## Supplemental material: Simulation of multi-shell fullerenes using Machine-Learning Gaussian Approximation Potential

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## Sect. S1. Description of Animations produced for the buckyonion models

To aid visualizing some of the discussions in the paper, we have produced some animations for some buckonion models.

- 1. BO300\_mov.mp4 and BO1374\_mov.mp4 files show the buckyonion formation process for BO300 and BO1374 respectively.
- 2. BO\_growthProcess.mp4 show the clustering and growth process for 540 randomly distributed atoms within a 720-atom fullerene isomer.

## Sect. S2. Supporting Tables and Figures

| Models                    | Layers | Ring size: |     |     |    |   |
|---------------------------|--------|------------|-----|-----|----|---|
|                           |        | 5          | 6   | 7   | 8  | 9 |
| <b>BO</b> <sub>60</sub>   | $s_1$  | 12         | 20  | 0   | 0  | 0 |
| <b>BO</b> <sub>300</sub>  | $s_1$  | 15         | 22  | 3   | 0  | 0 |
|                           | $s_2$  | 26         | 74  | 14  | 0  | 0 |
| <b>BO</b> <sub>540</sub>  | $s_1$  | 26         | 50  | 12  | 1  | 0 |
|                           | $s_2$  | 62         | 85  | 28  | 8  | 2 |
| <b>BO</b> <sub>840</sub>  | $s_1$  | 6          | 1   | 0   | 2  | 1 |
|                           | $s_2$  | 34         | 84  | 20  | 3  | 0 |
|                           | $s_3$  | 68         | 148 | 40  | 11 | 0 |
| <b>BO</b> 1374            | $s_1$  | 27         | 34  | 14  | 3  | 1 |
|                           | $s_2$  | 25         | 189 | 13  | 2  | 0 |
|                           | $s_3$  | 50         | 292 | 36  | 3  | 0 |
| <b>BO</b> <sub>2160</sub> | $s_1$  | 15         | 20  | 10  | 2  | 1 |
|                           | $s_2$  | 36         | 291 | 25  | 0  | 5 |
|                           | $s_3$  | 55         | 380 | 32  | 2  | 5 |
| BO <sub>3774</sub>        | $s_1$  | 31         | 91  | 17  | 4  | 4 |
|                           | $s_2$  | 47         | 183 | 26  | 5  | 4 |
|                           | $s_3$  | 112        | 333 | 80  | 18 | 6 |
|                           | $s_4$  | 179        | 422 | 123 | 24 | 7 |

TABLE S1: Ring-size analysis for the buckyonion models.

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FIG. S1: Plots of the energy difference ( $\Delta E$ ) against the Conjugate gradient (CG) iterations carried out using VASP on BO<sub>300</sub> (a) and BO<sub>540</sub> (b). SIESTA was used for the BO<sub>300</sub> (c) and BO<sub>840</sub> (d) models as well. The insets in a-d represent the models before and after CG relaxation.



FIG. S2: Starting from (a) an initially random atomic configuration with a cylindrical shape, (b) a capped multi-wall carbon nanotube was formed. The figure in (c) show the nanotube from the z-axis with the cap sliced off. The system has 840 atoms and periodic boundaries only in the z-axis.



FIG. S3: Figure showing some of the buckyonions models simulated in this work



FIG. S4: Growth Process from  $C_{720}$  isomer with 540 atoms randomly distributed C atoms. The outermost shell (green) remained with 720 atoms at the end of the simulation. The heptagons in the outermost shell are coloured in black.



FIG. S5: Figure showing the distribution of the charge density for one end of  $BO_{300}$  model in -xy planar-slices starting with a pentagon (blue) in the inner shell (red) in a(i) to the first slice in the outer shell (green) showing a hexagonal structure (yellow) in a(v). The figures in b(i - v) and c(i - v) show the heat-map and contour values of the charge density for 5 slices from a(i) to a(v). The entire system is shown in a(iii) and charge distribution in the other end of the system is discussed in main manuscript.