INPP @OHIO UNIVERSITY INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Faddeev Approach to (d,p) Reactions As Tool to Study Exotic Nuclei

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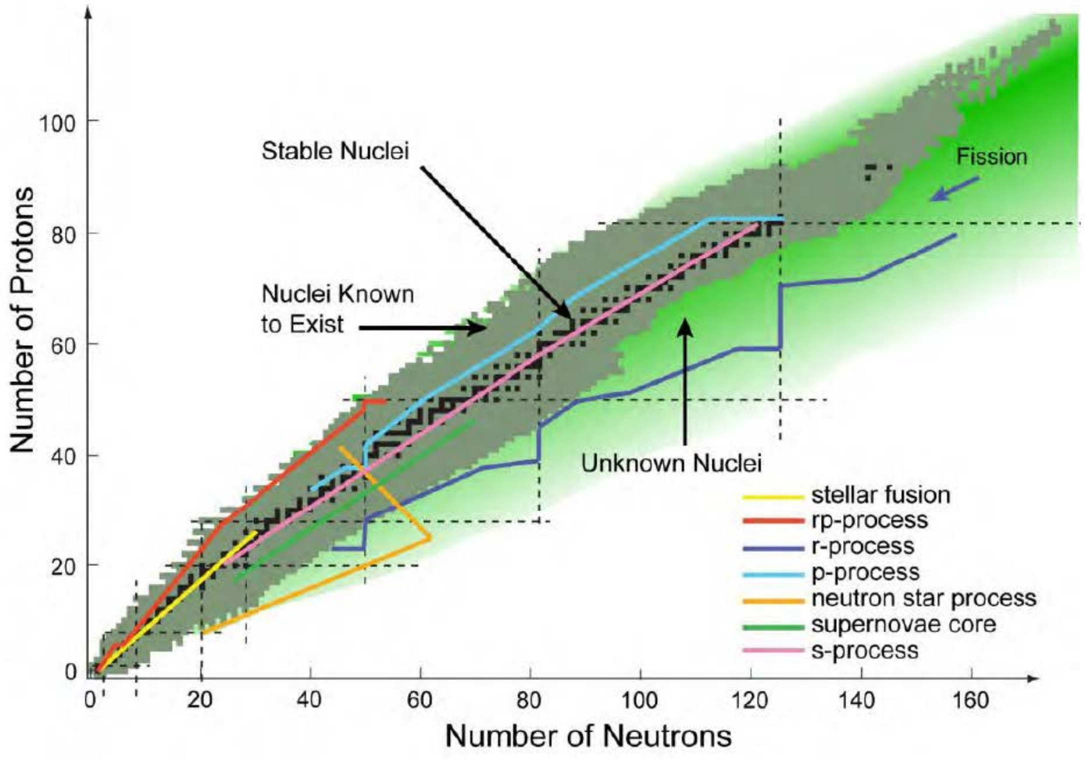
Astrophysics:
Stellar Evolution

Nuclear Physics:
Nuclear Synthesis



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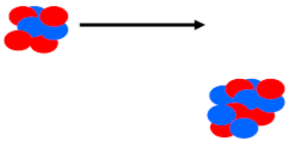
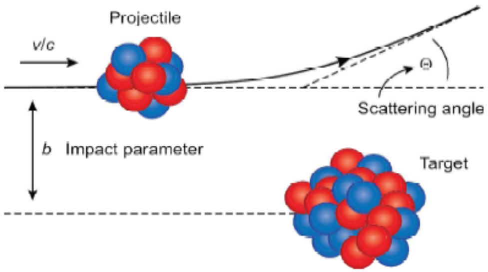
Why Reactions?

Elastic:



Traditionally used to extract optical potentials, rms radii, density distributions

Inlastic:



Traditionally used to extract electromagnetic transitions or nuclear deformations.

Transfer:

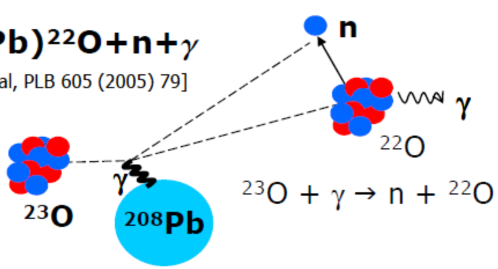
Traditionally used to extract spin, parity, spectroscopic factors
example: $^{132}Sn(d,p)^{133}Sn$

Traditionally used to study two-nucleon correlations and pairing
example: $^{11}Li(p,t)^{9}Li$

Breakup:

$^{23}O(Pb,Pb)^{22}O+n+\gamma$

[Nociforo et al, PLB 605 (2005) 79]



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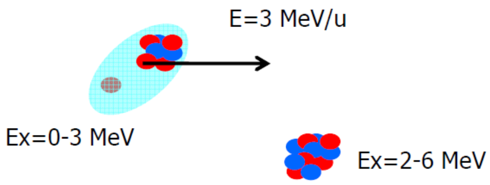
Challenge:

- In the continuum, theory can solve the few-body problem exactly.
- Reaction theories need to map onto the many-body problem!

It is not easy to develop effective field theories in reactions:



There is not always a clear separation of scales.

Direct Reactions with Nuclei:

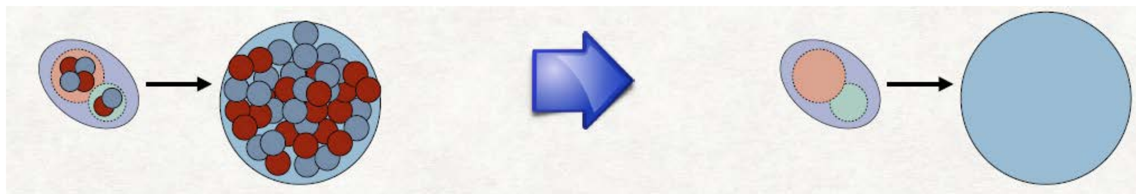
- Elastic & inelastic scattering
- Few-particle transfer (stripping, pick-up)
- Charge exchange
- Knockout

World of few-body methods

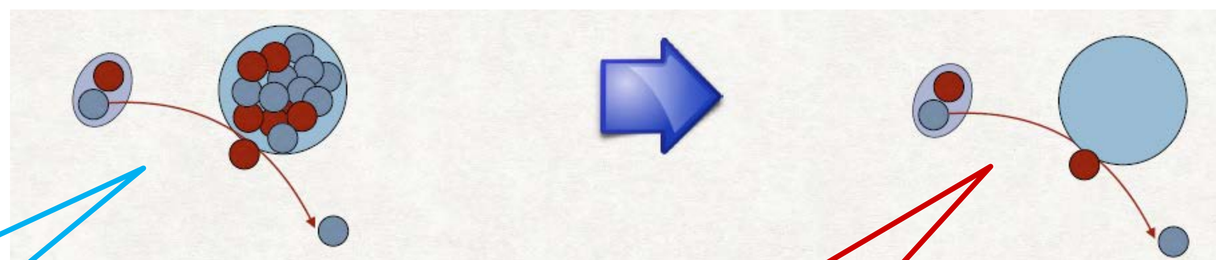
Faddeev Ansatz in Nuclear Reactions

Nuclear reactions study e.g.

Cluster structure in nuclei:



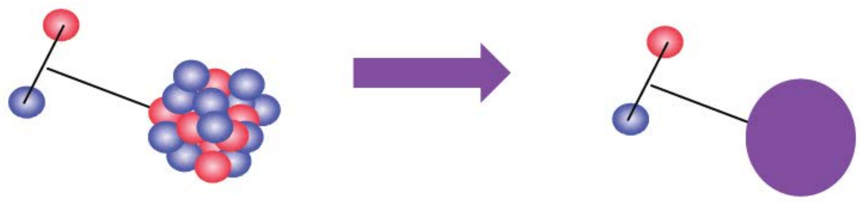
Single particle motion of the "last" nucleon in a nucleus near the dripline



Many-body problem

Few-body problem

Example: (d,p) Reactions:
Reduce Many-Body to Few-Body Problem



Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

np interaction (V_{np})

Effective (optical) potentials p+A and n+A ($V_{nA} + V_{pA}$)

Effective Three-Body Problem

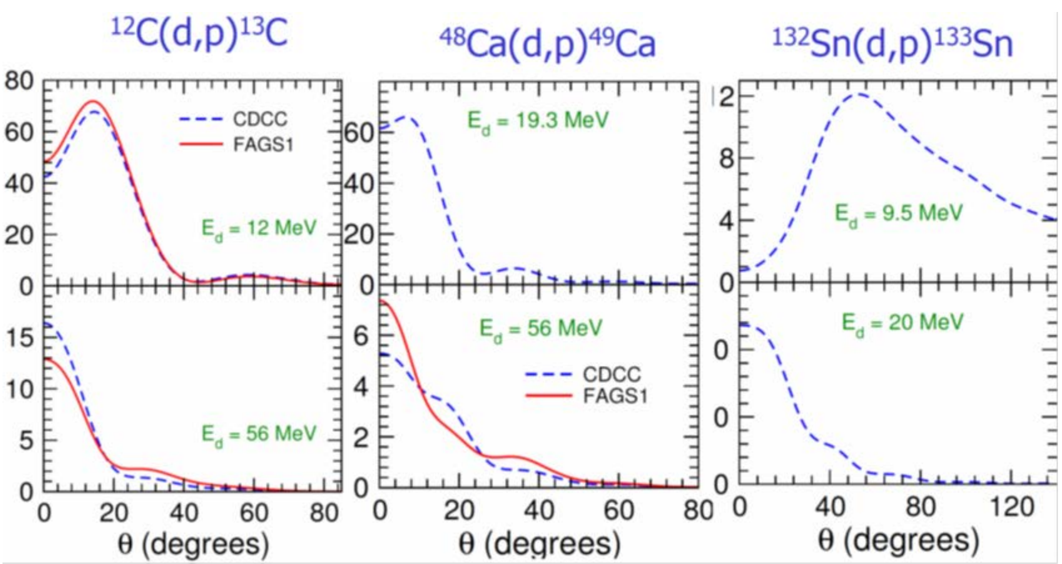
(d,p) Reactions as three-body problem

Faddeev equations: Exact solution of the three-body problem

Momentum space solution pioneered by:

Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Courtesy: F.M. Nunes

Issues:

- **current momentum space implementation of Coulomb interaction (shielding) does not converge for Z ≥ 20**
- **CDCC and Faddeev do not always agree in breakup up channels**

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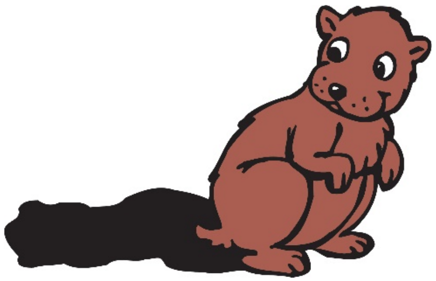
(d,p) Reactions: Reduce Many-Body to Few-Body Problem

“Shadow” ?



Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

Challenges & Opportunities

Nucleon-nucleon interaction well known:

today: chiral interactions, ‘high precision’ potentials

Effective proton (neutron) interactions:

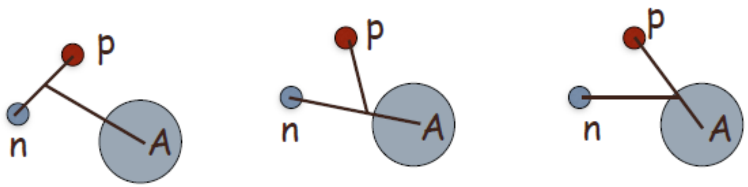
- purely phenomenological optical potentials fitted to data
- optical potentials with theoretical guidance
- microscopic optical potentials
- ab initio derivation of effective interaction being attempted

Solving the effective few-body problem

Faddeev equations:

Expand three-body wave function in three Jacobi systems

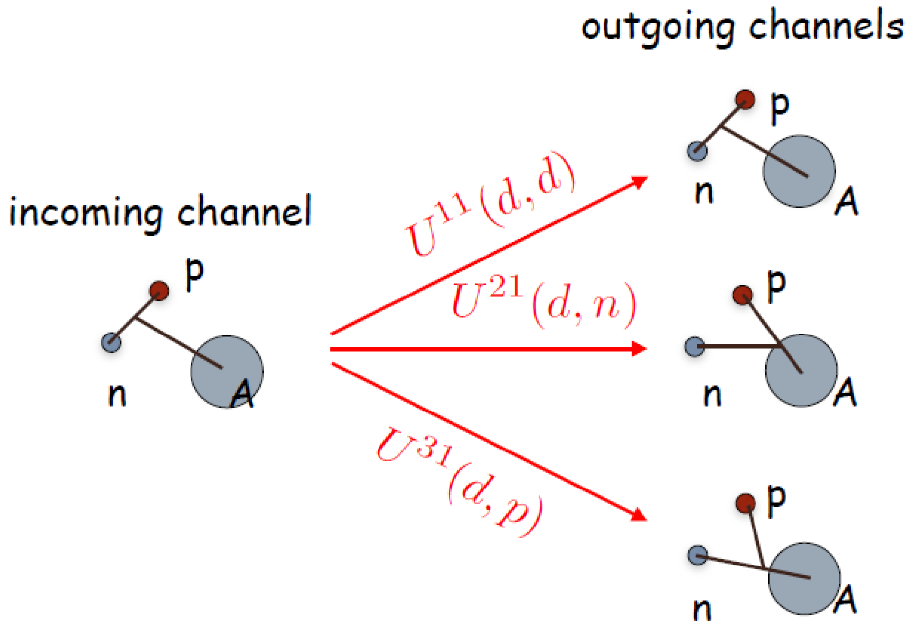
$$|\Psi\rangle = |\psi_{np}\rangle + |\psi_{nA}\rangle + |\psi_{pA}\rangle$$



Each sub-system specifies particular boundary conditions:
e.g. elastic scattering, transfer reaction

Momentum space: no difference if interactions are local or nonlocal

Solving Faddeev equations



Faddeev-AGS equations: [Alt et al., Nucl.Phys. B2 (1967) 167]

$$U^{ij} = \bar{\delta} G_0^{-1}(E) + \sum_k \bar{\delta}_{ik} t_k(E) G_0(E) U^{kj}$$

Cross sections: $\sigma_{j\leftarrow i} \propto |\langle \Psi_j | U^{ij} | \Psi_i \rangle|^2$

Considerations for two-body subsystems

Are described in momentum space by solutions of LS integral equations:

$$t_i\,(E) = V + V\,G_0\,(E)\,t_i\,(E)$$

Two-body potential V :

$V(p',p)$ ≡ non-separable

$V(p',p) = \sum_{nm} h_n(p')\,\lambda_{mn}\,h_m(p)$ ≡ separable

EST scheme: basis expansion of potential in scattering wave functions

$$V^{separable} = VP\,(PVP)^{-1}\,PV$$

EST: PRC 8, 46 (1973)
PRC 9, 1780 (1974)

With $P = \sum_n |\phi_{E_n,p_n}\rangle\langle\phi_{E_n,p_n}|$ and $|\phi_{E_n,p_n}\rangle = |p_n\rangle + G_0^{(+)}(E_n)V|\phi_{E_n,p_n}\rangle$

t-matrix $t_i(E) = \sum_{mn} |h_m^i\rangle\tau_{mn}^i(E)\langle h_n^i|$

In two-body system identical observables, PRC 88, 064608 (2013)

Why separable expansion?

Explicit inclusion of Coulomb interaction in momentum space (without screening):

Formulation of Faddeev equations in Coulomb basis instead of plane wave basis (separable interactions needed)

A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Target excitations:
Including specific excited states → **separable interactions preferred**

Faddeev-AGS equations with separable interactions

Matrix representation

$$\begin{bmatrix} X^{11} \\ X^{21} \\ X^{31} \end{bmatrix} = \begin{bmatrix} 0 \\ Z^{21} \\ Z^{31} \end{bmatrix} + \begin{bmatrix} 0 & Z^{12}\,\tau^{(2)} & Z^{13}\,\tau^{(3)} \\ Z^{21}\,\tau^{(1)} & 0 & Z^{23}\,\tau^{(3)} \\ Z^{31}\,\tau^{(1)} & Z^{(32)}\,\tau^{(2)} & 0 \end{bmatrix} \begin{bmatrix} X^{11} \\ X^{21} \\ X^{31} \end{bmatrix}.$$

Three components for three different subsystems

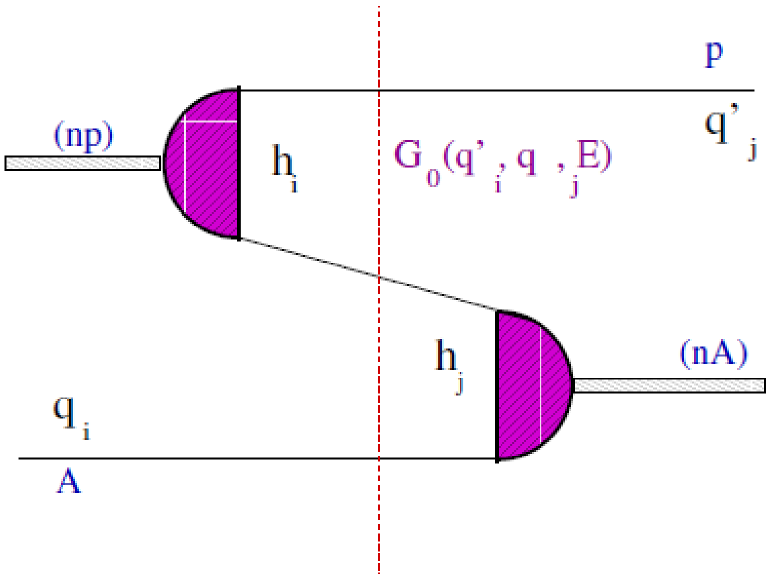
Radial part of
transition operators

τ^i generalized propagators

$Z^{(ij)}$ generalized transition amplitudes

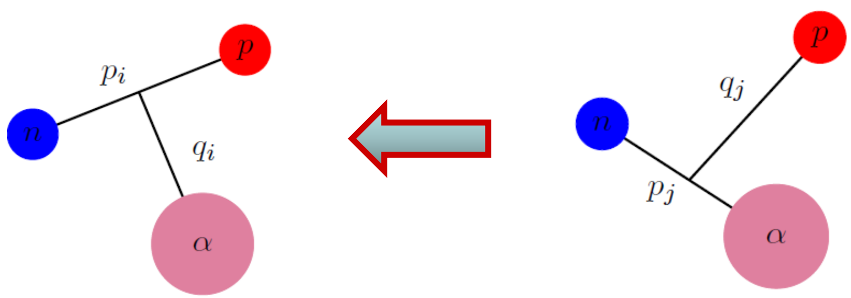
Bound state Faddeev equations have similar structure but are a set of homogeneous integral equations

‘transition amplitudes’ $Z^{(ij)}(q_i,q_j')$



Contains three-body dynamics

Describes transition between channels (j) and (i)



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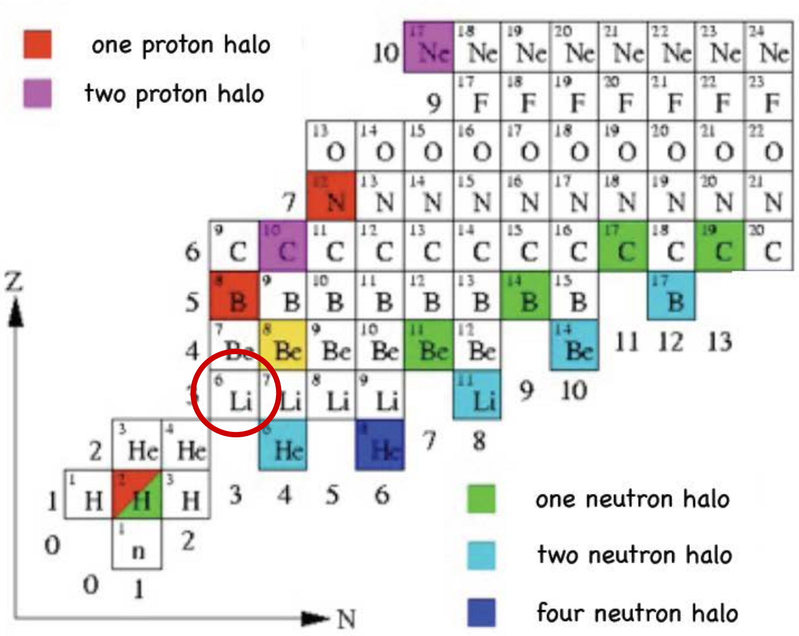
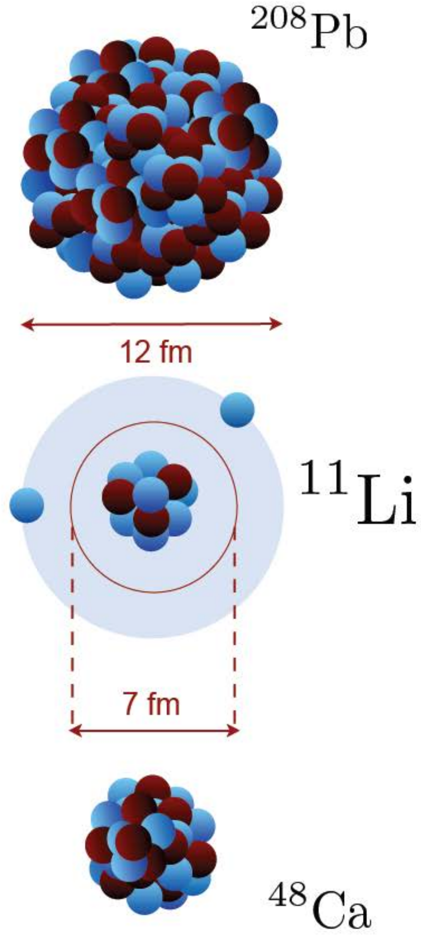
Suitable nucleus for development work: ^{6}Li as n+p+α system





Alpha tightly bound: $E_4[\alpha]$ = -28.3 MeV
n & p loosely bound: $E_3[^6\text{Li}]$ = -3.7 MeV

Several Faddeev type calculations exist → ideal for benchmarking

Two-body interactions

Deuteron channel: **CD-Bonn Potential** ($\chi^2/N \approx 1$)

[R. Machleidt, Phys. Rev. C63, 024001 (2001)]

n/p – α channel ($S_{1/2}$, $P_{1/2}$, $P_{3/2}$): **Bang Potential**

[J. Bang *et al.*, Nucl. Phys. A405, 126 (1983)]

I. J. Thompson et al, .Phys. Rev. C, 61, 024318 (2000)

$$v(r) = -\frac{V_0}{1+\exp\left(\frac{r-R_0}{a_0}\right)} + \left(\frac{1}{r}\right)\frac{d}{dr}\frac{V_{so}}{1+\exp\left(\frac{r-R_{so}}{a_{so}}\right)}\,\mathbf{l}\cdot\boldsymbol{\sigma}$$

$V_0 = 44$ MeV $a_0 = 0.65$ fm, $R_0 = 2$ fm, $V_{so} = 40$ MeVfm
$a_{so} = 0.37$ fm,$R_{so} = 1.5$ fm

Projecting out Pauli-forbidden state

- $S_{1/2}$ partial wave supports Pauli-forbidden state $|\phi\rangle$ (unphysical)
- To project out the state $|\phi\rangle$: $V \longrightarrow \tilde{V} = V + \lim_{\Gamma\to\infty} |\phi\rangle\, \Gamma\, \langle\phi|$
- Corresponding t-matrix:

$$\tilde{t}(E) = t(E) - (E - H_0)\, \frac{|\phi\rangle\langle\phi|}{(E-E_b)[1-(E-E_b)/\Gamma]} (E - H_0)$$

- Γ limit can be taken analytically

$$\tilde{t}(p', p; E) = t(p', p; E) - (E - E_{p'})\, \frac{\phi(p')\phi(p)}{E-E_b} (E - E_p)$$

Can be generalized to arbitrary number of Pauli-forbidden states

Particularly well suited for momentum space Faddeev equations

Projecting out Pauli-forbidden state

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- Corresponding t-matrix:

$$\tilde{t}(E) = t(E) - (E - H_0)\,\frac{|\phi\rangle\langle\phi|}{(E-E_b)[1-(E-E_b)/\Gamma]}(E - H_0)$$

- Γ limit can be taken analytically

$$\tilde{t}(p', p; E) = t(p', p; E) - (E - E_{p'})\,\frac{\phi(p')\phi(p)}{E-E_b}(E - E_p)$$

Separable Expansion

- Separable expansion of V also supports bound state $|\phi\rangle$, must be removed
- Convenient approach: expand $\tilde{V}$ instead of V
- Advantages: (1) straightforward implementation and (2) does not increase rank

Convergence of the ^{6}Li binding energy

we developed 2 codes for our benchmark (Phys.Rev. C96 (2017) no.6, 064003)

CD-Bonn np potential

label	rank	E_3 [MeV]
EST5-1	5	-**3.78**47
EST5-2	5	-**3.78**48
EST5-3	5	-**3.78**55
EST6-1	6	-**3.78**67
EST6-2	6	-**3.78**68
EST6-3	6	-**3.78**71
EST7-1	7	**-3.7867**
EST7-2	7	**-3.7867**
EST7-3	7	**-3.7867**
EXACT:		**-3.787**

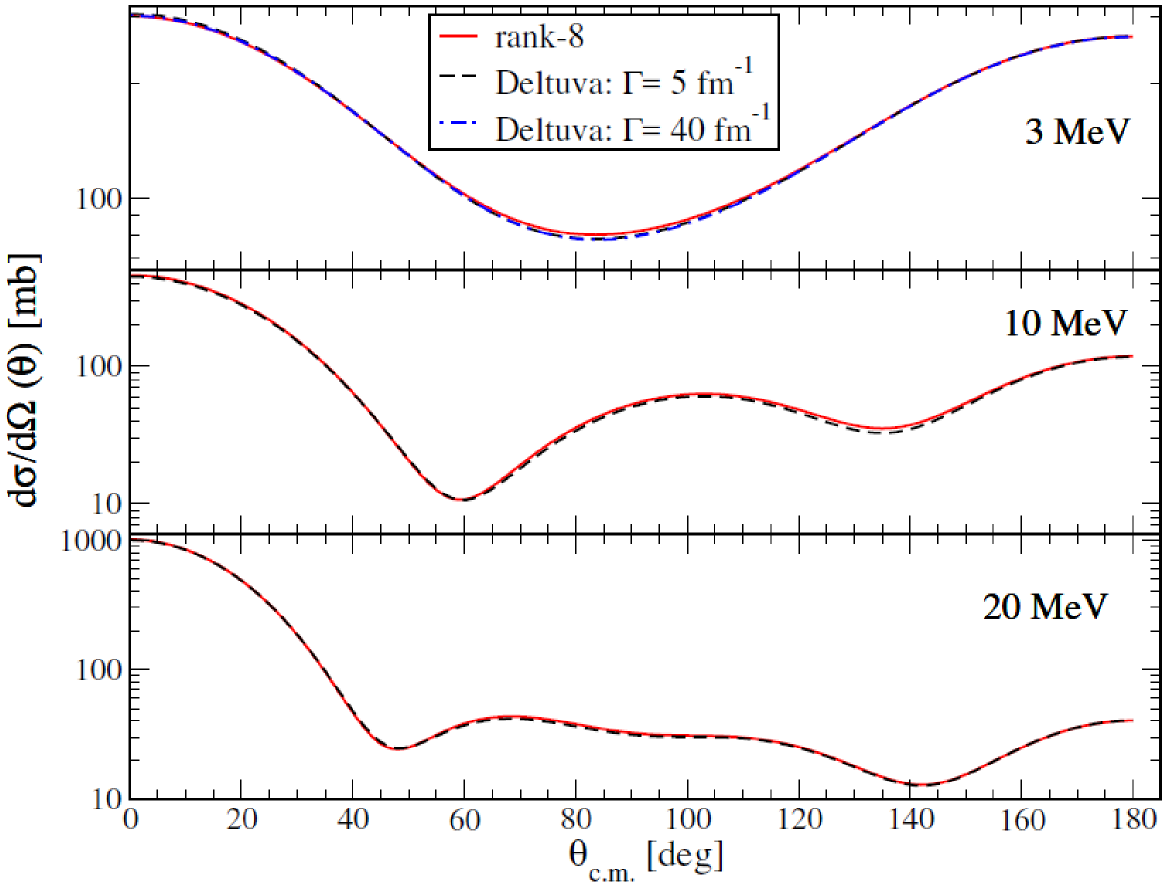
Bang $n\alpha$ potential

label	rank	E_{3b} [MeV]
EST6-1	6	-**3.78**56
EST6-2	6	-**3.78**52
EST6-3	6	-**3.78**52
EST7-1	7	-**3.786**8
EST7-2	7	-**3.786**4
EST7-3	7	-**3.786**7
EST8-1	8	-**3.78**70
EST8-2	8	-**3.78**70
EST8-3	8	-**3.78**66
EXACT:		**-3.787**

♦ **Four significant figures** stable w.r.t (1) choice of $\{E_m\}$ and (2) rank; agrees with **exact** calculation; with Coulomb $E_3 = -2.777$ MeV

Elastic scattering: d+α

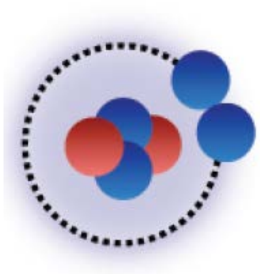
Benchmark our code with Deltuva's code:



Coulomb force not included

n+p+α system at low energy

Reminder: ^{6}He ≡ n+n+α system ≡ **2 neutron halo system**

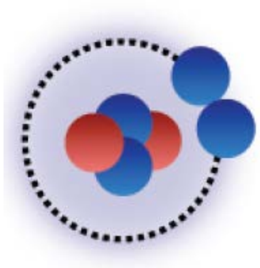


Borromean system

many studies on universal behavior

n+p+α system at low energy

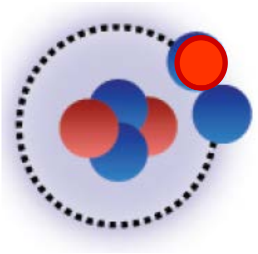
Reminder: ^{6}He ≡ n+n+α system ≡ **2 neutron halo system**



Borromean system

many studies on
universal behavior

^{6}Li



≡ n+p+α system

`deuteron' halo ?

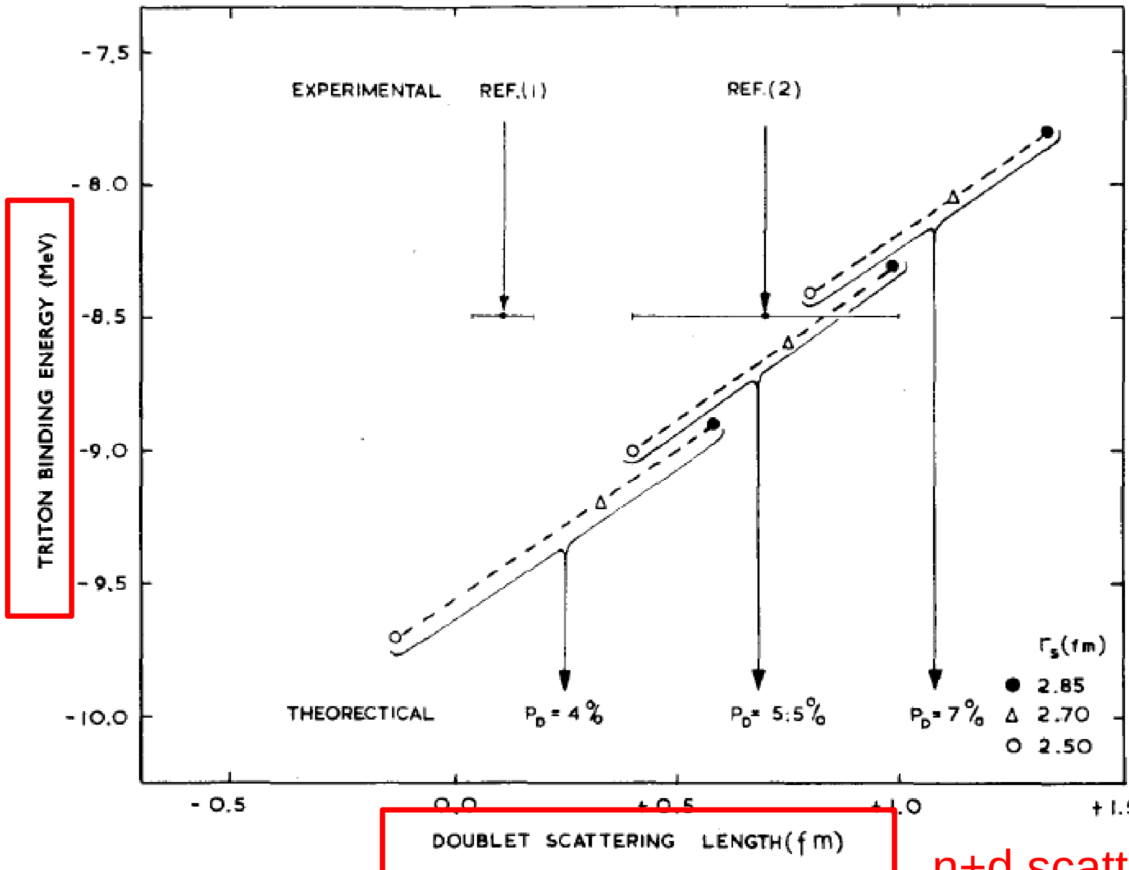
universal behavior at low energies?

n+n+p system:

A.C.Phillips, Nucl. Phys. A **107** 209 (1968)

well known phenomenon in low energy n+d system:

Phillips line



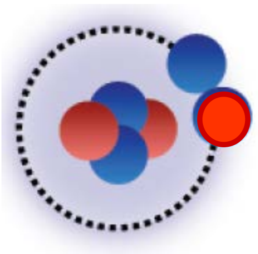
n+d scattering

Many studies using conventional forces or EFT methods see this

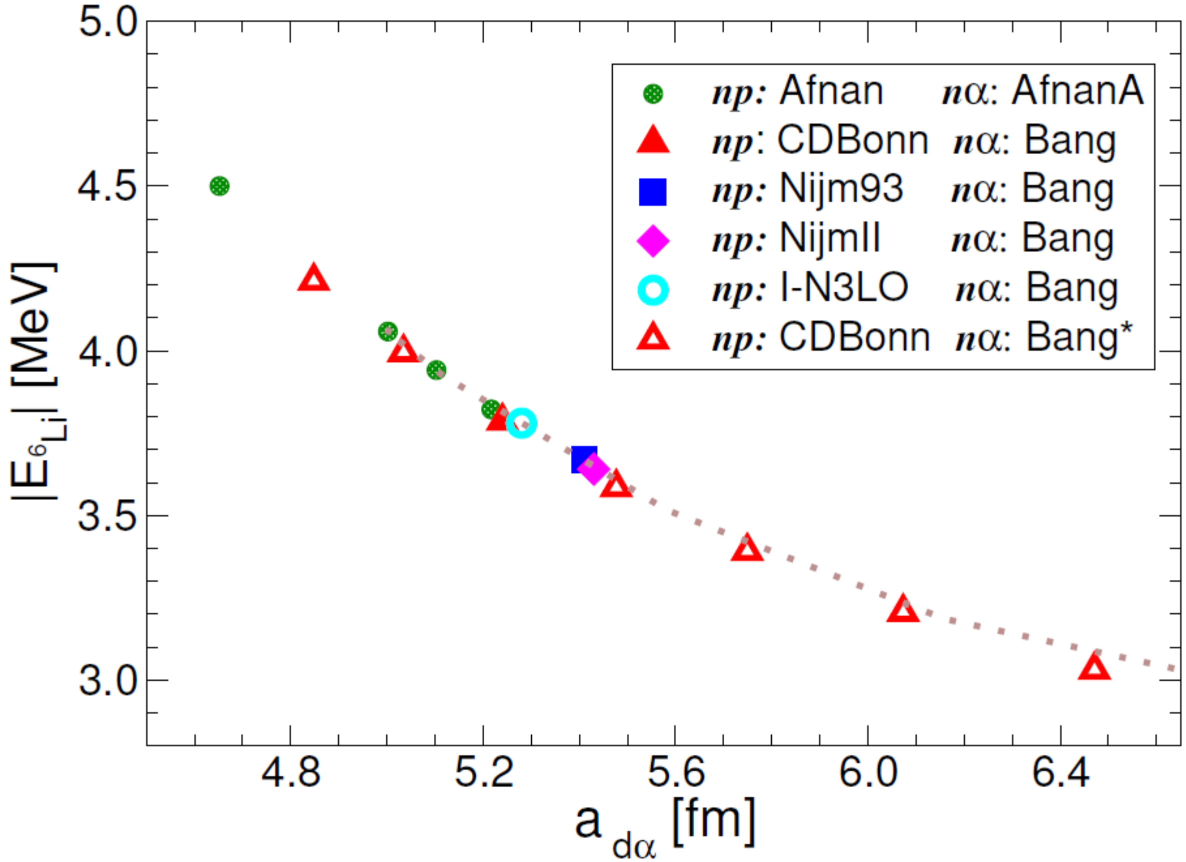
What about the n+p+a system

n+p+α system at low energy

^{6}Li



Binding energy vs scattering length in ^{6}Li channel



one parameter curve

independent of

- np interaction
- nα interaction

Summary

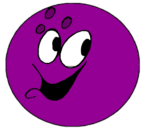
Benchmark of Faddeev equations for bound states and elastic scattering

- for n+p+α system directly
- with separable expansion of interactions successfully completed.

Projecting out Pauli-forbidden states:

- Procedure easily implemented in momentum space Faddeev equations
- For non-separable and separable forces alike
- Straightforward generalization for systems with several Pauli-forbidden states (heavy nuclei)

Universal behavior of the low energy n+p+α system



Benchmark of Faddeev equations for breakup scattering in progress

Next: Faddeev-AGS equations in Coulomb basis

Outlook and Challenges

Can we test this picture?

Scattering **d+α** can be calculated as many body problem by NCSM+RGM

Heavier nuclei:

Is this too simple?

Better?

Nucleus can be deformed

Our approach can solve the effective three-body problem for (d,p) reactions for nuclei across the nuclear chart

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