Phys 2053: Homework XII

due April 20, 2016

1. (*1 pts*) Estimate the sizes of the nuclei ${}^{4}_{2}\mathrm{He}$, ${}^{16}_{8}\mathrm{O}$, ${}^{56}_{26}\mathrm{Fe}$, and ${}^{208}_{82}\mathrm{Pb}$.

2. (*4 pts*)
(a) Using the mass of the neutron given in Appendix A and the atomic masses from Appendix B, calculate the total binding energy and the binding energy per nucleon of the following nuclei: ${}^{4}_{2}\mathrm{He}$, ${}^{16}_{8}\mathrm{O}$, ${}^{56}_{26}\mathrm{Fe}$, and ${}^{208}_{82}\mathrm{Pb}$.
(b) Use the semi-empirical mass formula (Morrison 14.6) to calculate the total binding energy and the binding energy per nucleon of the above nuclei.

3. (*2 pts*) Using the semi-empirical mass formula without the pairing term, derive an explicit expression for the binding energy per nucleon of a nucleus with atomic mass number A and atomic number $Z = N = A/2$. Show that the expression for the binding energy per nucleon you obtain has a maximum for $Z = A/2 = 26$.

4. (*2 pts*) Tritium (${}^{3}_{1}\mathrm{H}$) has a half-life of 12.3 years. What fraction of the tritium atoms would remain after 40 years?

5. (*4 pts*) The carbon isotope (${}^{14}_{6}\mathrm{C}$) is continuously produced in the atmosphere by the reaction

$$n +{}^{14}_{7}\mathrm{N} \rightarrow p +{}^{14}_{6}\mathrm{C}\ , \tag{1}$$

where the neutron is due to cosmic rays. ${}^{14}_{6}\mathrm{C}$ decays back to ${}^{14}_{7}\mathrm{N}$ by the reaction

$${}^{14}_{6}\mathrm{C} \rightarrow{}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}_e\ , \tag{2}$$

with a half-life of 5730 years. Since living organisms continually exchange carbon with the atmosphere, they have the same amount of the ${}^{14}_{6}\mathrm{C}$ isotope in a given sample of carbon as does the atmosphere.

- Using the fact that a gram of carbon in the atmosphere or in a living organism on the average emits 15.3 beta rays every minute, calculate proportion of (${}^{14}_{6}\mathrm{C}$ in carbon.
- What rate count would you expect from one gram of carbon extracted from a bone fragment that was 20,000 years old?