# Online Appendix
# A Simple Model to Teach Business Cycle Macroeconomics for Emerging Market and Developing Economies

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## The Two-Period AG Model

### The consumers

The consumer's problem consists of choosing $C$, $C'$, $N$, $N^{'}$, and $B'$ to:[1]

$$\max_{C,C',N,N',B'} \quad ln\left(C - \frac{N^2}{2}\right) + \beta ln\left(C' - \frac{N'^2}{2}\right)$$

s.t.:

$$C + B' = wN + \pi$$

$$C' = w'N^{'} + \pi' + (1+r)B'$$

where the initial stock of net assets ($B$) is given. We assume the initial and the terminal conditions $B = 0$ and $B'' = 0$.

The Lagrangian function is

$$\mathcal{L} = ln\left(C - \frac{N^2}{2}\right) + \beta ln\left(C' - \frac{N'^2}{2}\right) + \lambda[wN + \pi - C - B'] + \lambda'[w'N^{'} + \pi + (1+r)B' - C']$$

The first-order conditions (FOCs) are

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{1}{C - \frac{N^2}{2}} - \lambda = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial C'} = \frac{\beta}{C' - \frac{N'^2}{2}} - \lambda' = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial N} = -\frac{N}{C - \frac{N^2}{2}} + \lambda w = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial N'} = -\frac{\beta N'}{C' - \frac{N'^2}{2}} + \lambda' w' = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial B'} = -\lambda + \lambda'(1+r) = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = wN + \pi - C - B' = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda'} = w'N' + \pi' + (1+r)B' - C' = 0 \tag{7}$$

**The firms**

Assuming a Cobb-Douglas production function, the firm's current and future profits $(\pi, \pi')$ are

$$\pi = zK^{\alpha}N^{1-\alpha} - wN - [K' - (1-\delta)K] \tag{8}$$

$$\pi' = z'K'^{\alpha}N'^{1-\alpha} - w'N' + (1-\delta)K' \tag{9}$$

where $0 < \alpha < 1$ is the capital share, $0 < \delta < 1$ is the depreciation rate, and we impose the terminal condition $K'' = 0$.

Given the initial stock of capital $(K)$, the unconstrained optimization problem consists of choosing $N$, $N'$, and $K'$ to maximize the firm's value —the present value of profits $(V)$. That is,

$$\max_{N,N',K'} \quad V = \pi + \frac{\pi'}{1+r}$$

The FOCs are

$$\frac{\partial V}{\partial N} = (1-\alpha)zK^{\alpha}N^{-\alpha} - w = 0 \tag{10}$$

$$\frac{\partial V}{\partial N'} = \frac{(1-\alpha)z'K'^{\alpha}N'^{-\alpha} - w'}{1+r} = 0 \tag{11}$$

$$\frac{\partial V}{\partial K'} = -1 + \frac{\alpha z'K'^{\alpha-1}N'^{1-\alpha} + (1-\delta)}{1+r} = 0 \tag{12}$$

**Market-clearing conditions**

Markets clear in the present and the future periods if

$$N^s = N^d \tag{13}$$

$$N^{s'} = N^{d'} \tag{14}$$

$$Y^s = Y^d \tag{15}$$

$$Y^{s'} = Y^{d'} \tag{16}$$

**Solution**

Given that $r = r^w$, the solution of the model is

$$N^* = w^* = \Big[(1-\alpha)zK^\alpha\Big]^{\frac{1}{1+\alpha}} \tag{17}$$

$$Y^* = zK^\alpha\Big[(1-\alpha)zK^\alpha\Big]^{\frac{1-\alpha}{1+\alpha}} \tag{18}$$

$$K'^* = \Big(\frac{\alpha z'}{r+\delta}\Big)^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)z' \tag{19}$$

$$I^* = K'^* - (1-\delta)K \tag{20}$$

$$N'^* = w'^* = (1-\alpha)z'\Big(\frac{\alpha z'}{r+\delta}\Big)^{\frac{\alpha}{1-\alpha}} \tag{21}$$

$$Y'^* = z'K'^{*\alpha}N'^{*1-\alpha} \tag{22}$$

$$C^* = \frac{1}{(1+\beta)(1+r)}\Big[\frac{(N'^*)^2}{2}\Big] + \frac{1}{(1+\beta)}(\alpha Y^* + (1-\delta)K) + \frac{2+\beta}{1+\beta}\Big[\frac{(N^*)^2}{2}\Big] \tag{23}$$

$$NX^* = Y^* - C^* - I^* \tag{24}$$

$$B'^* = NX^* \tag{25}$$

$$C'^{*} = \frac{1+2\beta}{1+\beta}\left[\frac{(N'^{*})^2}{2}\right] + \frac{\beta(1+r)(\alpha Y^{*} + (1-\delta)K)}{1+\beta} + \frac{\beta(1+r)}{1+\beta}\left[\frac{(N^{*})^2}{2}\right] \tag{26}$$

### Supply and demand functions

Using equations (1) and (3), we can derive the current labor supply

$$N^s = w \tag{27}$$

Equation (10) yields the current labor demand

$$N^d = \left[\frac{(1-\alpha)zK^{\alpha}}{w}\right]^{\frac{1}{1-\alpha}} \tag{28}$$

It is easy to prove that $\partial N^d/\partial z > 0$, $\partial N^d/\partial K > 0$, $\partial N^d/\partial w < 0$. Recall that the current production function does not depend on the real interest and constitutes the current output supply:

$$Y^s = zF(K, N) = zK^{\alpha}N^{1-\alpha} \tag{29}$$

where, by assumption, $\partial Y^s/\partial z > 0$, $\partial Y^s/\partial K > 0$, and $\partial Y^s/\partial N > 0$. The equations for the current demand for consumption goods, the future capital stock, and the current demand for investment goods ((23), (19), (20)) yield

$$C^d = \frac{1}{(1+\beta)(1+r)}\left[\frac{(N'^{*})^2}{2}\right] + \frac{1}{(1+\beta)}(\alpha Y^{*} + (1-\delta)K) + \frac{2+\beta}{1+\beta}\left[\frac{(N^{*})^2}{2}\right]$$

$$I^d = \left(\frac{\alpha z'}{r+\delta}\right)^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)z' - (1-\delta)K$$

It is straightforward to prove that $\partial C^d/\partial z > 0$, $\partial C^d/\partial z' > 0$, $\partial C^d/\partial r < 0$, $\partial C^d/\partial K > 0$, $\partial I^d/\partial z' > 0$, $\partial I^d/\partial r < 0$, and $\partial I^d/\partial K < 0$. Putting all of these elements together we have

$$Y^d = C^d(\underset{-}{r}, \underset{+}{z}, \underset{+}{z'}, \underset{+}{K}) + I^d(\underset{-}{r}, \underset{+}{z'}, \underset{-}{K}) + NX \tag{30}$$

Finally, rewriting equations (27)-(30) in general form, jointly with the parity of interest rates, we have the equations shown in the second section:

$$N^s = w$$

$$N^d = N^d(\underset{-}{w}, \underset{+}{z}, \underset{+}{K})$$

$$Y^s = zF(\underset{+}{K}, \underset{+}{N})$$

$$Y^d = C^d(\underset{-}{r}, \underset{+}{z}, \underset{+}{z'}, \underset{+}{K}) + I^d(\underset{-}{r}, \underset{+}{z'}, \underset{-}{K}) + NX$$

$$r = r^w$$

# Other Applications and Extensions

Next we show other applications and simple extensions of the AG model. We give priority to simplicity and view these applications as complementary explanations to others given in the macroeconomic literature for EMDEs.

## Output Drops and Capital Outflows

Some economists have observed that large reductions in the flow of international capital into an EMDE are accompanied by deep recessions. Kaminsky, Reinhart and Végh (2005) find that capital flows to EMDEs are procyclical.[2] How can we understand a sharp capital outflow, its corresponding current account reversal, and an output drop in the NP model?[3] Consider an economy that experiences a persistent fall in TFP ($\Delta z < 0$, $\Delta z' < 0$, $|\Delta z| > |\Delta z'|$). Based on our analysis, we can show that the effects on output and other endogenous variables have just the opposite signs that we see in Figure 2 in the main text.

To see the effects on the current account more clearly, let us use an alternative diagram. We can express the current account surplus as the difference between national saving and investment: $CA = S - I$, where $S = Y - C$. Keeping in mind equations (**??**) and the current demand for consumption goods, we can formulate national saving as a function of the real interest rate, current and future TFPs, among other determinants. Recalling the expressions for the current demand for investment and the country interest rate (repeated here for convenience), we have:

$$S = S(\underset{+}{r}, \underset{+}{z}, \underset{-}{z'}, ...) \tag{31}$$

$$I = I(\underset{-}{r}, \underset{+}{z'}, ...) \tag{32}$$

$$r = r^w + p(\underset{-}{z'}) \tag{33}$$

These expressions are represented in Figure 8, which initially shows a current account deficit ($CA_1 = S_1 - I_1 < 0$) at the real interest rate $r_1$. The persistent fall in TFP contracts both the saving curve and the investment curve, generating net capital outflows from the economy and a current account surplus. This surplus is enlarged by the jump in the real interest rate, which is a result of the rise in the country risk premium. At interest rate $r_2$, the current account surplus is positive and large ($CA_2 = S_2 - I_2 > 0$).

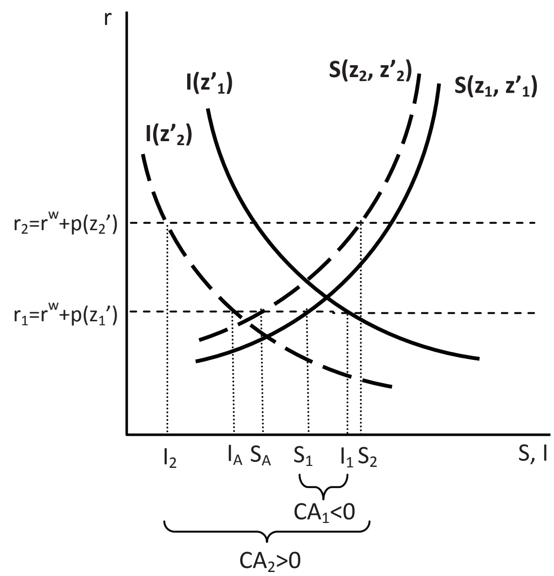


Figure 1: Output drops and capital outflows.

The graph is also useful to distinguish the effects of a persistent TFP reduction of the same magnitude on the current account of an advanced SOE and understand why current account balances are highly countercyclical in EMDEs.[4] If the risk premium is not affected by future TFP in an advanced SOE, then there is no change in the country interest rate. At rate $r_1$, the saving and investment curves shift such that the economy runs a current account surplus given by $CA_A = S_A - I_A > 0$. Thus, the graph shows that the current account surplus is larger in the EMDE than in the advanced SOE ($CA_2 = S_2 - I_2 > S_A - I_A = CA_A$).

## The Cyclical Behavior of Prices

Rand and Tarp (2002) find that the cyclical component of the CPI is negatively correlated with (detrended) output in most developing countries. In other words, prices are countercyclical in EMDEs.[5] Let us introduce a money market to the model using a similar approach as in Williamson (2013). For simplicity, the money supply ($M^s$) is exogenously controlled by the central bank, and the money demand ($M^d$) depends on total income and the interest rate as follows:

$$M^S = M \tag{34}$$

$$M^d = PL(\underset{+}{Y}, \underset{-}{r}) \tag{35}$$

where $P$ is the domestic price level determined in this market (see Figure 10).[6] In Figure 10, we represent the money supply as a vertical line at the initial level $M_1$. The money demand is represented by the straight upward-sloping line $P = M/L(Y,r)$, with slope $1/L(Y,r)$.

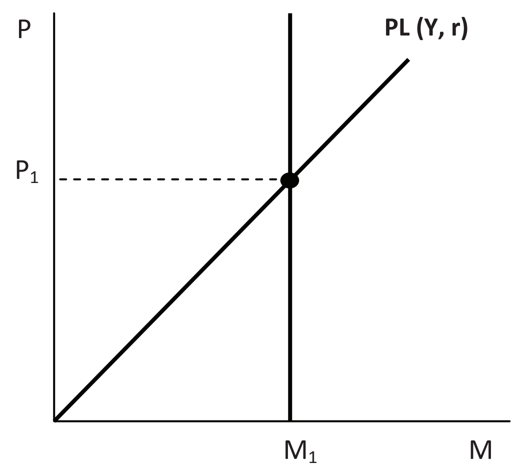


Figure 2: The equilibrium in the money market and the determination of the price level.

As previously shown, a persistent TFP increase raises income and lowers the interest rate. Unambiguously, money demand expands. This can be observed in Figure 11. Given a constant money supply, the subsequent excess demand at the initial price level ($P_1$) is only satisfied by a fall in prices ($P_1 > P_2$). If the economy is systematically shocked by persistent TFP changes, this result yields to a negative comovement between output and prices.[7]

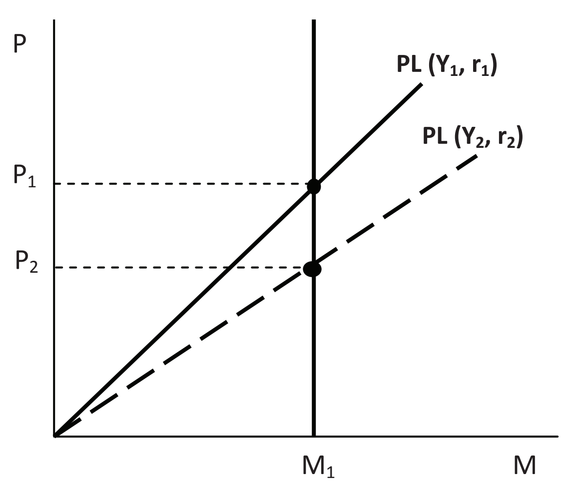


Figure 3: Countercyclical prices.

## Currency Depreciations and Output

Recent empirical evidence reported by Cordella and Gupta (2014) shows that currencies in EMDEs tend to depreciate when GDP growth is low and appreciate when GDP growth is high. That is, nominal exchange rates –defined as the relative price of a foreign currency in terms of the domestic currency– are countercyclical (currencies are procyclical) in most of the developing world.[8] One possibility is that the nominal exchange rate is indirectly affected by output fluctuations, which in turn are caused by a third factor such as TFP changes. If this mechanism is empirically correct, then how can we explain the negative correlation between currency depreciations and output fluctuations using any of the models? Following Williamson (2013), assume that the purchasing power parity condition holds:[9]

$$P = eP^* \tag{36}$$

where $P^*$ is the foreign price level and $e$ is the nominal exchange rate defined such that an increase represents a depreciation of the domestic currency. If we plug equation (36) in (35), we obtain

$$M^d = eP^*L(\underset{+}{Y}, \underset{-}{r}) \tag{37}$$

Equations (34) and (37) determine the nominal exchange rate in the money market under a flexible exchange rate regime.[10] Figure 12 shows the interaction between supply and demand and the determination of the nominal exchange rate.

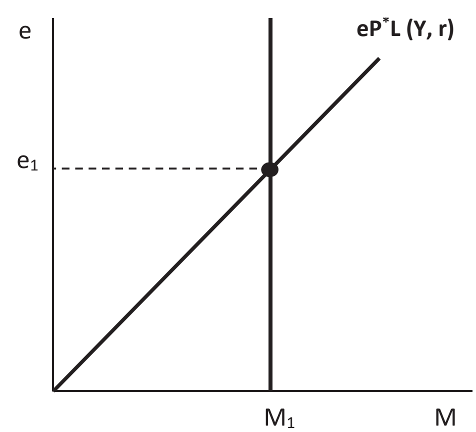


Figure 4: The money market and the determination of the nominal exchange rate.

Consider a persistent reduction in TFP that contracts output (in both models) and raises the interest rate (in the NP model). The reduced income leads to a lower demand for money and, as a result, a nominal depreciation of the currency (see Figure 13).

Therefore, if the economy is regularly hit by persistent TFP changes, we should expect a negative comovement between the nominal exchange rate and output.

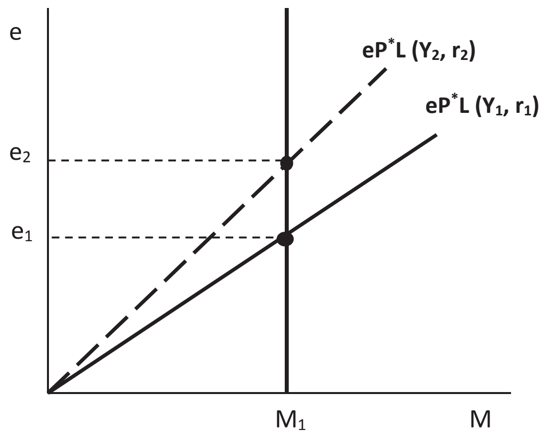


Figure 5: The negative comovement between the nominal exchange rate and output.

# Suggested Exercises

1. **A change in the world interest rate.** Show that a reduction in the world interest rate does not produce all of the predictions that we obtain from a persistent TFP fall in the AG model.

2. **The macroeconomic effects of natural disasters.** According to Raddatz (2007), climatic disasters (which include floods, droughts, extreme temperatures, and wind storms) result in declines in real GDP (per person) of 2% in low-income countries. This rate looks modest in absolute value, but the declines are significantly greater than the median growth rates of such countries (0.4%). Analyze the macroeconomic effects of a climatic disaster that destroys part of the current stock of capital using the AG model.

3. **Procyclical prices and monetary policy: The cases of Malaysia and Peru.** In the fourth section, we show that if we extend the model with a money market, a persistent TFP increase that raises income and lowers the interest rate will contract money demand, causing a countercyclical behavior of the price level. However, there exists evidence that prices are procyclical in certain EMDEs such as Malaysia (Rand and Tarp 2002) and Peru (Castillo, Montoro and Tuesta 2007). In addition, several works conclude that monetary policy tends to be procyclical in EMDEs (see Kaminsky, Reinhart and Végh 2005, Duncan 2014). Show that if we include a procyclical monetary rule that positively links the money supply to

output, $M^s = M^s(\underset{+}{Y})$, it is possible to observe an increase in the price level and, therefore, procyclical prices.

4. **Does risk matter?** Fernández-Villaverde, Quintana, Ramírez and Uribe (2011) show that changes in risk, measured by the volatility of the real interest rate, can account for the behavior of output, consumption, investment, and hours worked in Argentina, Brazil, Ecuador, and Venezuela. Suppose we extend the AG model by replacing the output demand equation by

$$Y^d = C^d(\underset{-}{r}, \underset{-}{\sigma_r}, ...) + I^d(\underset{-}{r}, \underset{-}{\sigma_r}, ...) + NX$$

where $\sigma_r$ is the standard deviation of the real interest rate. The intuition behind the negative link is that an increase in $\sigma_r$ raises precautionary savings and, thus, lowers consumption. In addition, physical capital becomes riskier and, hence, investment falls. Analyze the effects of an increase in $\sigma_r$ on the main endogenous variables.

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# Notes

[1]Strictly speaking, we should include $N^s$ and $N^{s'}$ in the consumer's problem and $N^d$ and $N^{d'}$ in the firm's problem. To lighten the notation, we impose the market clearing condition and just use $N$ and $N^{'}$.

[2]There is a related literature of sudden stops and crises (see, e.g., Calvo, 2003). This hypothesizes a causal link from sudden stops to output drops. We restrict the analysis to the cases in which sudden stops are the result of a persistent negative TFP shock, most likely caused by problems of external solvency and the anticipation of a financial turmoil.

[3]In practice, the countercyclicality of the current account balance does not necessarily mirror the procyclicality of the capital account balance. It depends on the cyclical behavior of the change in international reserves.

[4]Assuming the same fraction $\theta$ in the working-capital requirement.

[5]Backus and Kehoe (1992) show international evidence of the countercyclicality of prices after World War II in industrialized economies.

[6]To simplify the analysis, we set the inflation rate equal to zero; thus, the nominal equals the real interest rate.

[7]In this case, we assume that the monetary policy is acyclical or not used to stabilize prices. One of the suggested exercises in Appendix C addresses the (unusual) case of procyclical prices that might be understood if the central bank performs a (highly) procyclical monetary policy. Procyclical monetary policies are frequently observed in EMDEs (see, e.g., Kaminsky, Reinhart and Végh 2005).

[8]There also exists a large literature that tries to explain and verify the negative link by establishing a causal relationship from the nominal exchange rate to output in EMDEs. This is sometimes called the contractionary devaluation hypothesis. See, e.g., Kim and Ying (2007) and the references therein.

[9]Taylor (2002) finds that PPP holds in the long run for Argentina, Brazil, and Mexico during the 1870-1990 period. Calderón and Duncan (2003) find that PPP holds in Chile for the 1810-2002 period.

[10]In this case, $e$ is endogenous. Naturally, the quantity of money ($M$) becomes endogenous under a fixed exchange rate.