

**Exploring the ANOVA assumption of homoscedasticity  
 and its relationship to Type I Error using MC4G**

1. What are the three main assumptions that need to be met when utilizing the ANOVA *F* test?

A.

B.

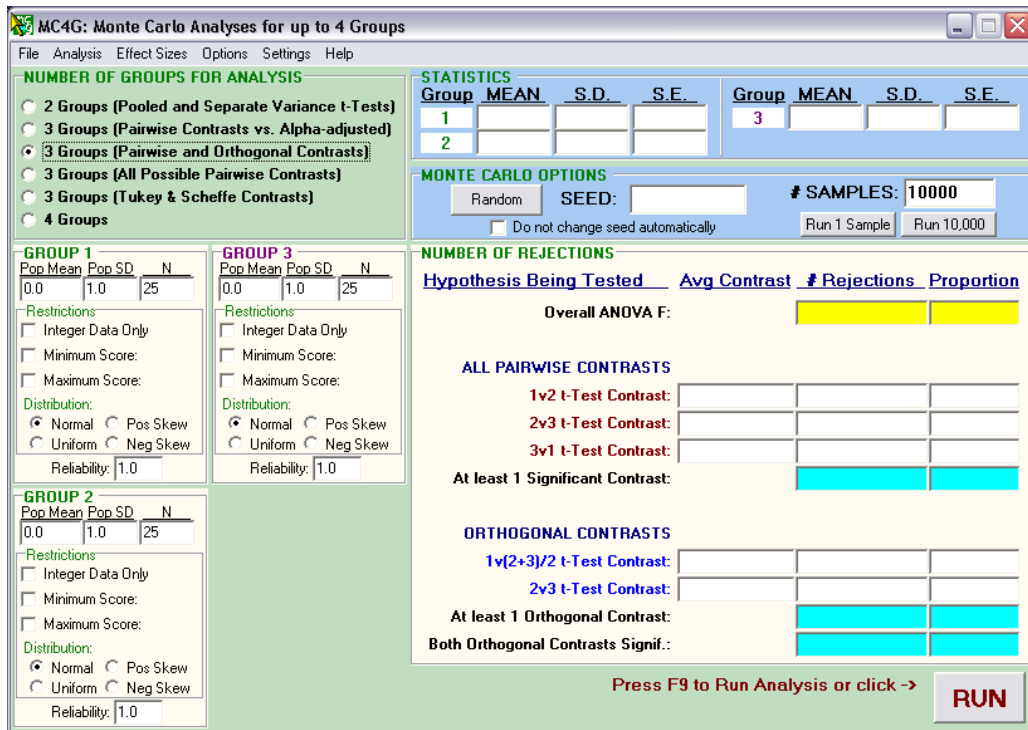
C.

2. A. What is the definition of Type I error?

B. How is Type I error symbolized?

C. Using an example, explain Type I error in your own words.

3. For these exercises, we will be focusing on the Assumption of Homoscedasticity. To complete these exercise, you will need to use the MC4G program.



**Exercise A:**

**ANOVA without Violating the Assumption of Homoscedasticity, Balanced Design**

1. SETTING THE PARAMETERS FOR THE POPULATION. Set your screen to match the conditions described below.

- A. Number of Groups: Select “3 Groups (Pairwise and Orthogonal Contrasts)”
- B. Mean: Because we are examining Type I error, the null hypothesis must be true. Therefore, you may choose any value for the mean; however all means must be equal.
- C. Standard Deviation: Recall that standard deviation is the square root of variance ( $\sqrt{\sigma^2} = \sigma$ ). In this first example, we will not violate the assumption of homoscedasticity. Therefore, you may choose any value for the standard deviation; however all standard deviations must be equal.
- D. Sample Size: In this exercise, we will be using a *balanced* design. Therefore, you may choose any value for sample size; however all sample sizes must be equal.
- E. Number of Samples: To make calculations simple, set the number of samples to 100.

**MC4G: Monte Carlo Analyses for up to 4 Groups**

File Analysis Effect Sizes Options Settings Help A. Set number of groups

**NUMBER OF GROUPS FOR ANALYSIS**

- 2 Groups (Pooled and Separate Variance t-Tests)
- 3 Groups (Pairwise Contrasts vs. Alpha-adjusted)
- 3 Groups (Pairwise and Orthogonal Contrasts)
- 3 Groups (All Possible Pairwise Contrasts)
- 3 Groups (Tukey & Scheffe Contrasts)
- 4 Groups

**STATISTICS**

Group	MEAN	S.D.	S.E.	Group	MEAN	S.D.	S.E.
1				3			
2							

**MONTE CARLO OPTIONS**

Random SEED:  # SAMPLES: 100

Do not change seed automatically

If left blank, a seed will automatically be chosen.

**GROUP 1**

Pop Mean: 0.0 Pop SD: 1.0 N: 25

**GROUP 2**

Pop Mean: 0.0 Pop SD: 1.0 N: 25

**GROUP 3**

Pop Mean: 0.0 Pop SD: 1.0 N: 25

**Overall ANOVA F:**          

**ALL PAIRWISE CONTRASTS**

1v2 t-Test Contrast:          

2v3 t-Test Contrast:          

3v1 t-Test Contrast:          

**At least 1 Significant Contrast:**          

**ORTHOGONAL CONTRASTS**

1v(2+3)/2 t-Test Contrast:          

2v3 t-Test Contrast:          

Both Orthogonal Contrasts Signif.:          

Press F9 to Run Analysis or click -> **RUN**

2. What is the null hypothesis for this exercise? Write your answer in words and in symbols (note: We will be testing our hypothesis at  $\alpha = .05$ ).

3. A. What population mean did you choose for Group 1? \_\_\_\_\_

B. What population mean did you choose for Group 2? \_\_\_\_\_

C. What population mean did you choose for Group 3? \_\_\_\_\_

D. With regard to population, what information do you know about all three of your groups?

E. Given this information, is the null hypothesis true or false? \_\_\_\_\_

#### 4. RUNNING MC4G

MC4G allows us to simulate data. We have told MC4G that we would like it to create data sets based upon our population parameters. MC4G will create 100 samples of each group. After it creates **each** set of samples (one for each group), MC4G will test the samples using the ANOVA omnibus  $F$  test. MC4G will record the proportion of times that it **rejects** the **true** null hypothesis.

This process is called Monte Carlo simulation (hence the name of the program: *Monte Carlo Analyses for up to 4 Groups*). In the five exercises that follow, you will perform Monte Carlo simulations and respond to several questions about the results. As you respond to the questions, your focus will be on the ANOVA omnibus  $F$  test, which is highlighted on screen in yellow.

Note that the screen of newer versions of MC4G varies slightly from the figures in this workbook. You should have no trouble finding the proper information, though.

Recall that Type I error is **not** a measure of the overall probability of rejecting the null hypothesis in an experiment. It is a measure of the probability of rejecting the null hypothesis given that the null hypothesis is true. In our simulations, we have selected **equal** means for our populations; therefore the null hypothesis is **true**.

You will run 10 simulations of this exercise. What is the number of times that you expect MC4G to reject the null hypothesis? Defend your answer by referring to Type I error in your explanation.

- B. Run MC4G. To run MC4G, you will click on the “RUN” button on the screen or hit the F9 key. You will run MC4G 10 times. Each time that you run MC4G, record the following information:

Group 1 Mean	Group 2 Mean	Group 3 Mean	# of Rejections	Proportion

- C. What is your *nominal* Type I error rate (also called alpha)? (Hint: what did we arbitrarily set alpha to at the beginning of the exercise?)
- D. What is the average number of times MC4G rejected a *true* null hypothesis (made a Type I error)? To get a better estimate of the actual Type I error rate, we need to average the number of rejections across the 10 Monte Carlo simulations.
- E. What is the average *actual* alpha for the simulations? \_\_\_\_\_
- F. Is this average actual Type I error rate what you expected? Why or why not?
- G. Given the fact that you know the null hypothesis is true, how can you account for rejections of the null hypothesis?

**Exercise B:**

**ANOVA without Violating the Assumption of Homoscedasticity, Unbalanced Design**

1. SETTING THE PARAMETERS FOR THE POPULATION. Set your screen to match the conditions described below. **Changes from Exercise A will be noted in bold print.**

- A. Number of Groups: Select “3 Groups (Pairwise and Orthogonal Contrasts)”
- B. Mean: Because we are examining Type I error, the null hypothesis must be true. Therefore, you may choose any value for the mean; however all means must be equal.
- C. Standard Deviation: Recall that standard deviation is the square root of variance ( $\sqrt{\sigma^2} = \sigma$ ). In this second example, we will not violate the assumption of homoscedasticity. Therefore, you may choose any value for the standard deviation; however all standard deviations must be equal.
- D. Sample Size: In this exercise, we will be using an *unbalanced* design. Therefore, choose *one* of the groups on your screen and double the sample size.
- E. Number of Samples: To make calculations simple, keep the number of samples to 100.

**MC4G: Monte Carlo Analyses for up to 4 Groups**

File Analysis Effect Sizes Options Settings Help

**NUMBER OF GROUPS FOR ANALYSIS**

- 2 Groups (Pooled and Separate Variance t-Tests)
- 3 Groups (Pairwise Contrasts vs. Alpha-adjusted)
- 3 Groups (Pairwise and Orthogonal Contrasts)
- 3 Groups (All Possible Pairwise Contrasts)
- 3 Groups (Tukey & Scheffe Contrasts)
- 4 Groups

**STATISTICS**

Group	MEAN	S.D.	S.E.	Group	MEAN	S.D.	S.E.
1				3			
2							

**MONTE CARLO OPTIONS**

Random SEED:  # SAMPLES:

Do not change seed automatically Run 1 Sample Run 10,000

**GROUP 1**

Pop Mean	Pop SD	N
0.0	1.0	25

**GROUP 2**

Pop Mean	Pop SD	N
0.0	1.0	25

**GROUP 3**

Pop Mean	Pop SD	N
0.0	1.0	50

**NUMBER OF REJECTIONS**

Hypothesis Being Tested	Avg Contrast	# Rejections	Proportion
Overall ANOVA F:			
<b>ALL PAIRWISE CONTRASTS</b>			
1v2 t-Test Contrast:			
2v3 t-Test Contrast:			
3v1 t-Test Contrast:			
At least 1 Significant Contrast:			
<b>ORTHOGONAL CONTRASTS</b>			
1v(2+3)/2 t-Test Contrast:			
2v3 t-Test Contrast:			
At least 1 Orthogonal Contrast:			
Both Orthogonal Contrasts Signif.:			

**D. Sample Size:** For Exercise B, we will use an UNBALANCED design. Choose ONE of the groups and DOUBLE the sample size.

Press F9 to Run Analysis or click -> **RUN**

2. What is the null hypothesis for this exercise? Write your answer in words and in symbols (note: we will be testing our hypothesis at  $\alpha = .05$ ).

3. RUNNING MC4G

A. You will run 10 simulations of this exercise. What is the number of times that you expect MC4G to reject the null hypothesis? Do you think that an unbalanced design will influence Type I error? Defend your answer.

B. Run MC4G. You will run MC4G 10 times. Each time that you run MC4G, record the following information:

Group 1 Mean	Group 2 Mean	Group 3 Mean	# of Rejections	Proportion

C. What is your *nominal* alpha? (What did we arbitrarily set alpha to at the beginning of the exercise?)

D. What is the average number of times MC4G rejected a *true* null hypothesis (made a Type I error)?

E. What is the average *actual* alpha for the simulations? \_\_\_\_\_

F. Is this average error rate what you expected? Why or why not?

G. What effect did having an unbalanced design have on alpha? Why do you think the effect occurred?

**Exercise C:**

**ANOVA with a Violation of the Assumption of Homoscedasticity, Balanced Design**

1. SETTING THE PARAMETERS FOR THE POPULATION. Set your screen to match the conditions described below. **Changes from Exercise B will be noted in bold print.**

- A. Number of Groups: Select “3 Groups (Pairwise and Orthogonal Contrasts)”
- B. Mean: Because we are examining Type I error, the null hypothesis must be true. Therefore, you may choose any value for the mean; however all means must be equal.
- C. Standard Deviation: Recall that standard deviation is the square root of variance ( $\sqrt{\sigma^2} = \sigma$ ). In this third example, we will violate the assumption of homoscedasticity. Therefore, choose *one* group on your screen and multiply the standard deviation by 5. (If  $\sigma = 2$ , change it to  $\sigma = 10$ .)
- D. Sample Size: In this exercise, we will be using an *balanced* design. Therefore, return your sample sizes to equal values.
- E. Number of Samples: To make calculations simple, keep the number of samples to 100.

**MC4G: Monte Carlo Analyses for up to 4 Groups**

File Analysis Effect Sizes Options Settings Help

**NUMBER OF GROUPS FOR ANALYSIS**

- 2 Groups (Pooled and Separate Variance t-Tests)
- 3 Groups (Pairwise Contrasts vs. Alpha-adjusted)
- 3 Groups (Pairwise and Orthogonal Contrasts)
- 3 Groups (All Possible Pairwise Contrasts)
- 3 Groups (Tukey & Scheffe Contrasts)
- 4 Groups

**STATISTICS**

Group	MEAN	S.D.	S.E.	Group	MEAN	S.D.	S.E.
1				3			
2							

**MONTE CARLO OPTIONS**

Random SEED:  # SAMPLES:

Do not change seed automatically Run 1 Sample Run 10,000

**GROUP 1**

Pop Mean	Pop SD	N
0.0	1.0	25

**GROUP 2**

Pop Mean	Pop SD	N
0.0	1.0	25

**GROUP 3**

Pop Mean	Pop SD	N
0.0	5.0	25

**NUMBER OF REJECTIONS**

Hypothesis Being Tested	Avg Contrast	# Rejections	Proportion
Overall ANOVA F:			
<b>ALL PAIRWISE CONTRASTS</b>			
1v2 t-Test Contrast:			
2v3 t-Test Contrast:			
3v1 t-Test Contrast:			
At least 1 Significant Contrast:			
<b>ORTHOGONAL CONTRASTS</b>			
1st Contrast:			
2v3 t-Test Contrast:			
At least 1 Orthogonal Contrast:			
Orthogonal Contrasts Signif.:			

Press F9 to Run Analysis or click -> **RUN**

**D. Sample Size:** For Exercise C, we will use a BALANCED design. All sample sizes must be equal.

**C. Standard Deviation:** For Exercise C, we will violate the assumption of equal variances. Choose ONE group and MULTIPLY the standard deviation by 5.

2. What is the null hypothesis for this exercise? Write your answer in words and in symbols (note: we will be testing our hypothesis at  $\alpha = .05$ ).

3. RUNNING MC4G

A. You will run 10 simulations of this exercise. What is the number of times that you expect MC4G to reject the null hypothesis? Do you think that violating the assumption of homoscedasticity, will influence Type I error? Defend your answer.

B. Run MC4G. You will run MC4G 10 times. Each time that you run MC4G, record the following information:

Group 1 Mean	Group 2 Mean	Group 3 Mean	# of Rejections	Proportion

C. What is your *nominal* alpha? (What did we arbitrarily set alpha to at the beginning of the exercise?)

D. What is the average number of times MC4G rejected a *true* null hypothesis (made a Type I error)?

E. What is the average *actual* alpha for the simulations? \_\_\_\_\_

F. Is this average error rate what you expected? Why or why not?

G. What effect did violating the assumption of homoscedasticity have on alpha? Why do you think the effect occurred?



**Exercise D:**

**ANOVA with a Violation of the Assumption of Homoscedasticity, Unbalanced Design #1**

1. SETTING THE PARAMETERS FOR THE POPULATION. Set your screen to match the conditions described below. **Changes from Exercise C will be noted in bold print.**
  - A. Number of Groups: Select “3 Groups (Pairwise and Orthogonal Contrasts)”
  - B. Mean: Because we are examining Type I error, the null hypothesis must be true. Therefore, you may choose any value for the mean; however all means must be equal.
  - C. Standard Deviation: Recall that standard deviation is the square root of variance ( $\sqrt{\sigma^2} = \sigma$ ). In this third example, we *will* violate the assumption of homoscedasticity. Therefore, choose *one* group on your screen and multiply the standard deviation by 5. (If  $\sigma = 2$ , change it to  $\sigma = 10$ .)
  - D. Sample Size: In this exercise, we will be using an *unbalanced* design in which we pair the *larger* standard deviation value with a *smaller* sample size. Therefore, you will double the sample size of **ONE** of the groups with the smaller standard deviation. An example of this is in the screen capture below.
  - E. Number of Samples: To make calculations simple, keep the number of samples to 100.

**MC4G: Monte Carlo Analyses for up to 4 Groups**

File Analysis Effect Sizes Options Settings Help

**NUMBER OF GROUPS FOR ANALYSIS**

- 2 Groups (Pooled and Separate Variance t-Tests)
- 3 Groups (Pairwise Contrasts vs. Alpha-adjusted)
- 3 Groups (Pairwise and Orthogonal Contrasts)
- 3 Groups (All Possible Pairwise Contrasts)
- 3 Groups (Tukey & Scheffe Contrasts)
- 4 Groups

**STATISTICS**

Group	MEAN	S.D.	S.E.	Group	MEAN	S.D.	S.E.
1				3			
2							

**MONTE CARLO OPTIONS**

Random SEED:  # SAMPLES:

Do not change seed automatically Run 1 Sample Run 10,000

**GROUP 1**

Pop Mean	Pop SD	N
0.0	1.0	25

**GROUP 2**

Pop Mean	Pop SD	N
0.0	1.0	50

**GROUP 3**

Pop Mean	Pop SD	N
0.0	5.0	25

**NUMBER OF REJECTIONS**

Hypothesis Being Tested	Avg Contrast	# Rejections	Proportion
Overall ANOVA F:			
<b>ALL PAIRWISE CONTRASTS</b>			
1v2 t-Test Contrast:			
2v3 t-Test Contrast:			
3v1 t-Test Contrast:			
At least 1 Significant Contrast:			
<b>ORTHOGONAL CONTRASTS</b>			
1v2, 3 t-Test Contrast:			
2v3 t-Test Contrast:			
Orthogonal Contrast:			
Orthogonal Contrasts Signif.:			

Press F9 to Run Analysis or click -> **RUN**

Note that Group 3 has the largest standard deviation.

D. Sample Size: For Exercise D, we will use an UNBALANCED design. You will DOUBLE the sample size of ONE of the groups with a smaller standard deviation.

2. What is the null hypothesis for this exercise? Write your answer in words and in symbols (note: we will be testing our hypothesis at  $\alpha = .05$ ).

3. RUNNING MC4G

A. You will run 10 simulations of this exercise. What is the number of times that you expect MC4G to reject the null hypothesis? Do you think that violating the assumption of homoscedasticity in this *unbalanced* design will influence Type I error? Defend your answer.

B. Run MC4G. You will run MC4G 10 times. Each time that you run MC4G, record the following information:

Group 1 Mean	Group 2 Mean	Group 3 Mean	# of Rejections	Proportion

C. What is your *nominal* alpha?

D. What is the average number of times MC4G rejected a *true* null hypothesis (made a Type I error)?

E. What is the average *actual* alpha for the simulations? \_\_\_\_\_

F. Is this average error rate what you expected? Why or why not?

G. Compare your actual alpha to the actual alpha from Exercise C. What does this information tell you about the *robustness* of ANOVA to the Violation of Homoscedasticity?

**Exercise E:**

**ANOVA with a Violation of the Assumption of Homoscedasticity, Unbalanced Design #2**

1. SETTING THE PARAMETERS FOR THE POPULATION. Set your screen to match the conditions described below. **Changes from Exercise D will be noted in bold print.**
  - A. Number of Groups: Select “3 Groups (Pairwise and Orthogonal Contrasts)”
  - B. Mean: Because we are examining Type I error, the null hypothesis must be true. Therefore, you may choose any value for the mean; however all means must be equal.
  - C. Standard Deviation: Recall that standard deviation is the square root of variance ( $\sqrt{\sigma^2} = \sigma$ ). In this third example, we *will* violate the assumption of homoscedasticity. Therefore, choose *one* group on your screen and multiply the standard deviation by 5. (If  $\sigma = 2$ , change it to  $\sigma = 10$ .)
  - D. Sample Size: In this exercise, we will be using an *unbalanced* design in which we pair the *larger* standard deviation value with the *larger* sample size. Therefore, you will assign the larger standard deviation to the group with the larger sample size. An example of this is in the screen capture below.
  - E. Number of Samples: To make calculations simple, keep the number of samples to 100.

The screenshot shows the MS4G software interface with the following settings:

- NUMBER OF GROUPS FOR ANALYSIS:** 3 Groups (Pairwise and Orthogonal Contrasts)
- STATISTICS:** Group 1 (Mean: 0.0, S.D.: 1.0, N: 25), Group 2 (Mean: 0.0, S.D.: 1.0, N: 25), Group 3 (Mean: 0.0, S.D.: 5.0, N: 50)
- MONTE CARLO OPTIONS:** # SAMPLES: 100
- GROUP 1:** Pop Mean: 0.0, Pop SD: 1.0, N: 25
- GROUP 2:** Pop Mean: 0.0, Pop SD: 1.0, N: 25
- GROUP 3:** Pop Mean: 0.0, Pop SD: 5.0, N: 50
- NUMBER OF REJECTIONS:** Overall ANOVA F: (highlighted in yellow), ALL PAIRWISE CONTRASTS (1v2, 2v3, 3v1 t-Test Contrast), At least 1 Significant Contrast: (highlighted in cyan), ORTHOGONAL CONTRASTS (1v(2+3)/2, 2v3 t-Test Contrast), least 1 Orthogonal Contrast: (highlighted in cyan), Both Orthogonal Contrasts Signif.: (highlighted in cyan)

**Callout Box:** D. Sample Size: For Exercise E, we will use an UNBALANCED design. You will DOUBLE the sample size of the group with the LARGEST standard deviation.

2. What is the null hypothesis for this exercise? Write your answer in words and in symbols (note: we will be testing our hypothesis at  $\alpha = .05$ ).

3. RUNNING MC4G

A. You will run 10 simulations of this exercise. What is the number of times that you expect MC4G to reject the null hypothesis? Do you think that violating the assumption of homoscedasticity in this *unbalanced* design will influence Type I error? Defend your answer.

B. Run MC4G. You will run MC4G 10 times. Each time that you run MC4G, record the following information:

Group 1 Mean	Group 2 Mean	Group 3 Mean	# of Rejections	Proportion

C. What is your *nominal* alpha?

D. What is the average number of times MC4G rejected a *true* null hypothesis (made a Type I error)?

E. What is the average *actual* alpha for the simulations? \_\_\_\_\_

F. Is this average what you expected? Why or why not?

G. Compare your actual alpha to the actual alpha from Exercise C. What does this information tell you about the *robustness* of ANOVA to the Violation of Homoscedasticity?

***Summary and Review (try to answer questions 1-3 without looking them up)***

1. What is the definition of Type I error?
2. How is Type I error symbolized?
3. Using an example, explain Type I error in your own words.
4. Compare the actual alpha rate to the nominal alpha in the preceding exercises. In words, describe the consequences of having the actual alpha change from the nominal alpha in such a manner? Would you describe this change as a positive (a qualitative “good”) change or a negative (a qualitative “bad”) change?
5. Compare your actual Type I error rates from Exercises D and E to those from earlier exercises. The general rule is that ANOVA is robust to violations of homoscedasticity. After completing exercises C, D, and E, do you agree with this statement? Why or why not? (In what situations does this general rule hold true?)

***Reference***

Howell, D.C. (1999). *Fundamental statistics for the behavioral sciences* (4<sup>th</sup> ed.). Cincinnati, OH: Duxbury Press.