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Theory Challenges for describing Nuclear Reactions

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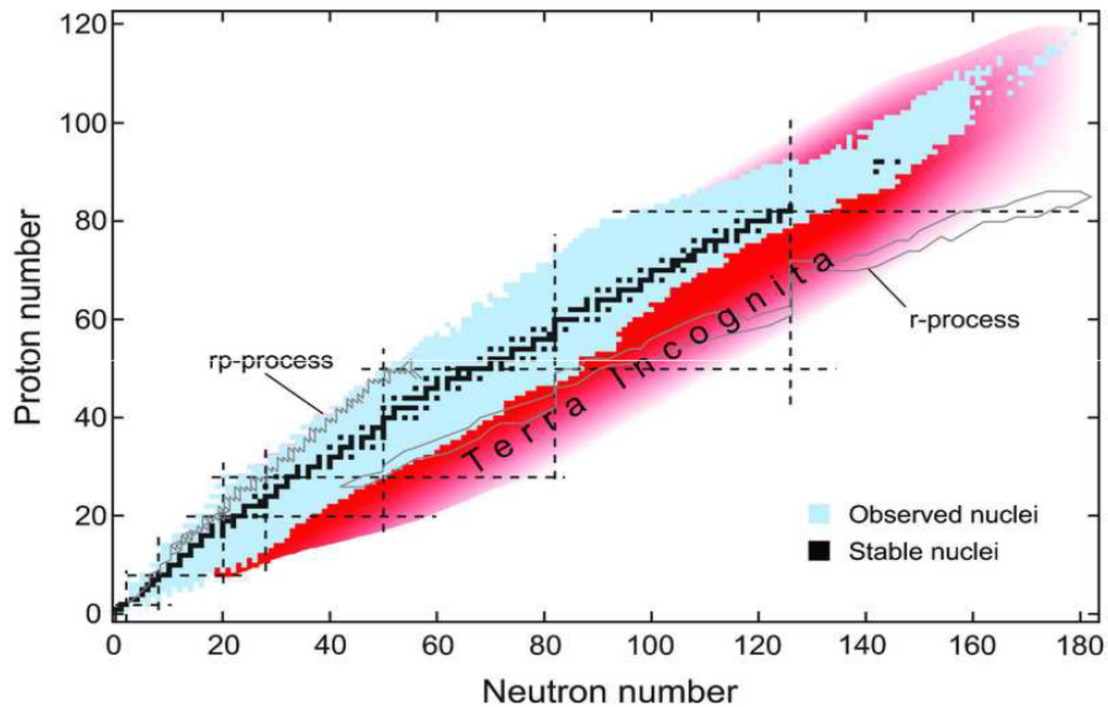
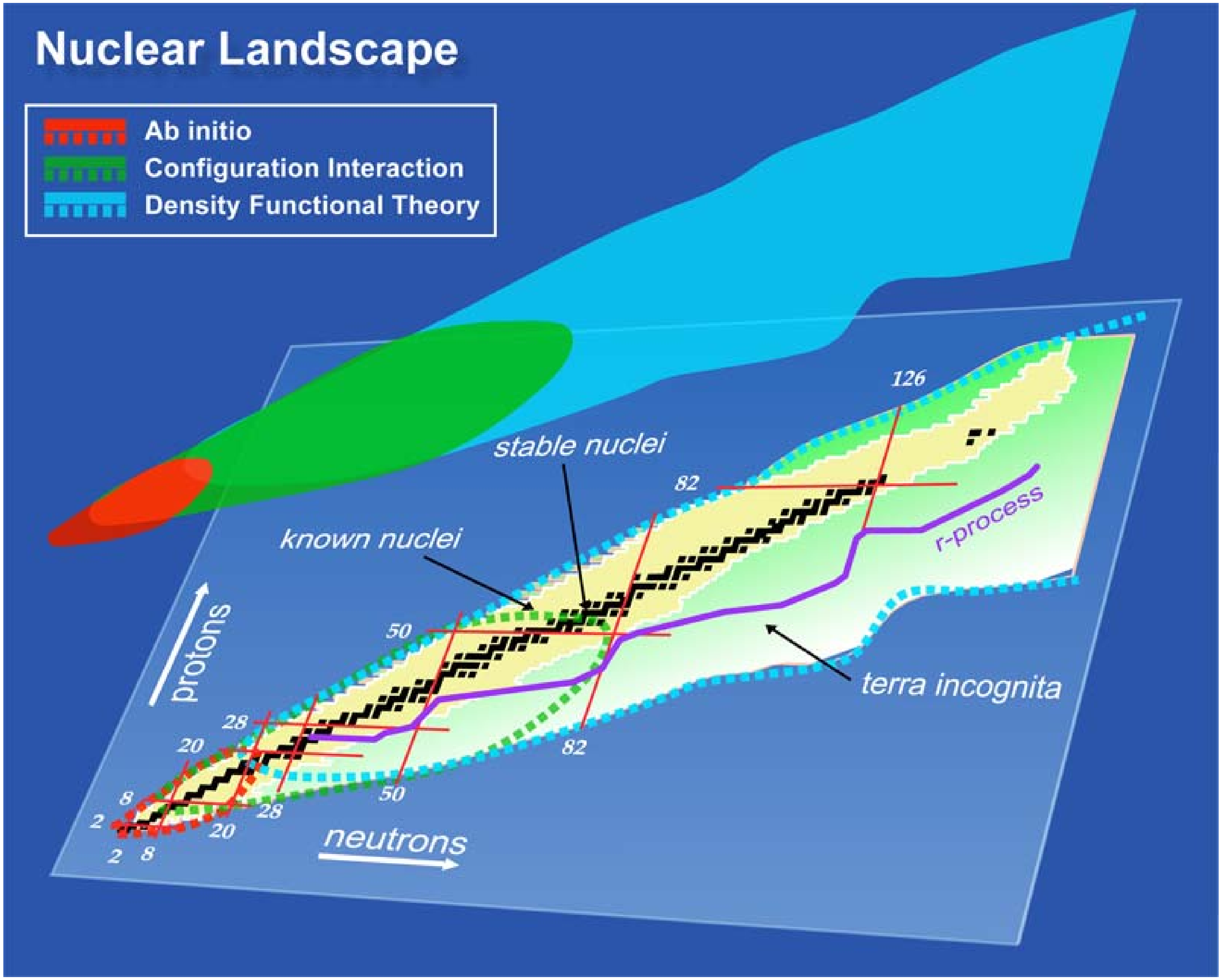
Astrophysics:
Stellar Evolution

Nuclear Physics:
Nuclear Synthesis





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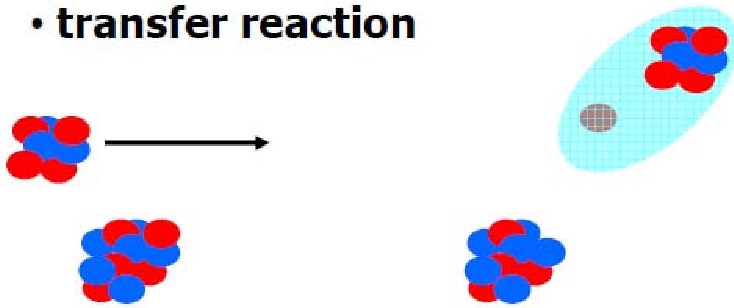
Indirect Methods: Nuclear Reactions

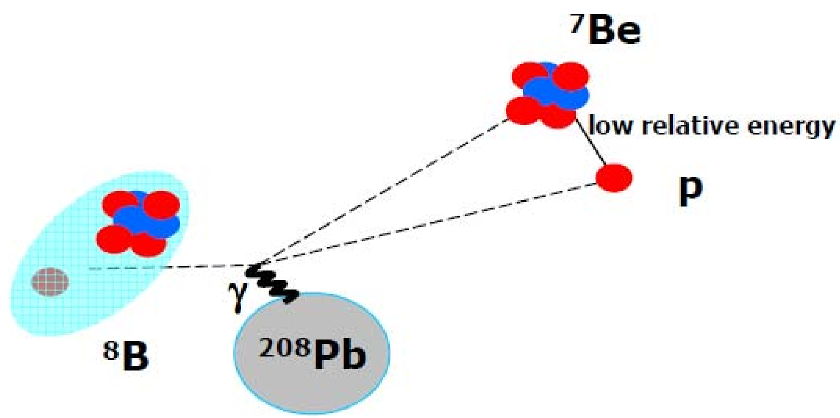
• direct measurement $^{7}Be(p,\gamma)^{8}B$

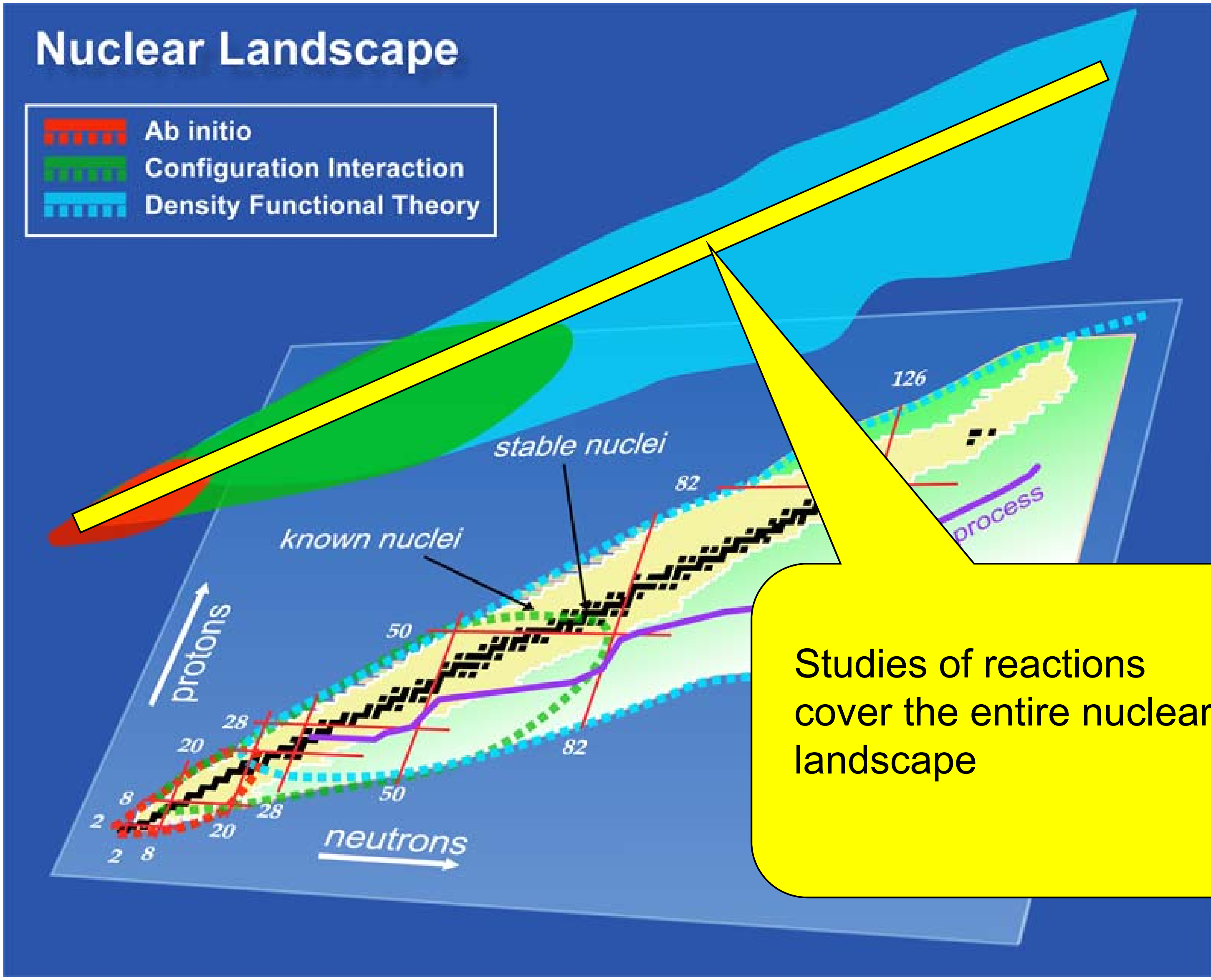
• transfer reaction



$^{14}N(^{7}Be,\ ^{8}B)^{13}C$

• Coulomb dissociation

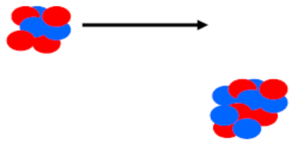




Studies of reactions cover the entire nuclear landscape

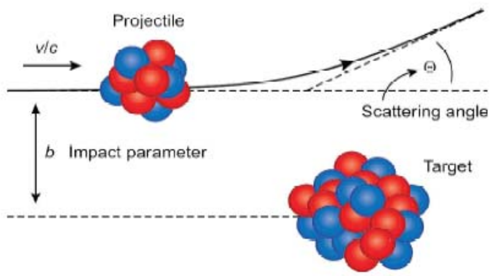
Why Reactions?

Elastic:



Traditionally used to extract optical potentials, rms radii, density distributions

Inlastic:



Traditionally used to extract electromagnetic transitions or nuclear deformations.

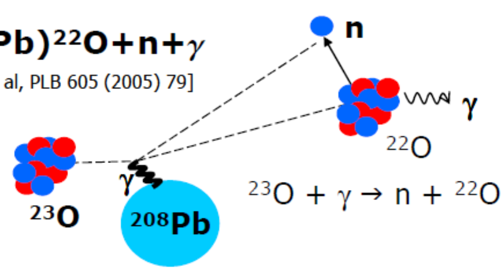
Transfer:

Traditionally used to extract spin, parity, spectroscopic factors
example: ^{132}Sn(d,p)^{133}Sn

Traditionally used to study two-nucleon correlations and pairing
example: ^{11}Li(p,t)^{9}Li

Breakup:

^{23}O(Pb,Pb)^{22}O+n+γ
[Nociforo et al, PLB 605 (2005) 79]



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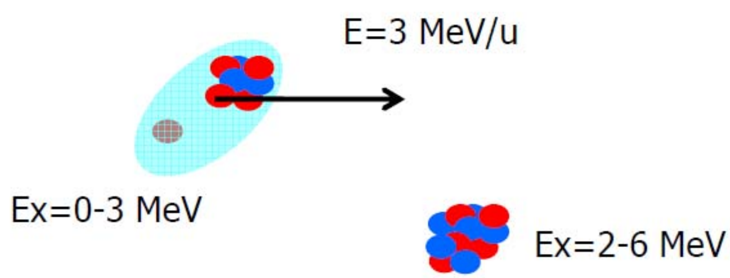
Challenge:

In the continuum theory can solve the few-body problem exactly.

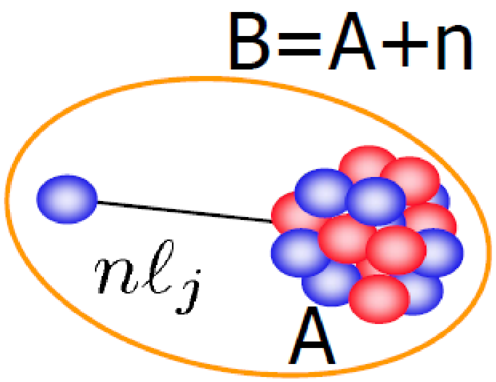
Reaction theories need to map onto the many-body problem!

It is not easy to develop effective field theories in reactions:



There is not always a clear separation of scales.

Often attempted meeting point between Structure and Reactions:



overlap function

$$I_{I_A:I_B}(\mathbf{r}) = \langle \Phi^A_{I_A}(\xi_A) | \Phi^B_{I_B}(\xi_A, \mathbf{r}) \rangle$$

spectroscopic factor (S_{nlj}):
norm of overlap function

Extracting spectroscopic factors can provide some handle on structure theory.

However: spectroscopic factors are not observables

Instead: asymptotic normalization coefficients

Direct Reactions with Nuclei:

- Elastic & inelastic scattering
- Few-particle transfer (stripping, pick-up)
- Charge exchange
- Knockout

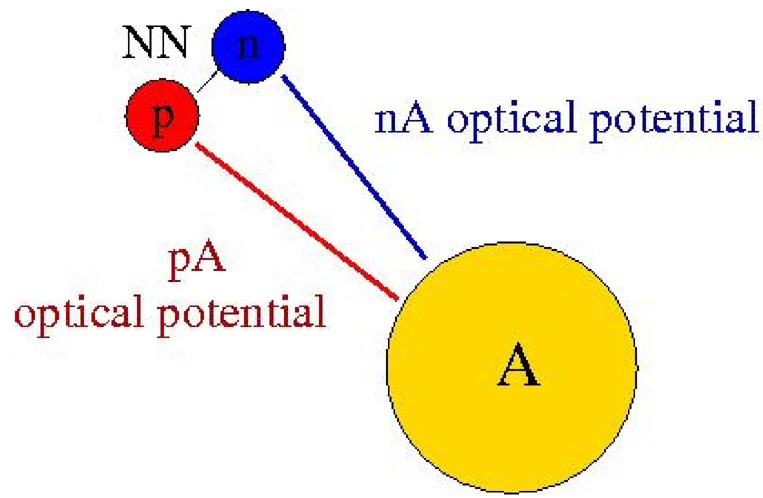
Task:

- **Isolate important degrees of freedom in a reaction**
- **Keep track of important channels**
- **Connect back to the many-body problem**

(d,p) Reactions: Effective Three-Body Problem

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

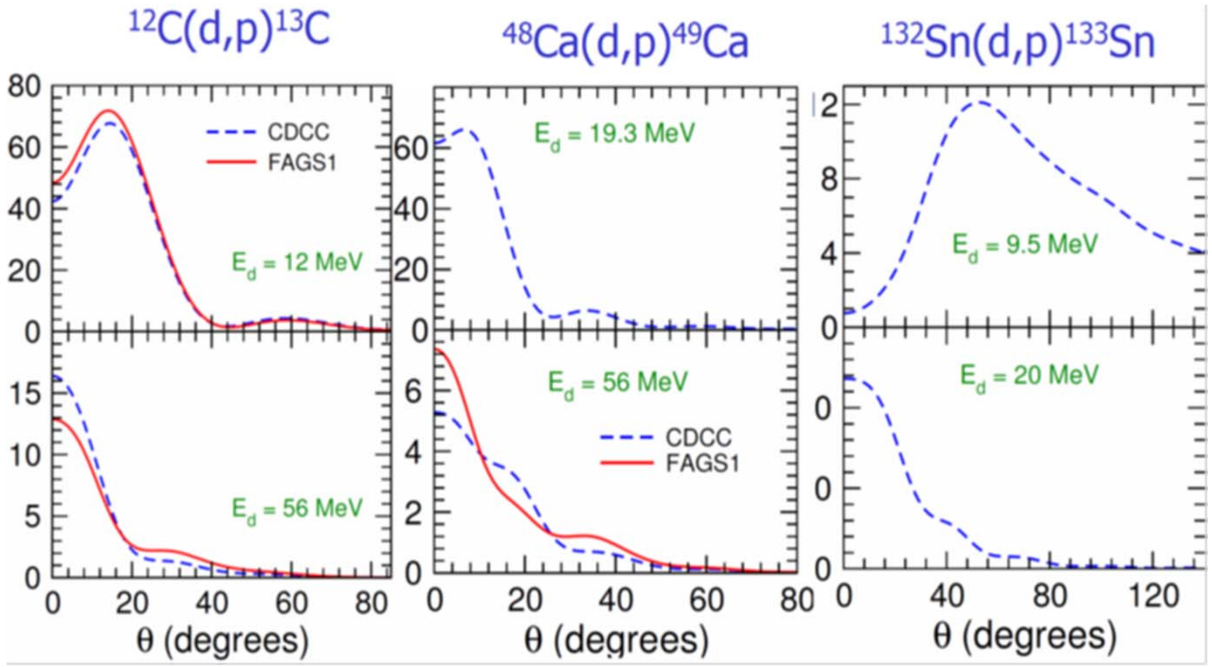


Many-body problem?

(d,p) Reactions as three-body problem

Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issues:

- **current momentum space implementation of Coulomb interaction (shielding) does not converge for Z ≥ 20**
- **CDCC and FAGS do not agree in breakup up**

Courtesy: F.M. Nunes

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Generalization of Faddeev-AGS approach needed :

Theory: **A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov**,
Phys.Rev. C86 (2012) 034001

Generalized Faddeev formulation of (d,p) reactions with:
(a) explicit inclusion of the Coulomb interaction (no screening)
(b) explicit inclusion of target excitations

- **Explicit inclusion of Coulomb interaction**:
  Formulation of Faddeev equations in Coulomb basis instead of plane waves
  Needs: Formulation with separable interactions to avoid pinch singularities

- **Target excitations:**
  Including specific excited states
  → Formulation with separable interactions also useful.

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Faddeev Equations in Coulomb Basis :

Three-body scattering state: $$|\Psi\rangle = |\phi\rangle + \sum_{a=1}^{3} g_0^C(E) V_a^S |\Psi\rangle$$

$$g_0^C(E) = \left(E - H_0 - V_{pA}^C + i0\right)^{-1}$$

Faddeev components ψ_a

$$|\Psi\rangle \equiv \sum_a |\psi_a\rangle = \sum_a \underbrace{\delta_{a,1}|\phi_1\rangle + g_0^C(E) V_a^S |\Psi\rangle}_{|\psi_a\rangle}$$

e.g. $|\phi_1\rangle$ is initial state $A + (pn)$

Faddeev-type equations

$$\begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) V_1^S \sum_a |\psi_a\rangle \\ |\psi_2\rangle = \qquad g_0^C(E) V_2^S \sum_a |\psi_a\rangle \\ |\psi_3\rangle = \qquad g_0^C(E) V_3^S \sum_a |\psi_a\rangle \end{cases} \Rightarrow \begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) t_1 \sum_{a\neq 1} |\psi_a\rangle \\ |\psi_2\rangle = \qquad g_0^C(E) t_2 \sum_{a\neq 2} |\psi_a\rangle \\ |\psi_3\rangle = \qquad g_0^C(E) t_3 \sum_{a\neq 3} |\psi_a\rangle \end{cases}$$

proof of principle work is here

A(d,p)B Reaction using Coulomb Green's functions

Coulomb-modified Green's function

$$V^C = \frac{Z_1 Z_2 \alpha^2}{r} \qquad g_0^C \equiv [E - H_0 + i0 - V_{pA}^C]^{-1}$$

$$g_0^C = \frac{|\psi_{\mathbf{k},\eta}^C\rangle\langle\psi_{\mathbf{k},\eta}^C|}{E - \overline{H}_0 + i0}$$

* É.I.Dolinskiĭ and A.M.Mukhamedzhanov. Sov. J. of Nucl. Phys. **3** (1966), 180.
* C.R.Chinn *et al.* Phys. Rev. C **44** (1991), 1569.

All matrix elements must be calculated in the Coulomb basis

Not trivial !

Highlights (up to now)

Single-channel optical potentials

1. n–A interaction:
    - OMPs are complex and energy dependent ⇒ developed a separable representation scheme for complex, energy dependent potentials
    - Scheme was successfully employed to construct separable representations for global phenomenological optical potentials for nuclei ranging from ^{4}He to ^{208}Pb
2. p–A interaction:
    - Extended n–A separable representation scheme to p–A interactions and applied it to global OMPs for nuclei ranging from ^{4}He to ^{208}Pb

Multichannel optical potentials

- Generalized separable representation to complex, energy dependent multichannel potentials and applied to a coupled-channels OMP for neutron inelastic scattering from ^{12}C

Based on Ernst-Shakin-Thaler (EST) Representation

Phys.Rev. C9, 1780 (1974)

Separable expansion of Hermitian operator, V, in basis $\{|\phi_i\rangle\}$:

Projection operator: $p = \sum_i |\phi_i\rangle\langle\phi_i|$

Projecting V onto p-subspace yields $v = Vp(pVp)^{-1}pV$

In the limit $p \longrightarrow 1$ we obtain $v = V$

EST scheme: choose basis vectors to be $|\psi_i^+\rangle$, the outgoing solutions of

$H = H_0 + V$ so that $v = \sum_{ij} V|\psi_i^+\rangle\lambda_{ij}\langle\psi_j^+|V$

Constraints $\Rightarrow$ $\delta_{kj} = \sum_i \langle\psi_k^+|V|\psi_i^+\rangle\lambda_{ij}$

$$\delta_{ik} = \sum_j \lambda_{ij}\langle\psi_j^+|V|\psi_k^+\rangle$$

Constraints ensure that at EST support points, E_i, wavefunctions corresponding to v and V are identical

Complex Potentials :

Given a time-reversal operator, $\mathcal{K}$, reciprocity is fulfilled if

$$\mathcal{K} v \mathcal{K}^\dagger = v^\dagger$$

Separable potential v does not satisfy this relation for complex potentials

Remedy: use 'in' states $|\psi_i^{(-)}\rangle$, eigenstates of $H' = H_0 + V^*$

For non-Hermitian potential V: $v = \sum_{ij} V|\psi_i^{(+)}\rangle \lambda_{ij} \langle \psi_j^{(-)}|V$

Constraints $\Rightarrow$ $\delta_{kj} = \sum_i \langle \psi_k^{(-)}|V|\psi_i^{(+)}\rangle \lambda_{ij}$

$$\delta_{ik} = \sum_j \lambda_{ij} \langle \psi_j^{(-)}|V|\psi_k^{(+)}\rangle$$

v fulfills reciprocity since λ is symmetric, i.e. $\lambda_{ij} = \lambda_{ji}$

t-matrix: $t(E) = \sum_{ij} V|\psi_i^{(+)}\rangle \tau_{ij}(E) \langle \psi_j^{(-)}|V$

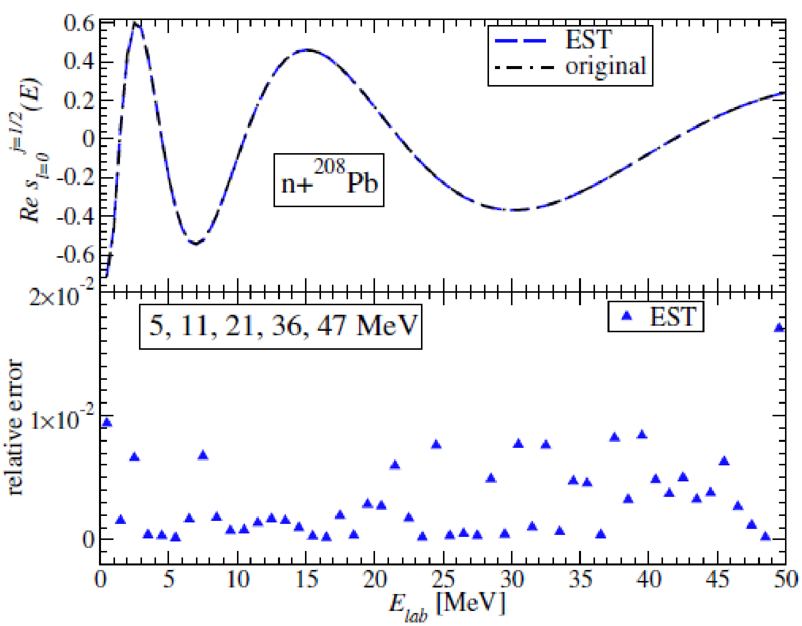
L. Hlophe, *et al.* Phys. Rev. C**88**, 064608 (2013)

Highlights for n-A potential: general set of EST support points

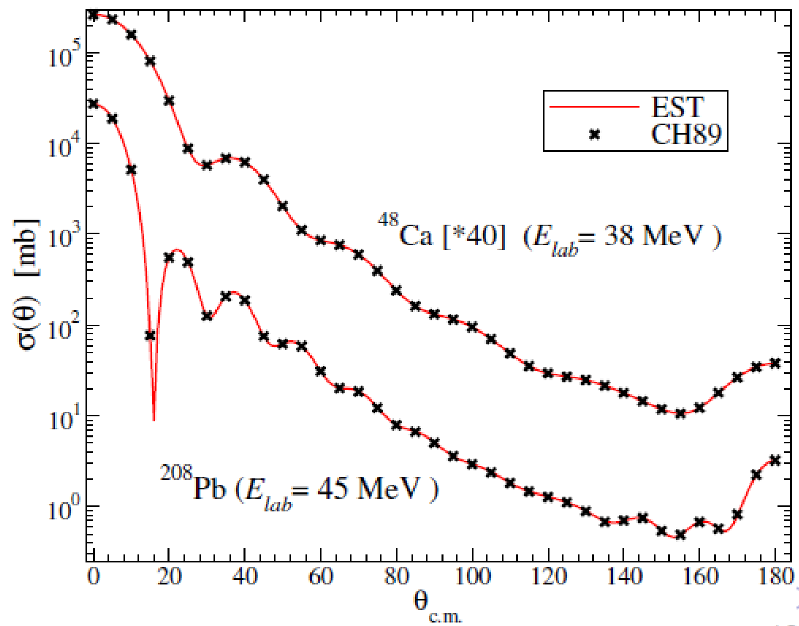
system	partial wave(s)	rank	EST support point(s) [MeV]
n+^{48}Ca	$l \geq 10$	1	40
	$l \geq 8$	2	29, 47
	$l \geq 6$	3	16, 36, 47
	$l \geq 0$	4	6, 15, 36, 47
n+^{132}Sn and n+^{208}Pb	$l \geq 16$	1	40
	$l \geq 13$	2	35, 48
	$l \geq 11$	3	24, 39, 48
	$l \geq 6$	4	11, 21, 36, 45
	$\mathbf{l \geq 0}$	**5**	**5, 11, 21, 36, 47** ⟵ Universal set

These EST support points provide good description of the s-matrices and cross sections from 0 to 50 MeV

Elastic scattering s-matrix



Differential cross section



Complex, energy dependent potentials

Use states $|\psi_i^{(+)}\rangle$, eigenstates of $H = H_0 + V(E_i)$

Constraints $\Rightarrow$ $\delta_{kj} = \sum_i \langle \psi_k^{(-)} | V(E_i) | \psi_i^{(+)} \rangle \lambda_{ij}$

$$\delta_{ik} = \sum_j \lambda_{ij} \langle \psi_j^{(-)} | V(E_j) | \psi_k^{(+)} \rangle$$

Energy dependence of V breaks symmetry: $\lambda_{ij} \neq \lambda_{ji} \Rightarrow$ violation of reciprocity

Energy dependent EST (eEST)

Define $v(E) = \sum_{ij} V(E_i) |\psi_i^+\rangle \lambda_{ij}(E) \langle \psi_j^- | V(E_j)$,

Constraint $\Rightarrow$ $\langle \psi_m^- | V(E) | \psi_n^+ \rangle = \langle \psi_m^- | v(E) | \psi_n^+ \rangle$

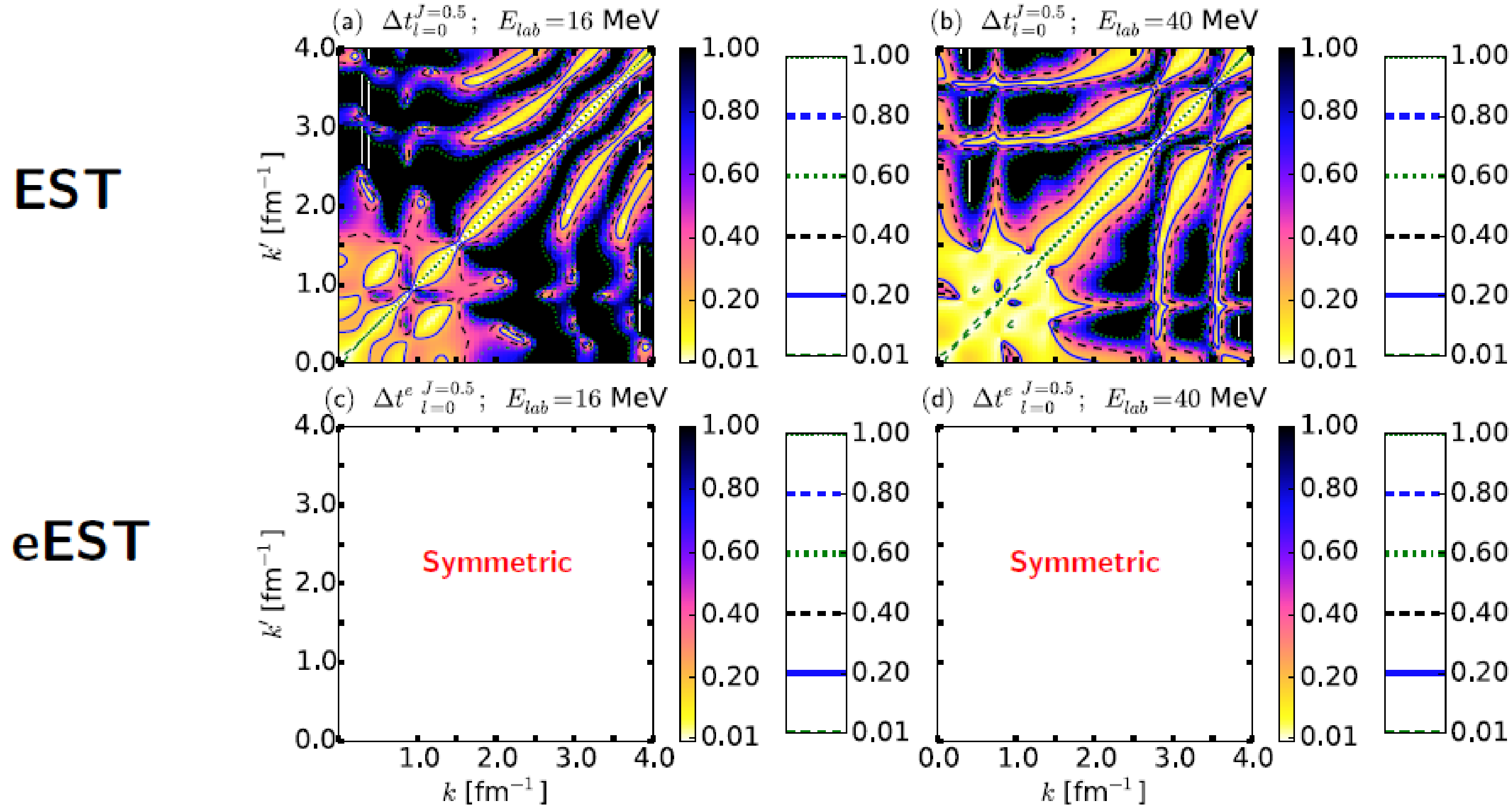
$$\Downarrow$$

Matrix elements of $v(E)$ and $V(E)$ are identical at all energies

Both EST constraints are satisfied and $\lambda_{ij} = \lambda_{ji}$

Off shell t-matrix: n+^{48}Ca, $l = 6$, $E_{lab} = 16$ MeV

Asymmetry: $\Delta t_l^J(k', k; E) = \left| \frac{t_l^J(k',k;E) - t_l^J(k,k';E)}{[t_l^J(k',k;E) + t_l^J(k,k';E)]/2} \right|$



- On shell momentum: $k_0 = 0.86$ fm^{-1} at 16 MeV,
  $k_0 = 1.36$ fm^{-1} at 40 MeV

EST and eEST for proton-nucleus scattering

(L. Hlophe, *et al.*, Phys. Rev. **C90**, 061602 (2014))

(1) Using Coulomb distorted neutron-nucleus form factors does not work for proton-nucleus scattering

(2) Use Coulomb-distorted nuclear scattering states, $|\psi_i^{sc(+)}\rangle$

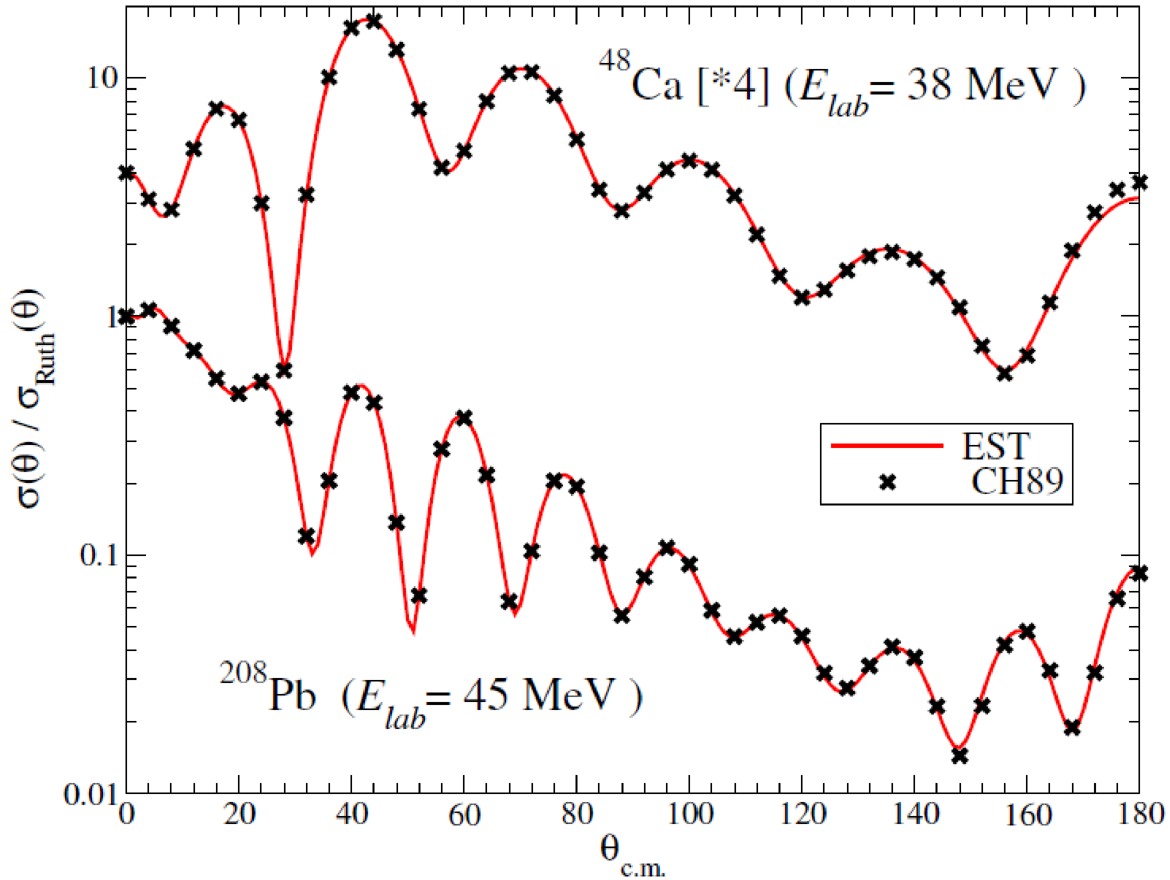
(3) Replace free propagator by the Coulomb Green's function

◆ Separable t-matrix becomes

$$t^{sc}(E) = \sum_{ij} V^s(E_i)|\psi_i^{sc(+)}\rangle \tau_{ij}^c(E)\langle\psi_j^{sc(-)}|V^s(E_j)$$

■ Evaluating form factor $V^s(E_i)|\psi_i^{sc(+)}\rangle$ requires matrix elements of V^s in Coulomb basis (C. Elster *et al.*, J.Phys. G**19**, 2123 (1993))

Proton-nucleus differential cross section



General set of EST support points also valid for p+A optical potentials

Multichannel Separable Potentials

- Including core-excitations leads to coupled-channels potentials

- Collective excitations result in couplings between members of a rotational or vibrational band

- EST form factors become the multichannel half-shell t-matrices

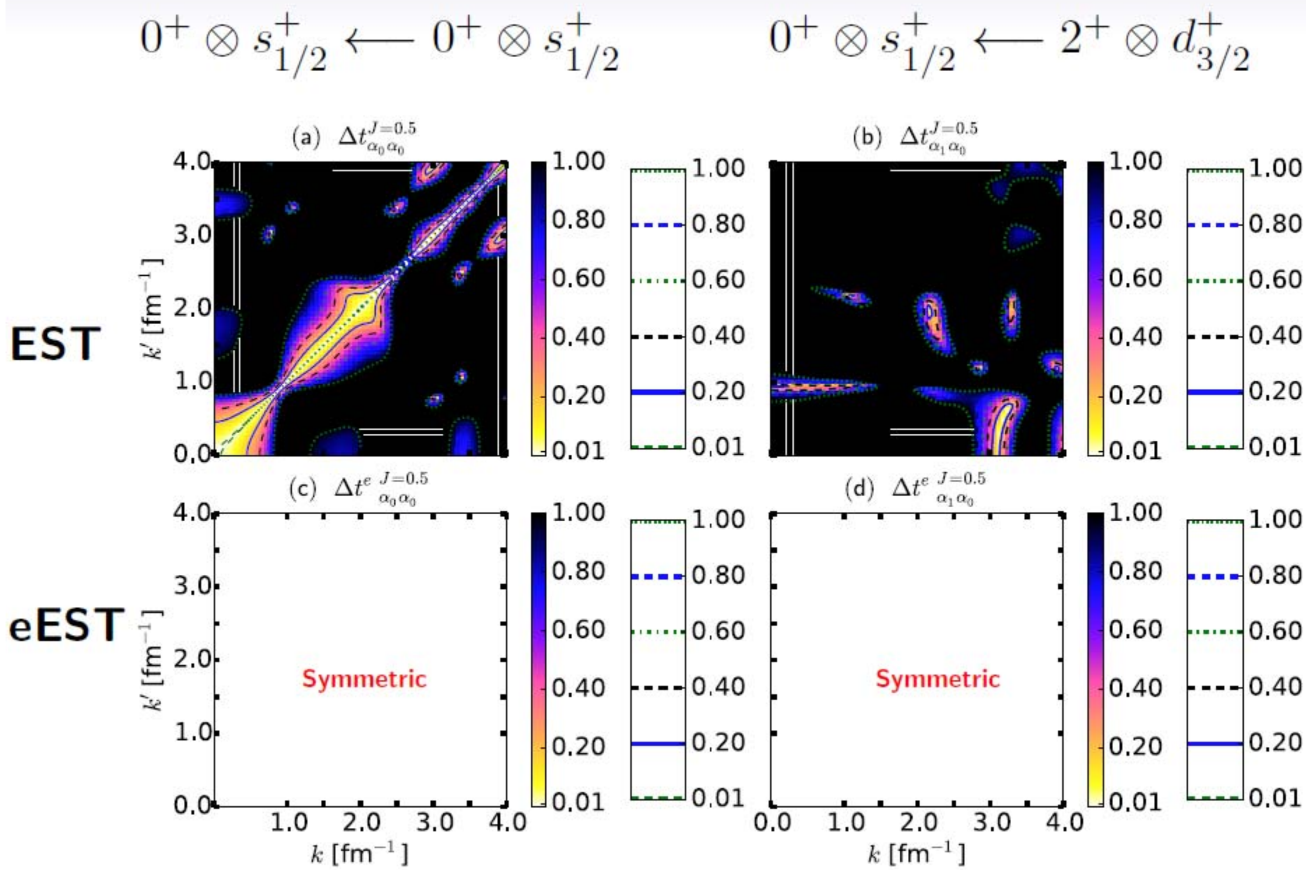
$$t^J_{\alpha_0\alpha}(E) = \sum_{\rho\sigma}\sum_{ij} T^J_{\alpha_0\rho}(E_i)\left|\phi_{l_\rho k_i^\rho}\right\rangle \tau^{\rho\sigma}_{ij}(E)\left\langle\phi_{l_\sigma k_j^\sigma}\right| T^J_{\sigma\alpha}(E_j)$$

- Quality of on-shell representation similar to single-channel case

- To check if reciprocity is satisfied define asymmetry

$$\Delta t^J_{\alpha\alpha_0}(k', k; E) = \left|\frac{t^J_{\alpha\alpha_0}(k',k;E) - t^J_{\alpha_0\alpha}(k,k';E)}{[t^J_{\alpha\alpha_0}(k',k;E) + t^J_{\alpha_0\alpha}(k,k';E)]/2}\right|$$

Asymmetry, $n+^{12}$C, $E_{lab} = 20.9$ MeV

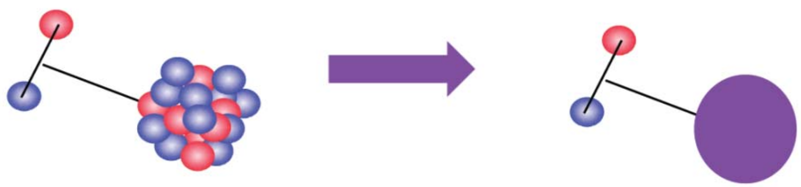


Asymmetry for EST more pronounced in multichannel scattering

eEST important when taking into account excitations

Near future: Numerical implementation of Faddeev-AGS equations

With this we can solve the effective three-body problem for (d,p) reactions for nuclei across the nuclear chart



Can we test this picture?

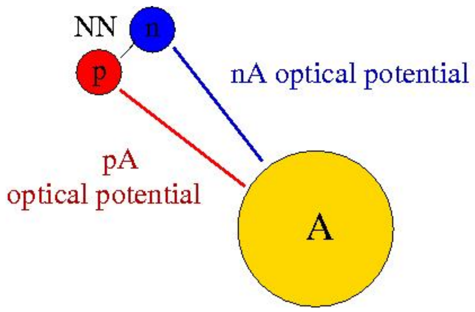
Scattering **d+^{4}He** can be calculated as many body problem by NCSM+RGM

Benchmark elastic and breakup scattering for d+^{4}He

Only reactions with light nuclei will allow benchmarks

Further Challenge:
Determine effective interactions V_{eff}

- V_{eff} is effective interaction between N+A and should describe elastic scattering



Hamiltonian for effective few-body poblem:

$$\mathbf{H = H_0 + V_{np} + V_{nA} + V_{pA}}$$

- V_{np} is well understood
- V_{nA} and V_{pA} are effective interactions
- Most used: phenomenological approaches
  - Global optical potential fits to elastic scattering data
  - Most data available for stable nuclei
  - Extrapolation to exotic nuclei questionable
- Microscopic approaches need to be developed or existing ones refined and adapted for exotic nuclei
  - Microscopic approaches were developed for A being a closed shell nucleus.

RIKEN:

^{6}He(p,p)^{6}He

and

^{8}He(p,p)^{8}He

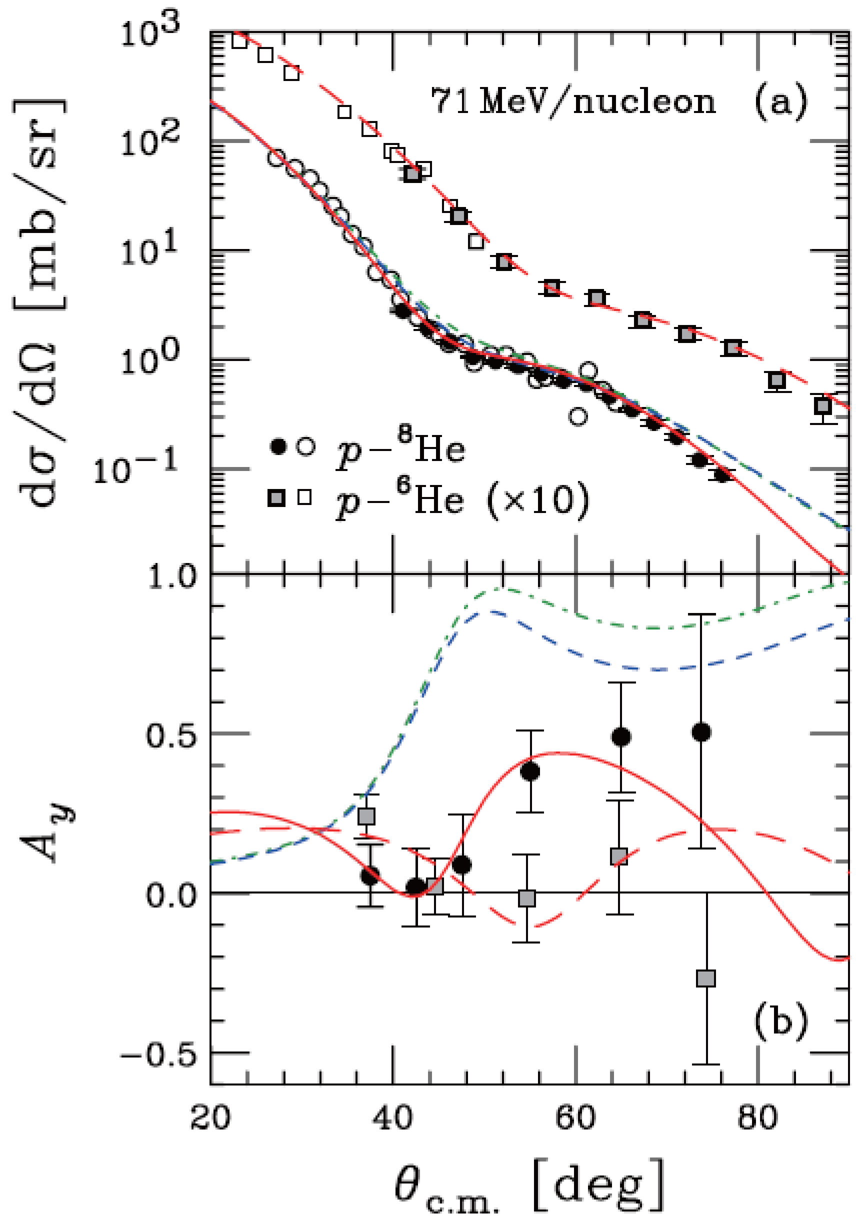
S. Sakaguchi et al.

PRC 87, 021601(R) (2013)

Standard Woods-Saxon type optical potential fit

Analyzing Powers of ^{6}He and ^{8}He behave differently!

A new A_y puzzle ?



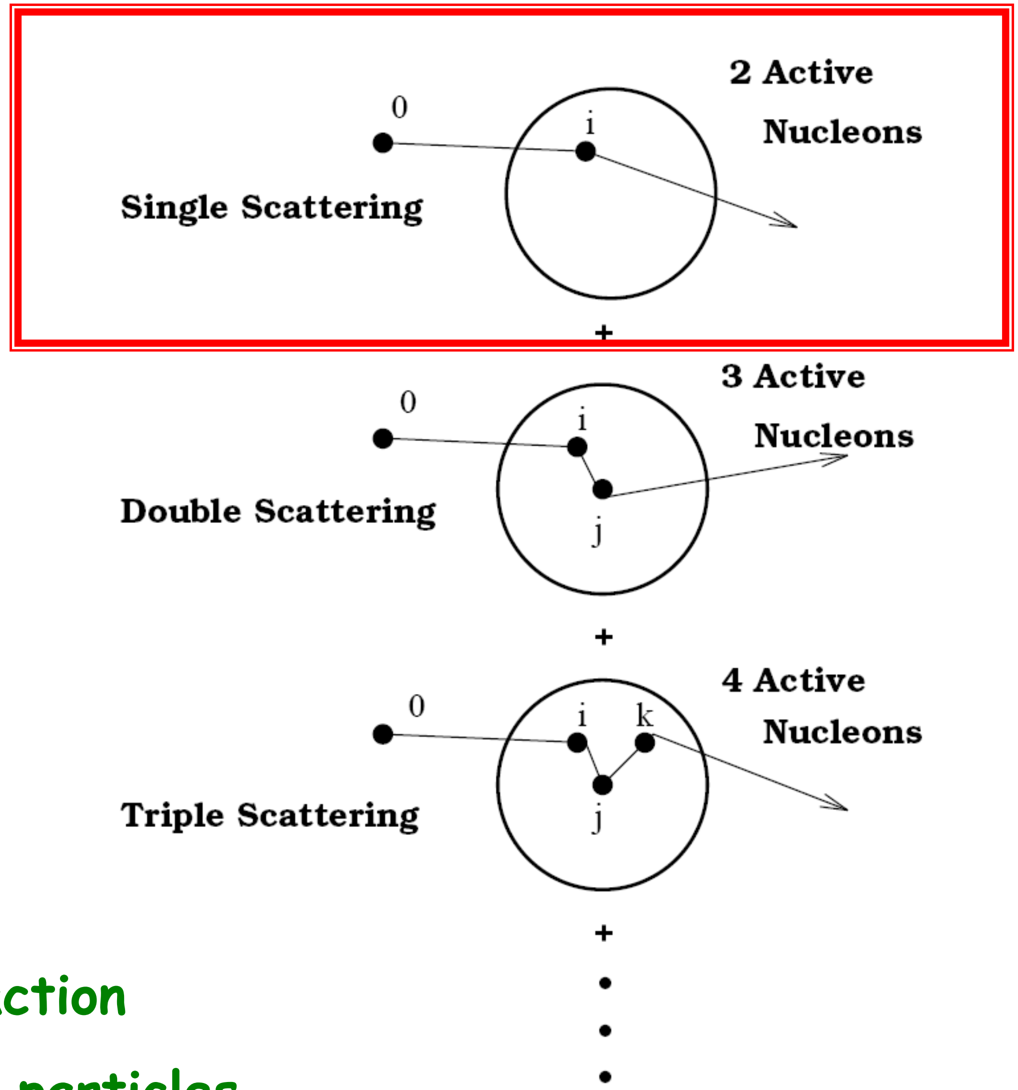
Multiple scattering approach to p+A scattering

Spectator Expansion:

Formulated by

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)



Expansion in:

- particles active in the reaction
- Antisymmetrized in active particles

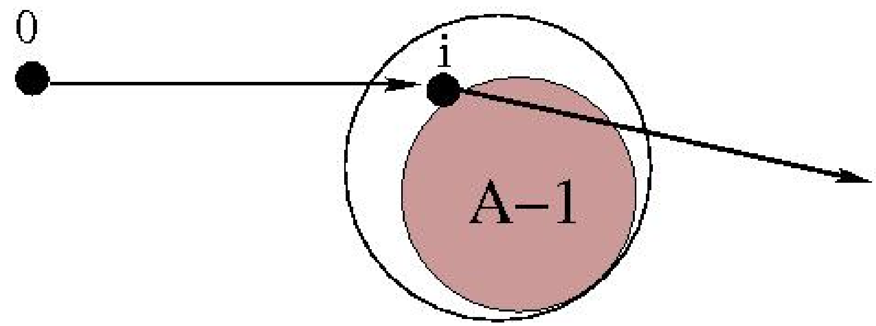
Closer look: Single Scattering

Three-body problem with particles:

o – i – (A-1)-core

o – i : NN interaction

i – (A-1) core : collective force



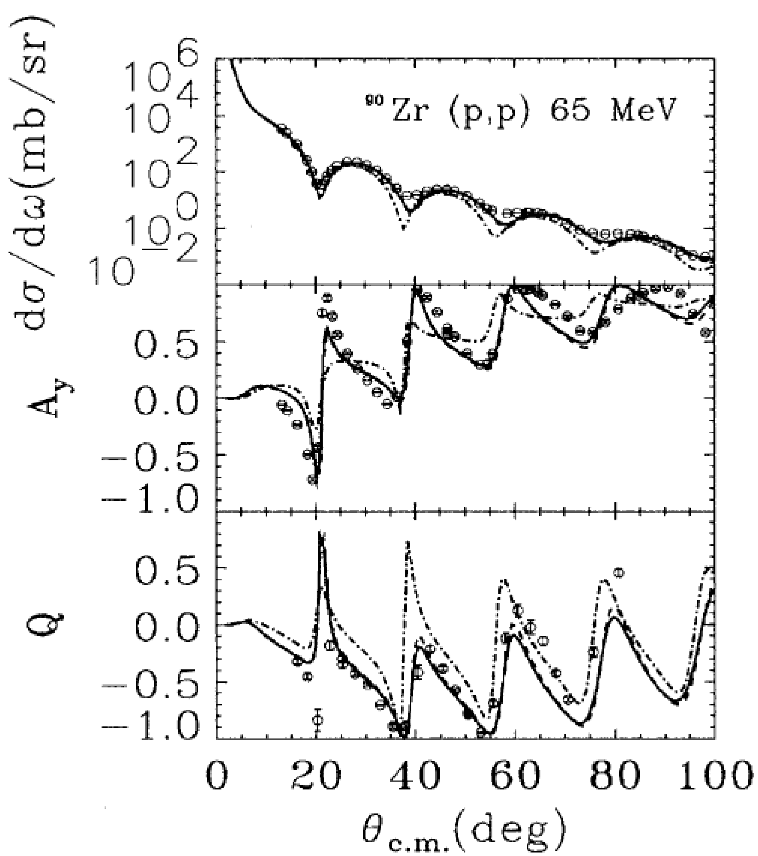
Scale of resolution?

Questions (new and old):

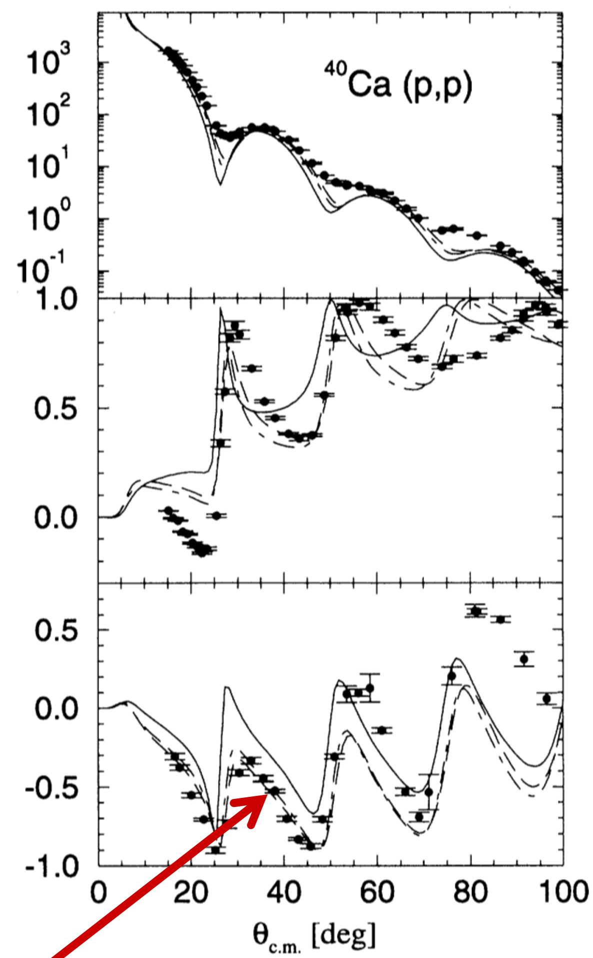
- Can the effective p+A or n+A force be derived from first principles?
- Is this problem a common ground for few- and many-body theory?

Needed as input for few-body description of reactions
Currently available for the energies of interest:
phenomenological descriptions

Parameter free calculations based on mean field HFB densities of Gogny CD-Bonn NN potential



65 MeV



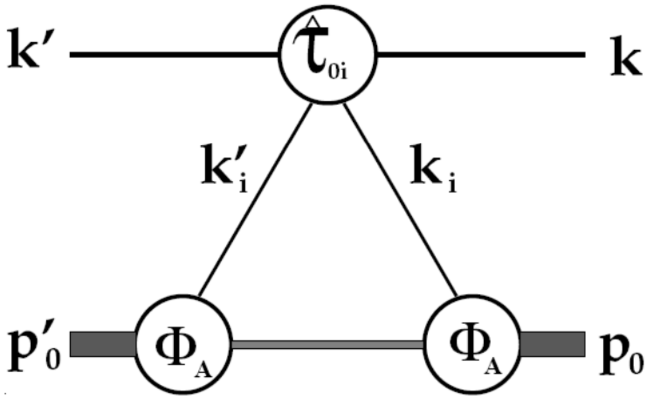
Improvement in spin-observables result from taking the mean field force explicitly into account

Microscopic :

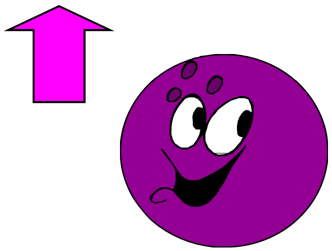
Chinn,Elster,Tandy, Redish, Thaler
Crespo, Johnson, Tostevin
Arrellano, Love

- **First order Optical Potential --- Full Folding**

$$\langle \mathbf{k}'|\langle\phi_A|PUP|\phi_A\rangle\mathbf{k}\rangle \equiv U_{el}(\mathbf{k}',\mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}'|\langle\phi_A|\hat{\tau}_{0i}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$



$$\langle\hat{\tau}_{01}\rangle \equiv \langle \mathbf{k}'|\langle\phi_A|\hat{\tau}_{01}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$



Proton scattering: $U_{el}(\mathbf{k}',\mathbf{k}) = Z\langle\hat{\tau}^{pp}_{01}\rangle + N\langle\hat{\tau}^{np}_{01}\rangle$

Optical Potential is non-local and depends on energy

Off-shell NN t-matrix and one-body nuclear density matrix

General Single Particle Density Matrix

Wave function ~ $\Phi_0(i) \sim f_l(i)\, Y_l^m(i)\, \chi_S^{(i)}$

Single particle density matrix from e.g. NCSM

Single particle density matrix

$$\rho_{I\,M_I,I\,M_I'}(i,i') \sim \sum_{k_l\,q_l;\,k_s\,q_s;\,k\,q} \begin{bmatrix} I & K & I \\ M_I' & q & M_I \end{bmatrix} \left\langle \Psi_I \,||\, \rho_{kq} \,||\, \Psi_I \right\rangle \chi^{ll'}_{k_l q_l}(i,i')\, f_l(i)\, f^*_{l'}(i')$$

$$\left\langle S\, m_s \,|\, \tau^{(i)}_{k_s q_s}(S) \,|\, S'\, m_s' \right\rangle \begin{bmatrix} k_l & k_s & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} l & l' & k_l \\ s & s & k_s \\ j & j' & k \end{Bmatrix}$$

Auxiliary tensor operator

$$\tau^{(i)}_{00} = 1$$

$$\tau^{(i)}_{k_s,q_s}\left(S = \tfrac{1}{2}\right):\quad \tau^{(i)}_{10} = 2s_z$$

$$\tau^{(i)}_{1,\pm1} = \mp\frac{2}{\sqrt{2}}(S_x \pm iS_y)$$

Orazbayev, Elster, Weppner

Phys.Rev. C88 (2013) , 034610

$$\chi^{ll'}_{k_l\,q_l}(i,i') = \sum_{l\,l'} (-1)^{l'-\lambda'} \begin{bmatrix} l & l' & k_l \\ \lambda & -\lambda' & q_l \end{bmatrix} Y_{l\lambda}(i)\, Y^*_{l'\lambda'}(i')$$

Orbital angular momentum

Reminder: calculate $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

NN interaction (t-matrix) in Wolfenstein representation:

$$\overline{\mathrm{M}}(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) = A(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, \mathbf{1} + i \cdot C(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, (\boldsymbol{\sigma}^{(0)} \otimes \mathbf{1} + \mathbf{1} \otimes \boldsymbol{\sigma}^{(i)}) \cdot \hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{n}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) - H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}})$$
$$+ (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) + H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN})$$

$$- -$$

$$+ D(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) \left[(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \right]$$

Off-shell

Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

Here may be an overlap where current structure models can be combined with reaction calculations.

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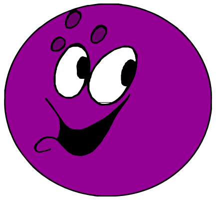
p+A and n+A effective interactions (optical potentials)



- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes

- **In the multiple scattering approach not even the first order term is fully explored: all work concentrates on closed-shell nuclei**

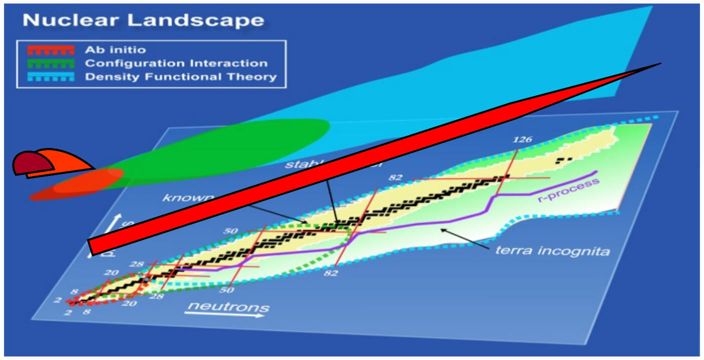
Via $\langle \Phi_A | \Phi_A \rangle$ results from nuclear structure calculations enter

- $\Rightarrow$ **Structure and Reaction calculations can be treated with similar sophistication**

Older microscopic calculations concentrated on closed shell spin-0 nuclei (ground state wave functions were not available)

- Today one can start to explore **importance of open-shells in light nuclei** full complexity of the NN interactions enters

Experimental relevance: Polarization measurements for $^6He \rightarrow p$ at RIKEN



Goal for Reaction Theory:

Determine the topography of the nuclear landscape according to reactions described in definite schemes

- At present `traditional' few-body methods are being successfully applied to a subset of nuclear reactions (with light nuclei)
    - Challenge: reactions with heavier nuclei
- Establish overlaps and benchmarks, where different approaches can be firmly tested.
- `cross fertilization' of different fields (structure and reactions) carries a lot of promise for developing the theoretical tools necessary for RIB physics.
- It is an exciting time to participate in this endeavor.

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