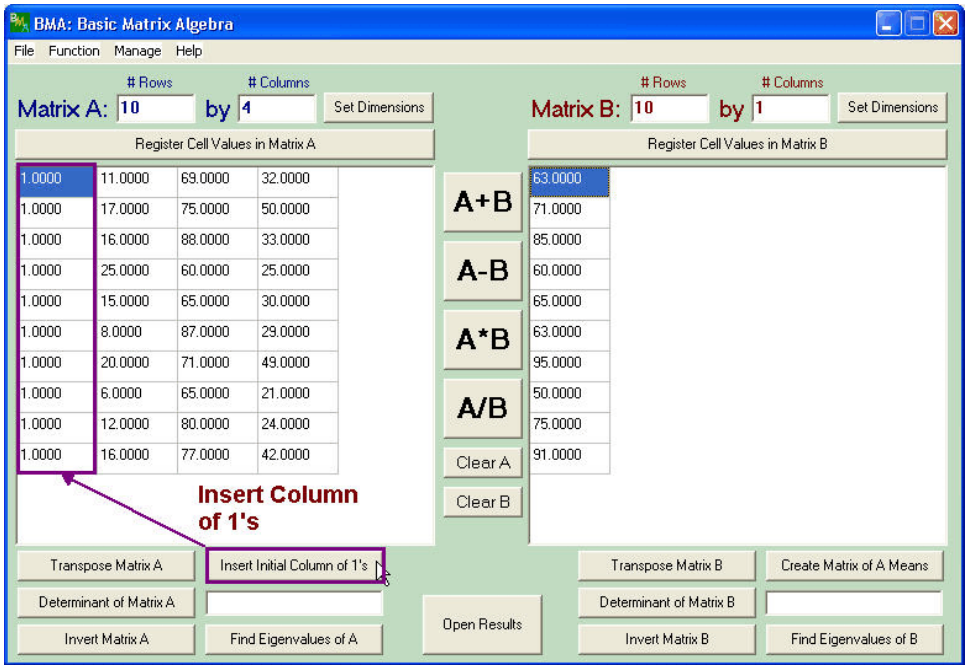
(Answer: 60)

Question02 Raw Regression Coefficients

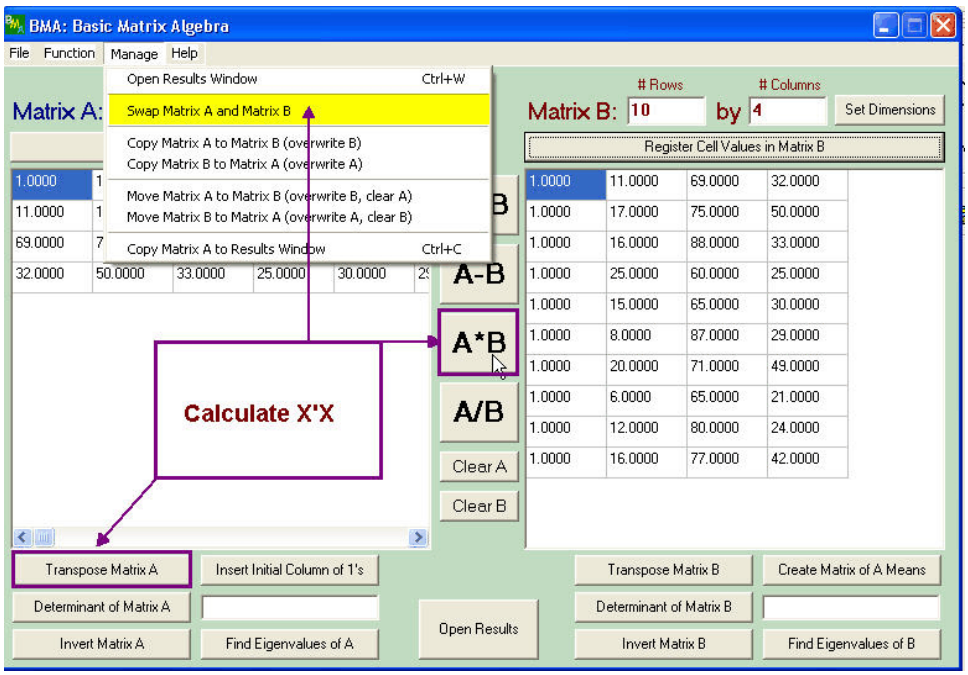
What is the raw regression coefficient for predictor variable X2?
(Hint: You need to run BMA to answer this question; $\beta = (x'x)^{-1}(x'y)$; create column vector of 1's and combine with x matrix to calculate intercept)

Answer:

Step1: Input X1, X2 and X3 into Matrix A box;
Step2: Click on the button of “Insert Initial Column of 1’s” to calculate intercept;

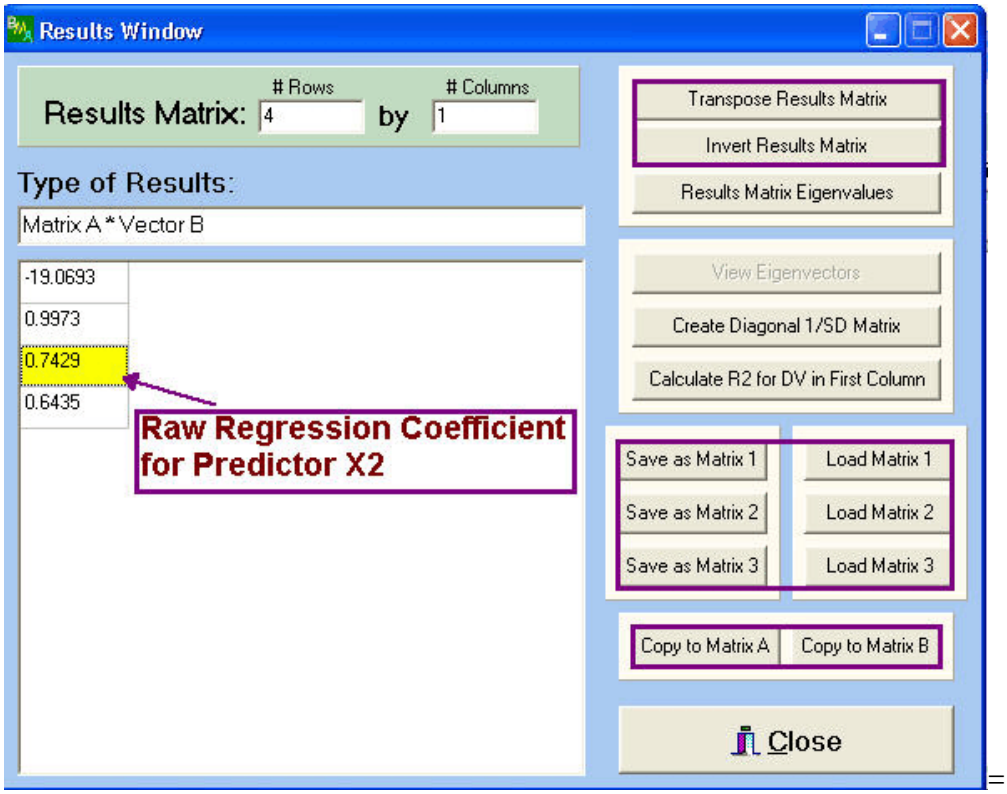


Step3: Calculate $(x'x)^{-1}$ using Transpose, Swap, Inverse, Temporary Save Functions provided in either Operation Window or Results Window;



Step 4: Calculate (x′y) in the similar manner as illustrated in Step3;

Step 5: Calulate $(x'x)^{-1}(x'y)$.



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Question03 Regression Model

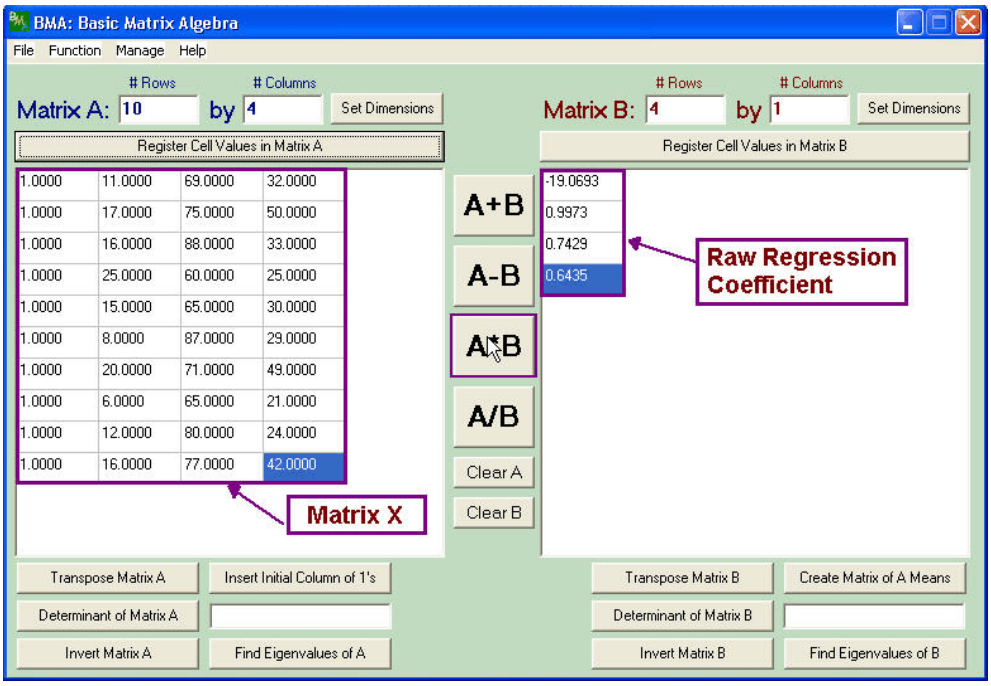
Write the regression model using raw regression coefficients (include intercept).(Hint: Refer to the results generated for Question 02)

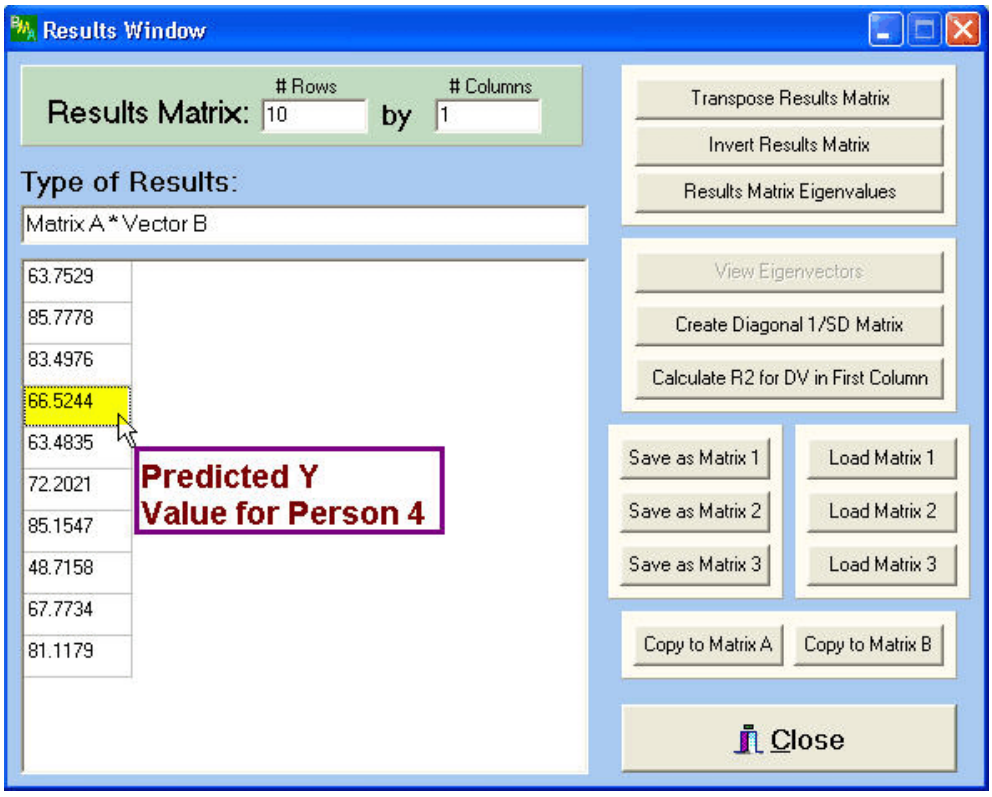
Answer: Y = -19.069 + .997X1 + .743X2 + .644X3

Question04 Predicted Values

What is the predicted Y value for Person_4? (Hint: Yhat = Xb or Yhat = $x*(x'x)^{-1}(x'y)$; refer back to the results produced for Question 02 and continue your calculation using BMA)

Answer:

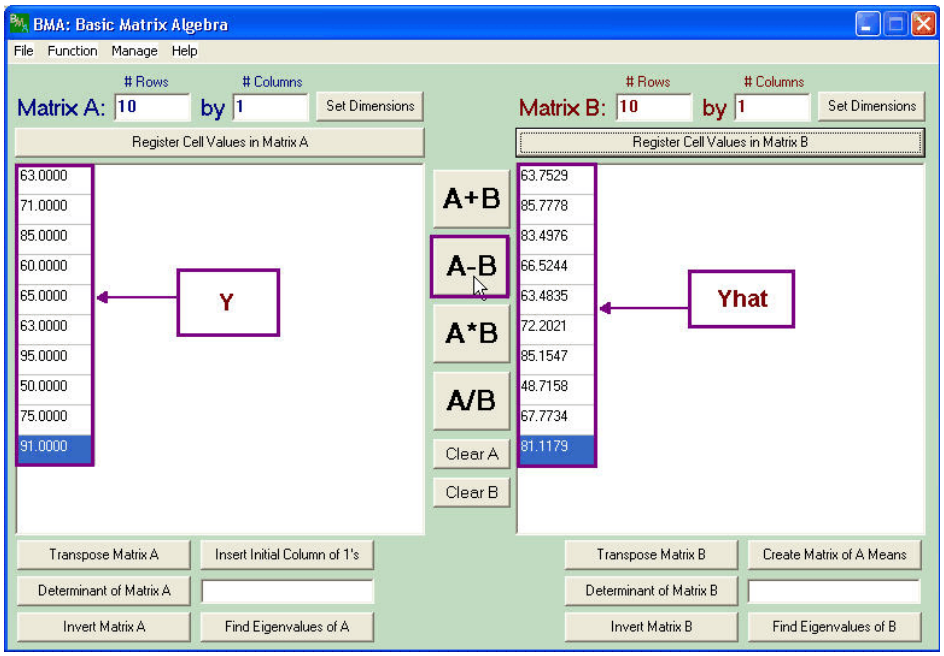

BMA: Basic Matrix Algebra
File Function Manage Help
Matrix A: # Rows 10 by # Columns 4 Set Dimensions
Register Cell Values in Matrix A
1.0000 11.0000 69.0000 32.0000
1.0000 17.0000 75.0000 50.0000
1.0000 16.0000 88.0000 33.0000
1.0000 25.0000 60.0000 25.0000
1.0000 15.0000 65.0000 30.0000
1.0000 8.0000 87.0000 29.0000
1.0000 20.0000 71.0000 49.0000
1.0000 6.0000 65.0000 21.0000
1.0000 12.0000 80.0000 24.0000
1.0000 16.0000 77.0000 42.0000
Matrix X
A+B
A-B
A*B
A/B
Clear A
Clear B
Matrix B: # Rows 4 by # Columns 1 Set Dimensions
Register Cell Values in Matrix B
-19.0693
0.9973
0.7429
0.6435
Raw Regression Coefficient
Transpose Matrix A
Insert Initial Column of 1's
Determinant of Matrix A
Invert Matrix A
Find Eigenvalues of A
Transpose Matrix B
Create Matrix of A Means
Determinant of Matrix B
Invert Matrix B
Find Eigenvalues of B

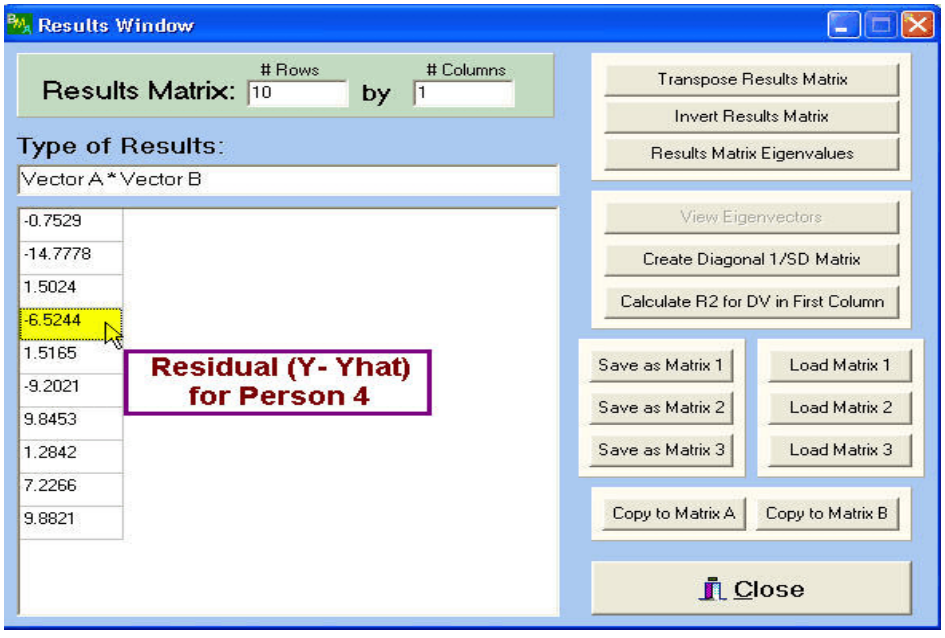

Results Window
Results Matrix: # Rows 10 by # Columns 1
Type of Results:
Matrix A * Vector B
63.7529
85.7778
83.4976
66.5244
63.4835
72.2021
85.1547
48.7158
67.7734
81.1179
Predicted Y Value for Person 4
Transpose Results Matrix
Invert Results Matrix
Results Matrix Eigenvalues
View Eigenvectors
Create Diagonal 1/SD Matrix
Calculate R2 for DV in First Column
Save as Matrix 1
Save as Matrix 2
Save as Matrix 3
Load Matrix 1
Load Matrix 2
Load Matrix 3
Copy to Matrix A
Copy to Matrix B
Close

Question05 Squared Residuals

What is the squared residual (Y-Yhat) for Person_4?
(Hint: Be careful!!! This is to calculate squared errors of prediction. You first have to calculate Y-Yhat using BMA. Use the results for Yhat in Question04)

Answer:





Question06 Degrees of Freedom for Residual

How many degrees of freedom are there for the residual?
(Hint: $df_{residual} = n - k - 1$)

Answer: 10 -3 -1 = 6

Question07 Degrees of Freedom for Regression

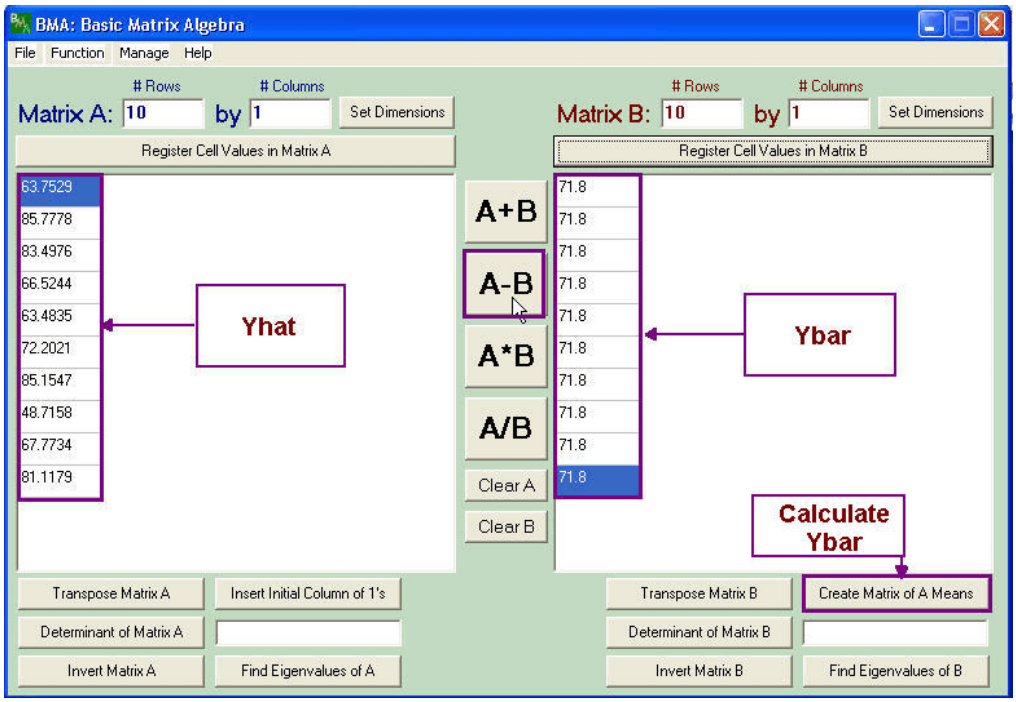
How many degrees of freedom are there for the regression?
(Hint: $df_{reg} = k$)

Answer: k = 3.

Question08 Mean Squares for Regression

What is the Mean Squares for the regression?
(Hint: $MS_{reg} = SS_{reg}$/number of predictors = [(Yhat - Ybar)′ (Yhat - Ybar)]/number of predictors; Ybar = (Sum of Y)/ number of cases)

Answer: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Rows
Columns
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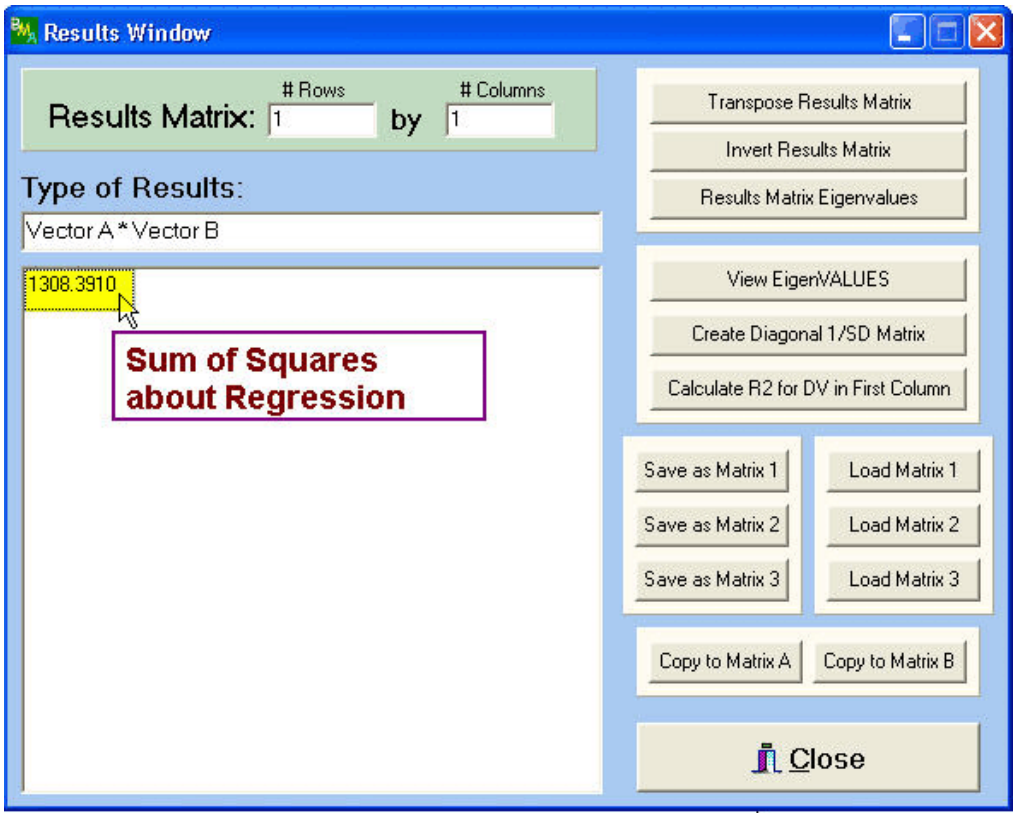
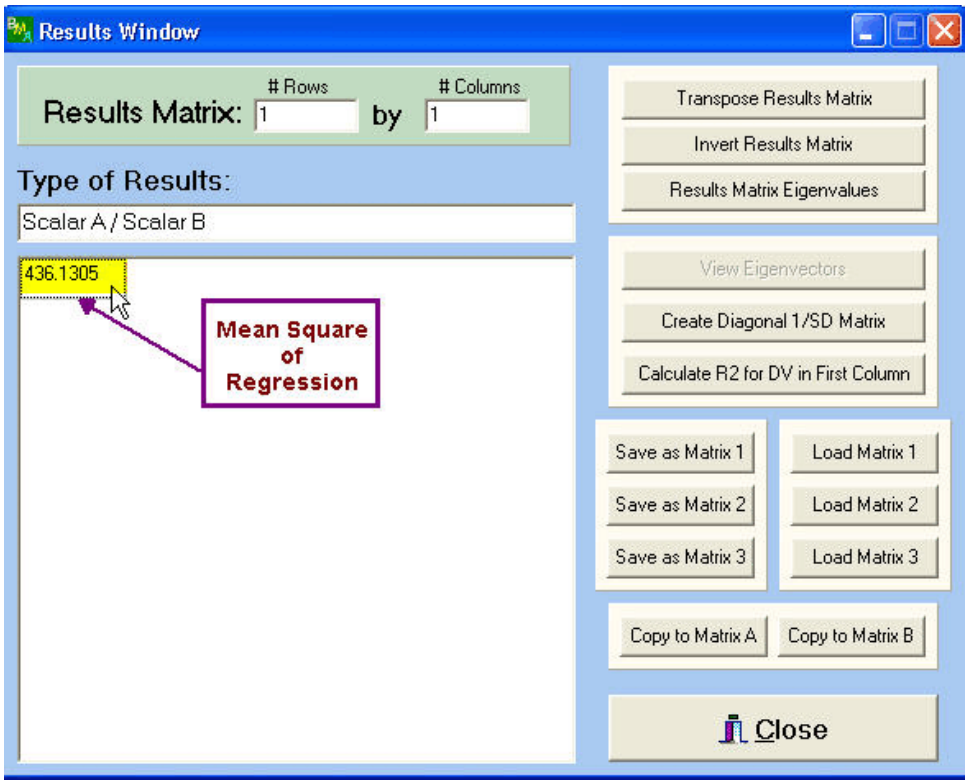
83.4976
66.5244
63.4835
72.2021
85.1547
48.7158
67.7734
81.1179
Yhat
A+B
A-B
A*B
A/B
Clear A
Clear B
Matrix B: 10 by 1 Set Dimensions
Register Cell Values in Matrix B
71.8
Ybar
Calculate Ybar
Transpose Matrix A
Insert Initial Column of 1's
Determinant of Matrix A
Invert Matrix A
Find Eigenvalues of A
Transpose Matrix B
Create Matrix of A Means
Determinant of Matrix B
Invert Matrix B
Find Eigenvalues of B

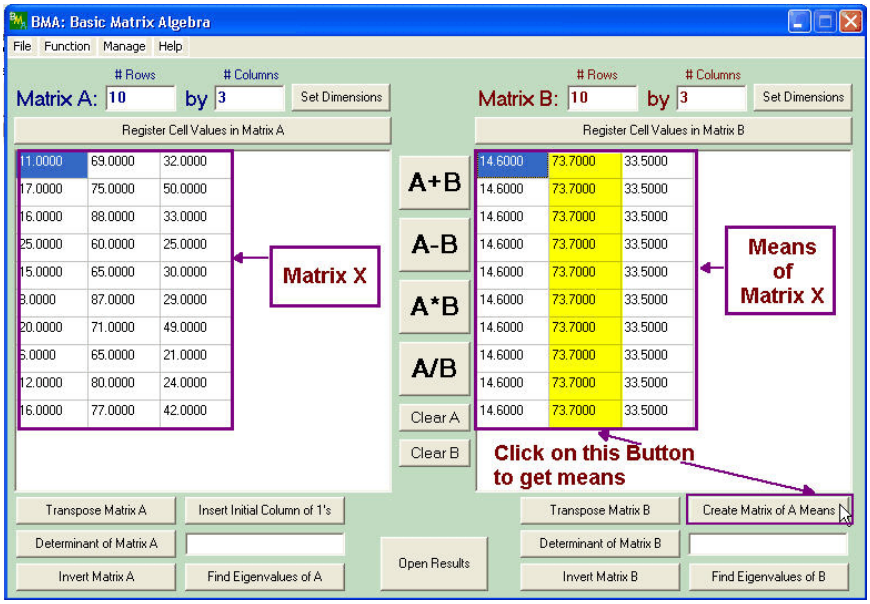

Results Window
Results Matrix: # Rows 1 by # Columns 1
Type of Results:
Vector A * Vector B
1308.3910
Sum of Squares about Regression
Transpose Results Matrix
Invert Results Matrix
Results Matrix Eigenvalues
View EigenVALUES
Create Diagonal 1/SD Matrix
Calculate R2 for DV in First Column
Save as Matrix 1
Save as Matrix 2
Save as Matrix 3
Load Matrix 1
Load Matrix 2
Load Matrix 3
Copy to Matrix A
Copy to Matrix B
Close



Question09 Means

What is the mean of predictor variable X2? (Hint: Use the function button under Matrix B box)

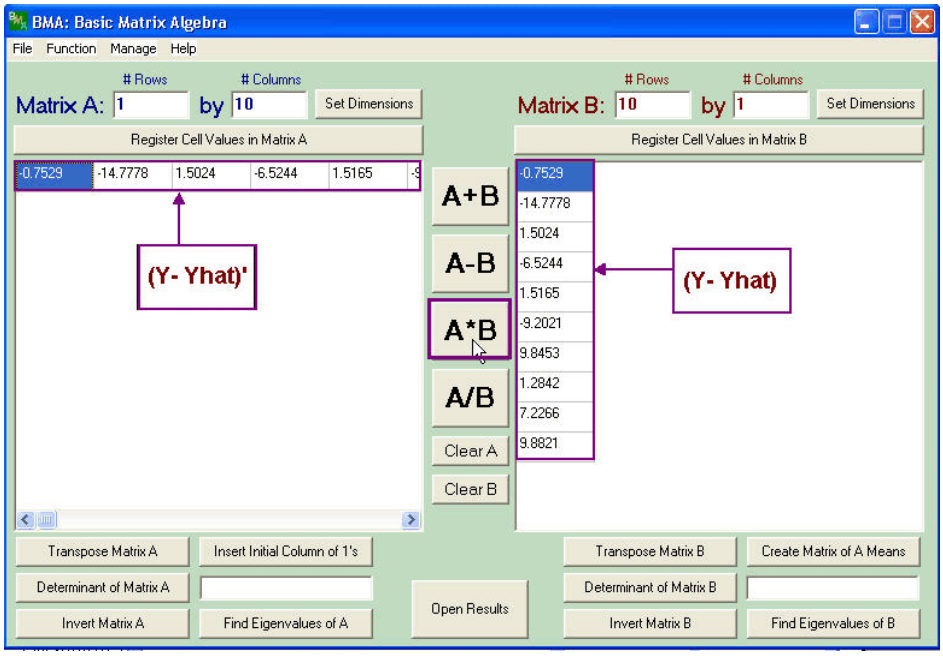
Answer:

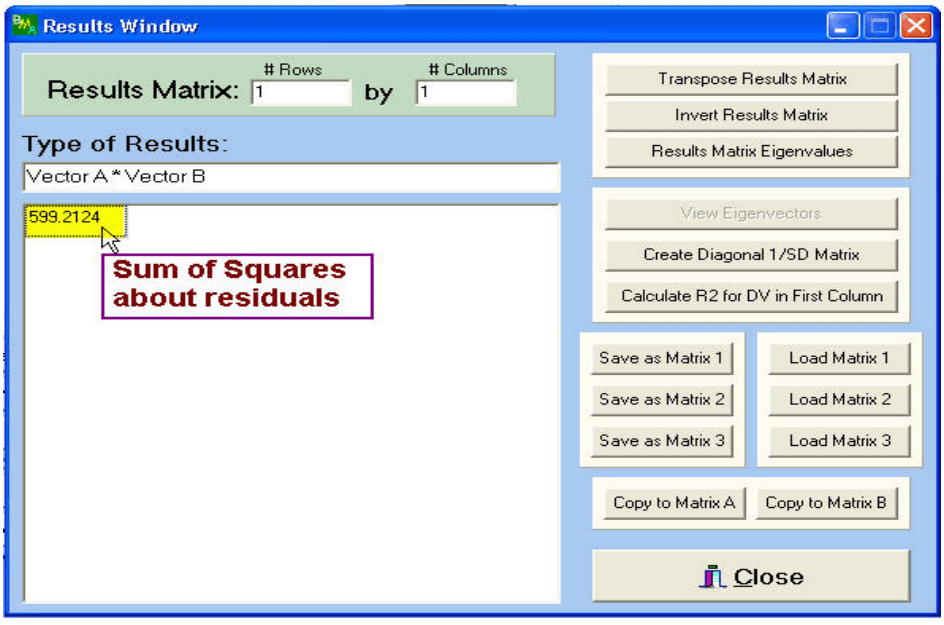


Question10 Mean Square of Residuals

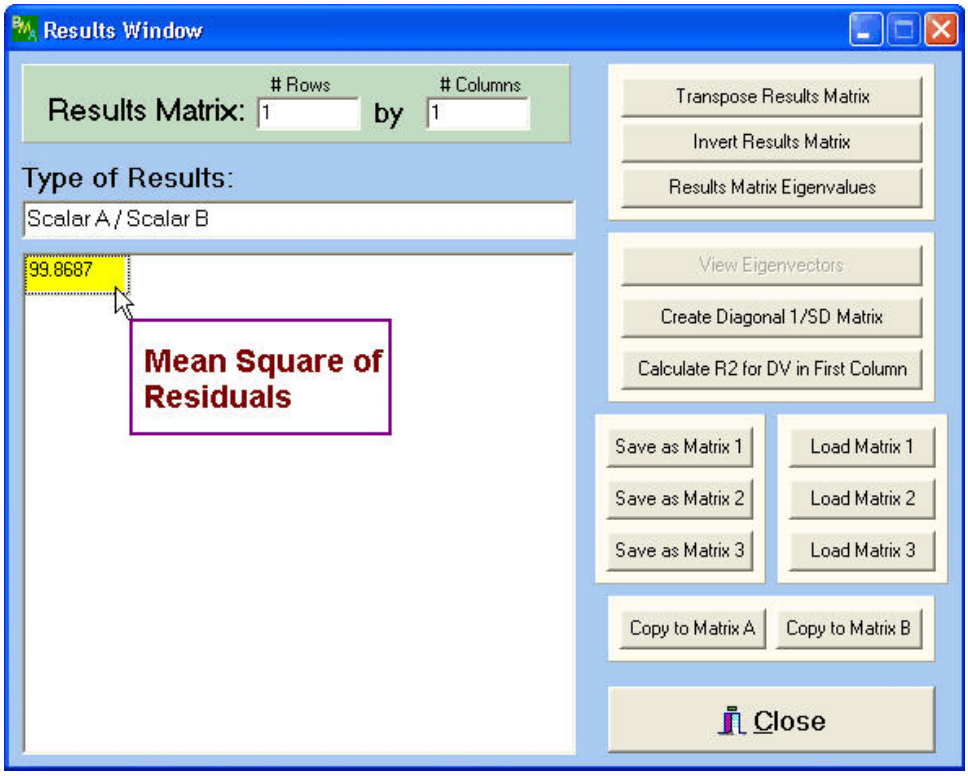
What is the Mean Square of residuals?
(Hint: $MS_{resid} = SS_{resid} / (\text{number of cases} - \text{number of predictors} - 1) = [(Y - Yhat)'(Y - Yhat)] / df_{resid}$).

Answer:
Step 1: Calculate $SS_{residual}$





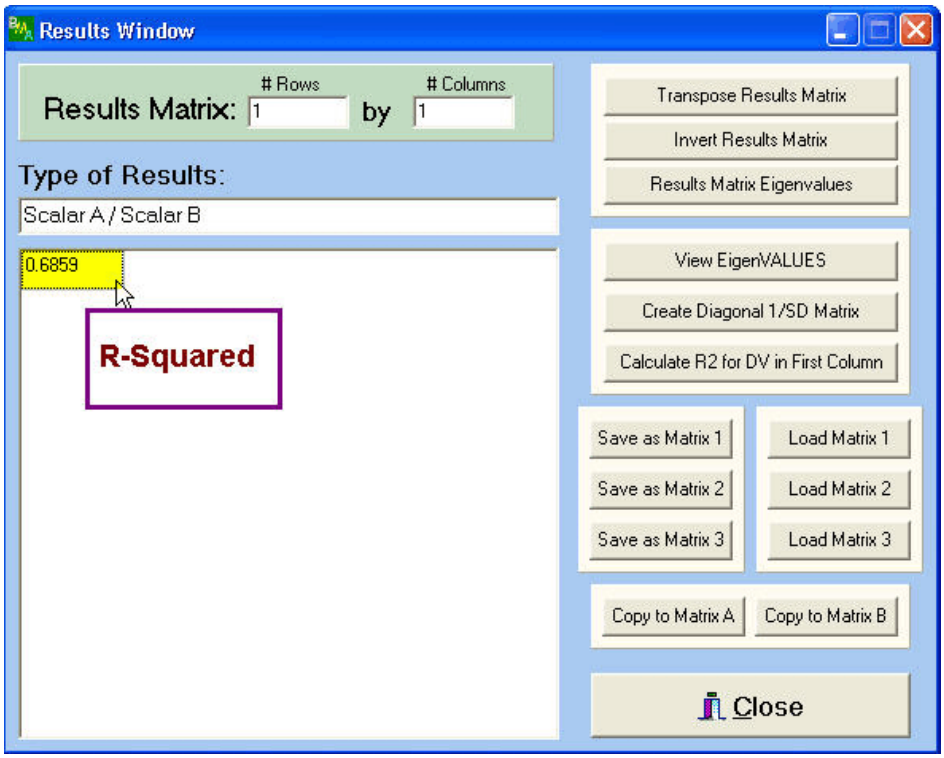
Step 2: Calculate $MS_{residual} = SS_{residual}/(10-3-1)$



Question11 R-squared

Calculate R-squared (be sure to show your work).
Hint: $R^2 = SS_{reg} / SS_{total} = [(Yhat - Ybar)' (Yhat - Ybar)]/ (SS_{residual}+SS_{reg})$; Refer to Question 8 and 10 to get SS_{reg} and $SS_{residual}$)

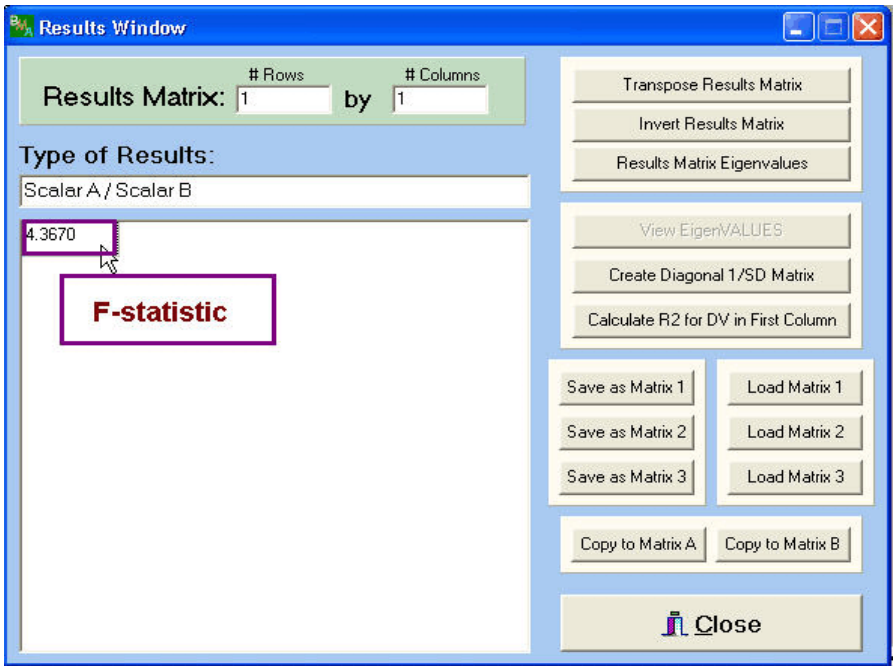
Answer:



Question11 F-statistic

What is the F-statistic?
(Hint: F-statistic= $MS_{reg}/MS_{residual}$; refer to results for Question 8 and 10)

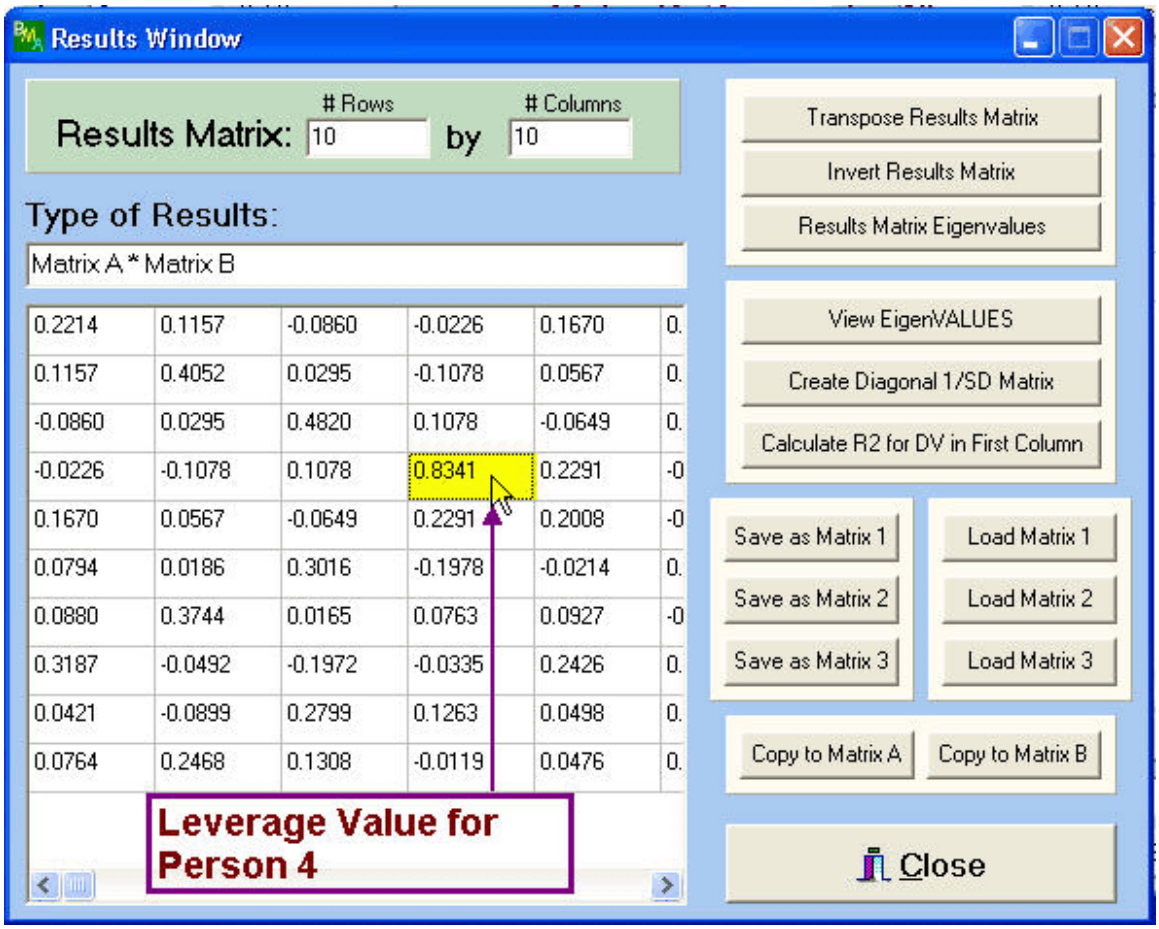
Answer:



Question13 Leverage Value

What is the leverage value for Person_4?
(Hint: Leverage value = $x(x'x)^{-1}x'$; be sure to insert initial column 1 in Matrix A box when calculating leverage values.)

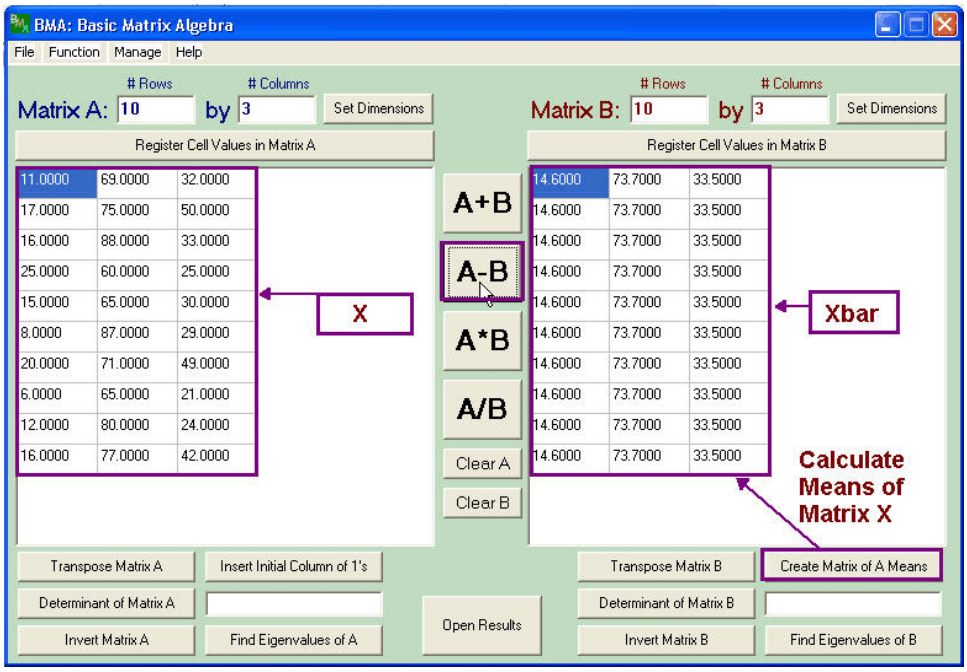
Answer:

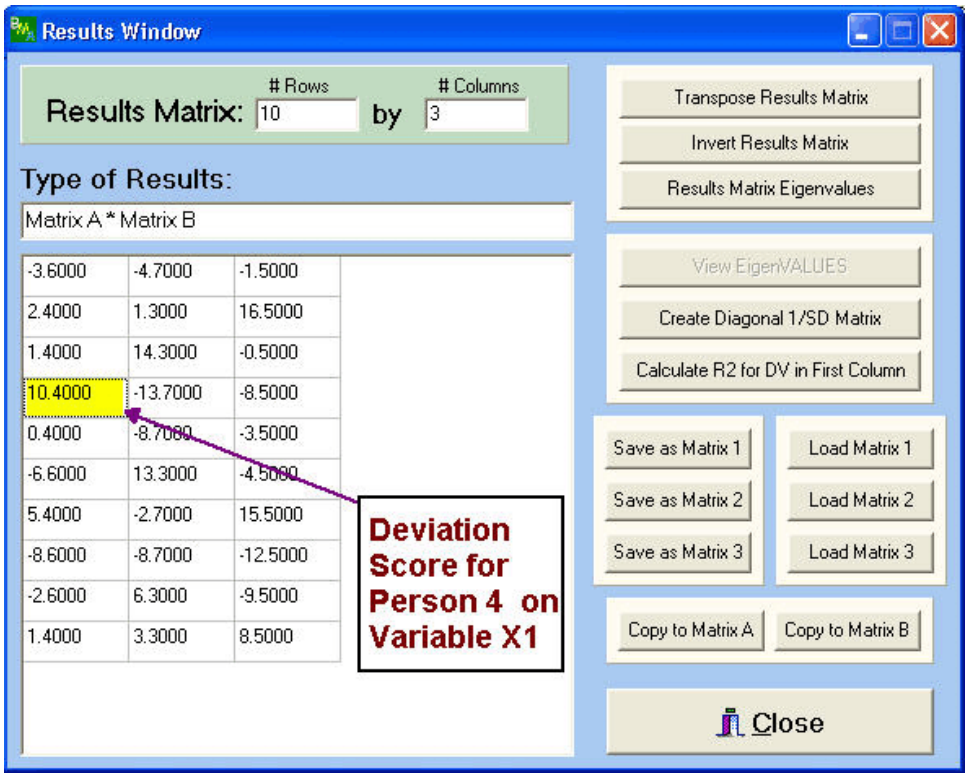


Question14 Deviation Score

What is the deviation score (X-Xbar) for Person_4 on dependent variable X1?

Answer:

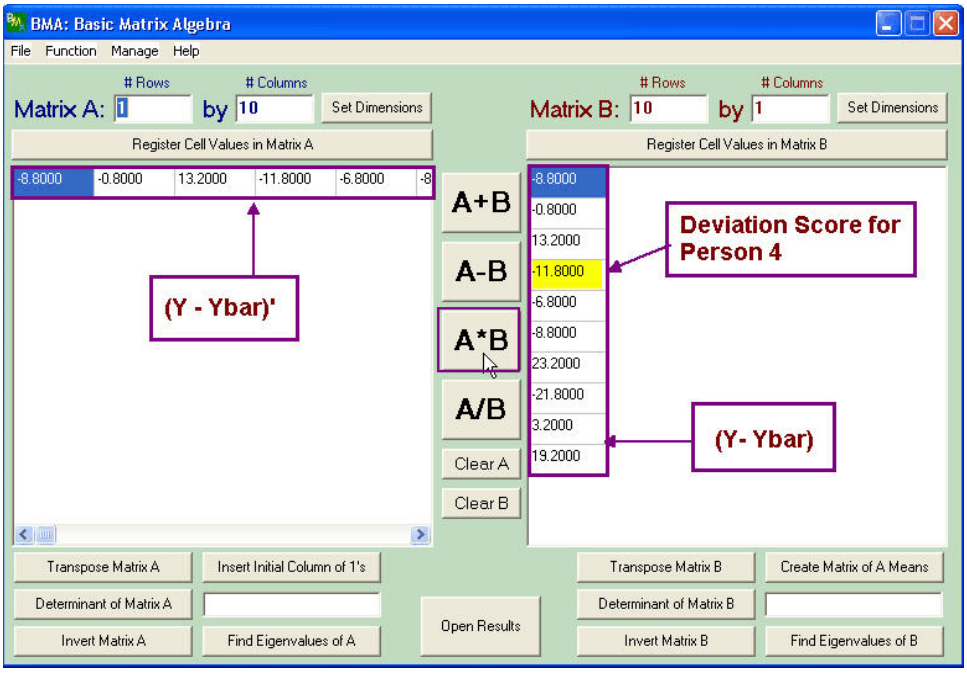


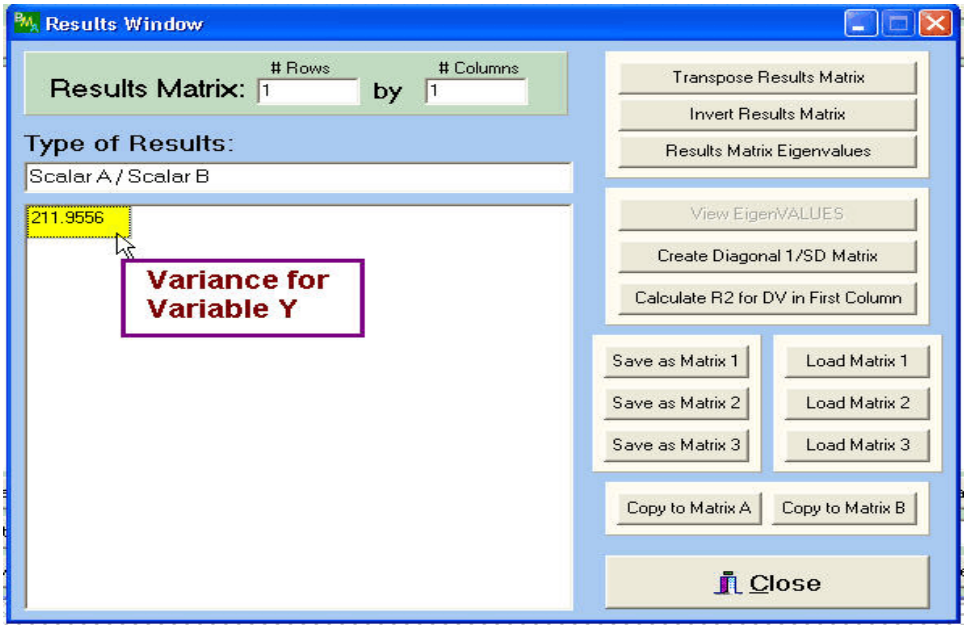


Question15 Z-scores

What is the z-score for Person_4 on dependent variable Y?
Hint: Z = (Y- Ybar)/SD_y; Get variance of variable Y first (Y - Ybar)′ (Y-Ybar)/(N-1).

Answer:





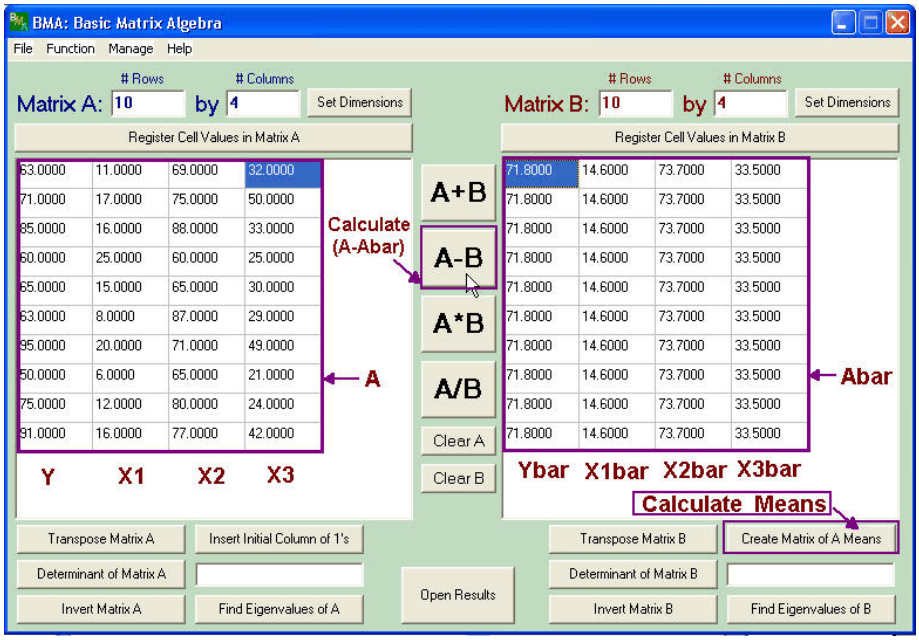
SD = Square Root of Variance = 14.587. Z = (-11.8)/14.587 = - 0.8105.

Question 16 Variances

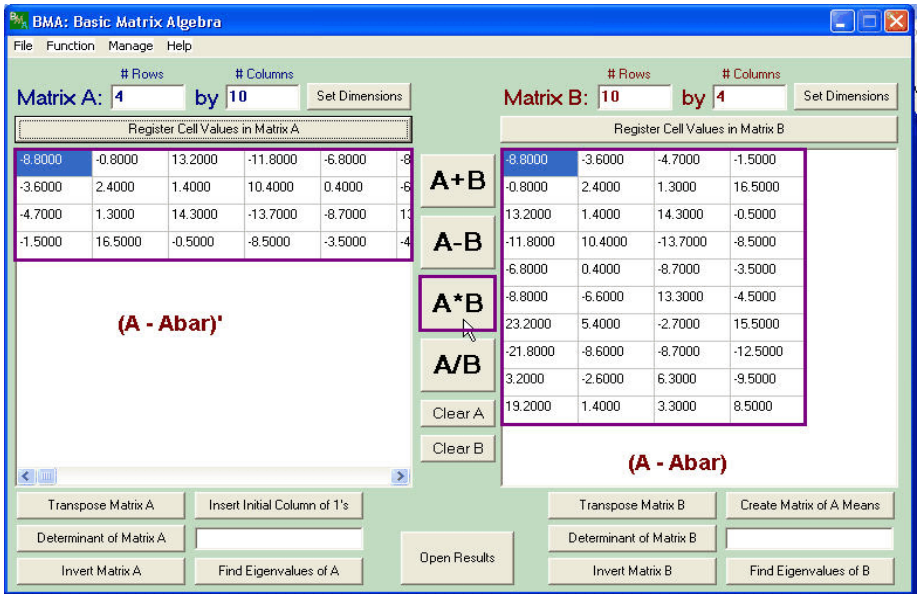
What are the variances for Y, X1, X2 and X3 respectively?
Hint: Create variance-covariance matrix for Y, X1, X2 and X3.

Answer:
Step1: Create a Matrix (A) for Y, X1, X2 and X3;

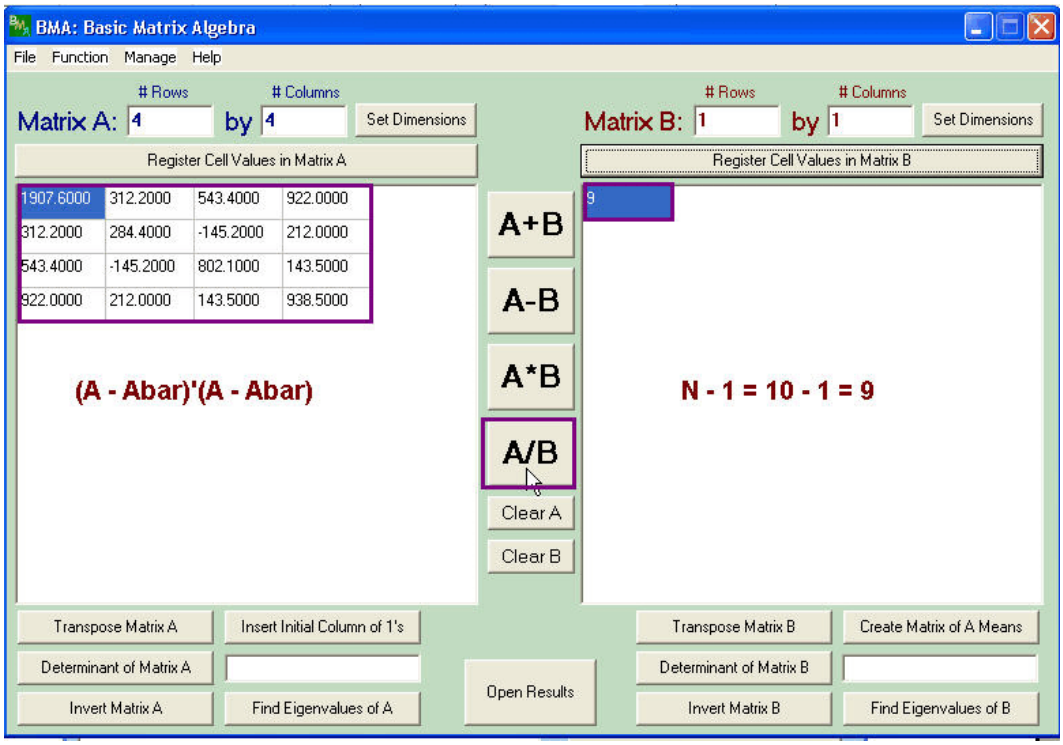
Step2: Calculate means for the four variables;

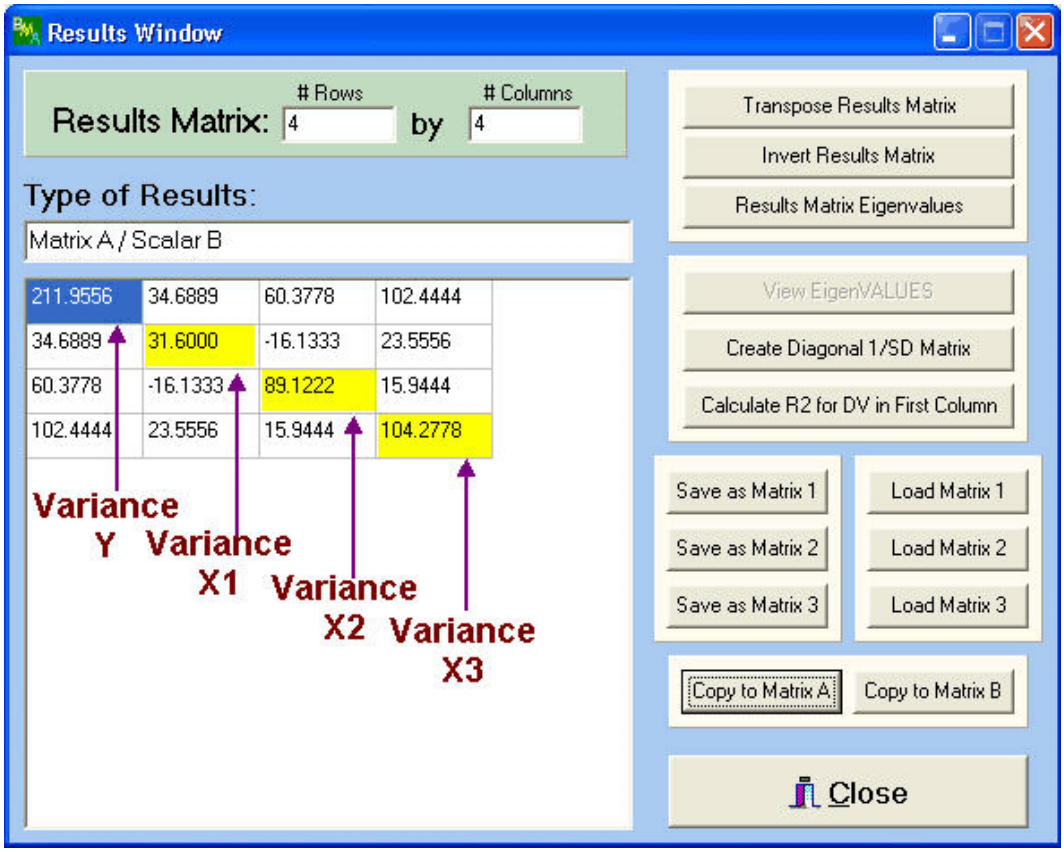


Step3: Calculate $(A - Abar)'(A - Abar)$;



Step4: Use (A - Abar)'(A - Abar)/ (N-1) to calculate variance;





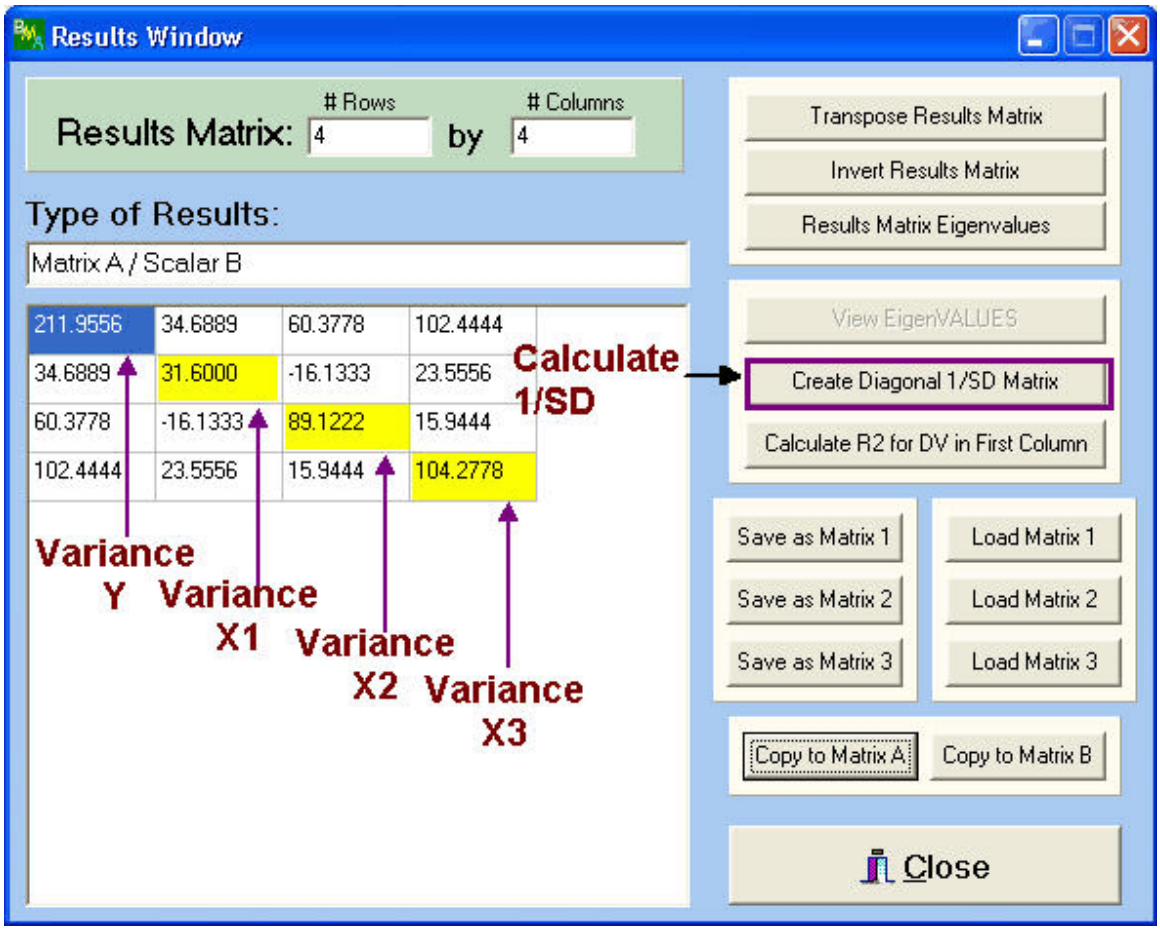
Question 17 Zero-order Correlation

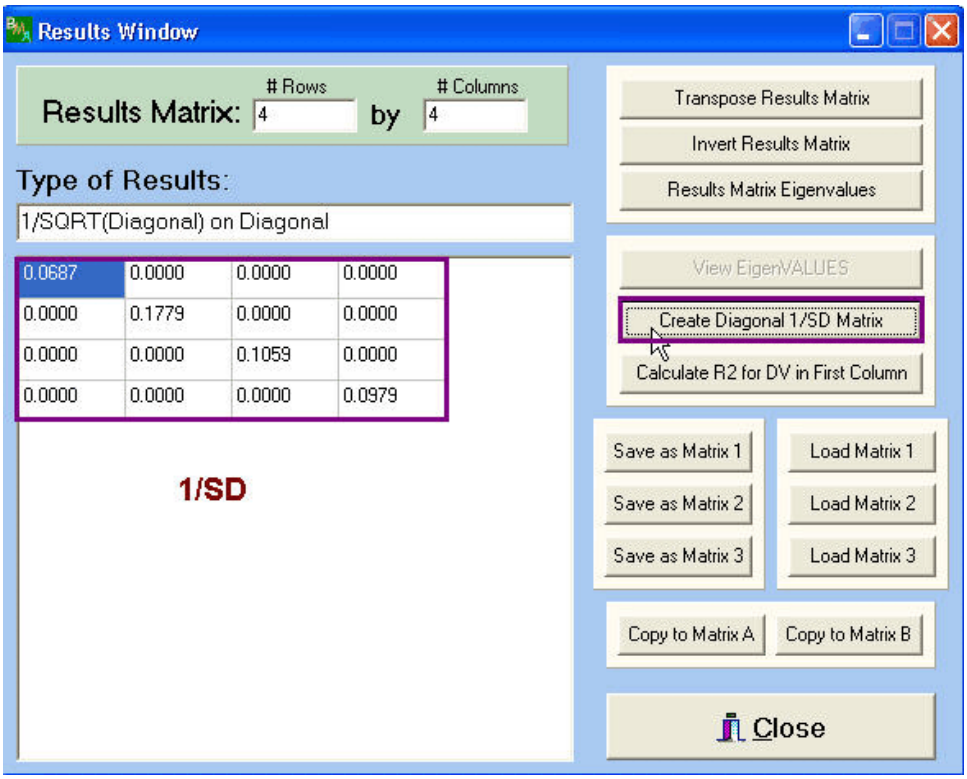
What is the zero-order correlation between variable X2 and variable Y? What is the correlation between predictor variable X1 and predictor variable X3?
(Hint: zero-order correlation = 1/SD * Variance * 1/SD; refer back to the results for Question 16)

Answer:

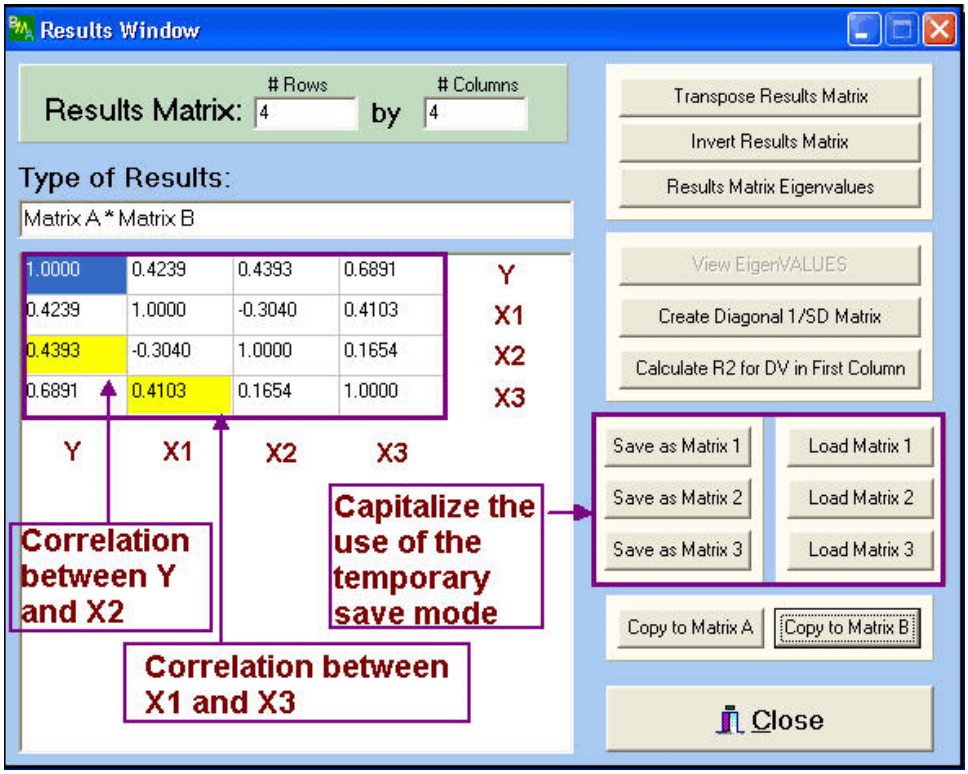
Step1: Once you obtain the variance matrix, click on 

to calculate 1/SD (you can use the Inverse function to get SD);





Step2: Capitalize the temporary save mode to calculate Correlation Matrix (1/SD * Variance * 1/SD)

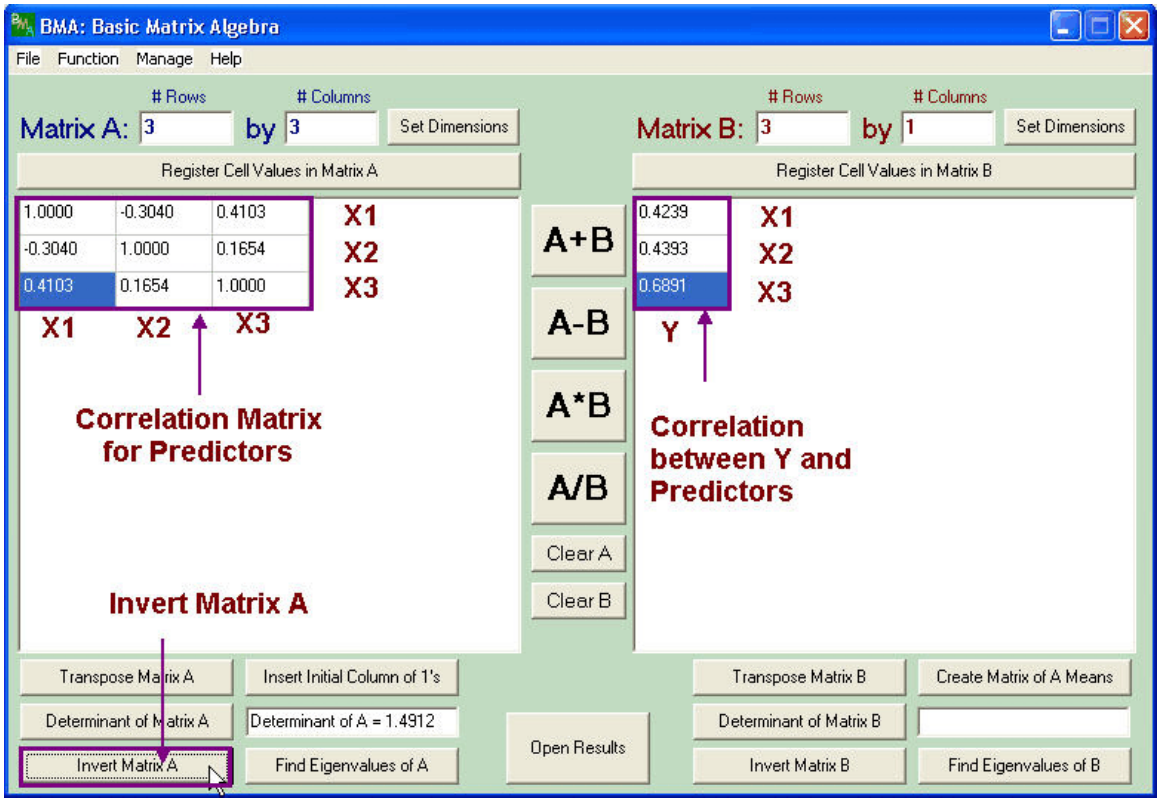


Question 18 Standardized Regression Coefficients

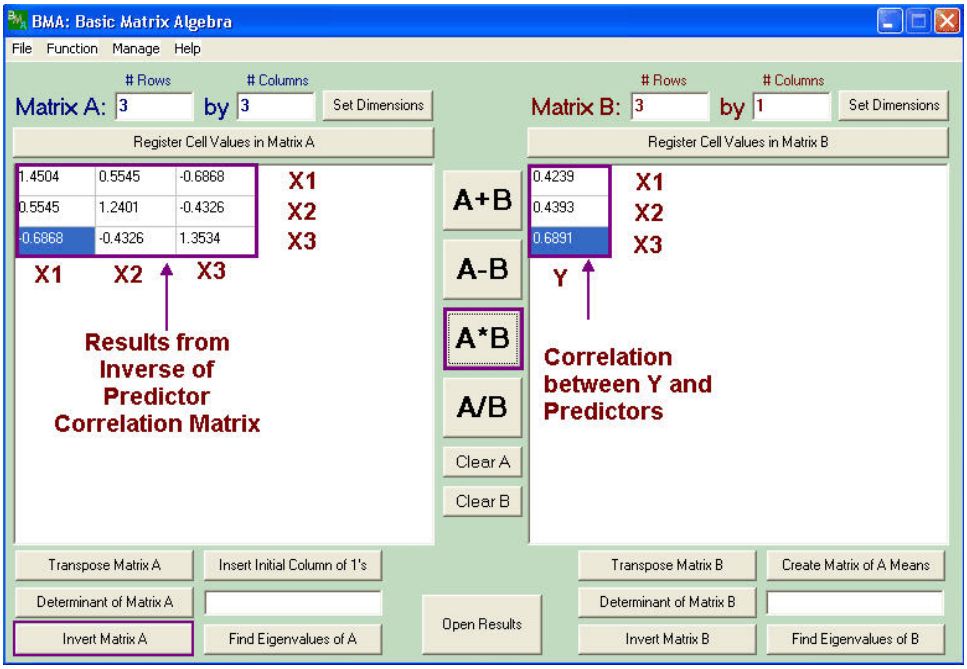
What is the standardized regression coefficient for predictor variable X3?
Hint: Standardized regression coefficient = (correlation matrix for predictors)$^{-1}$×correlation matrix for Y and predictors; refer to results for Question17 to get correlation matrix for predictors and that for Y and predictors.

Answer:

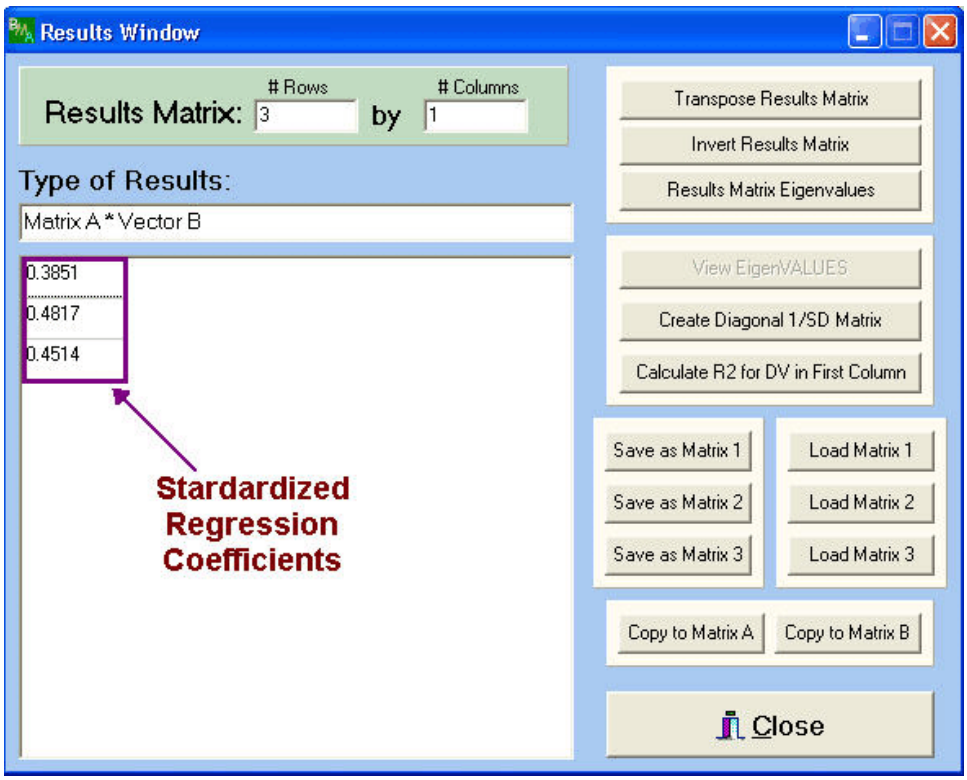
Step1: Enter Correlation Matrix for Predictors (Rxx) and Correlations between Y and Predictors (Rxy) respectively;



Step2: Calcualte the inverse of Predictor Correlation Matrix $(Rxx)^{-1}$;



Step3: Calculate B = $(Rxx)^{-1}(Rxy)$



Question 19 Intercept

What is the intercept for the regression model using standardized regression coefficients? (Hint: Be careful!)

Answer: 0.