Phys 2053: Homework IV

due February 10, 2016

Schrödinger Equation

1. *(4 p)*
In the region $0 \leq x \leq a$, a particle is described by the wave function $\psi_1(x) = -b(x^2 - a^2)$. In the region $a \leq x \leq w$, its wave function is $\psi_2(x) = (x - d)^2 - c$. For $x \geq w$, $\psi_3(x) = 0$.

(a) By applying the continuity conditions at $x = a$ find c and d in terms of a and b.

(b) Can you find w in terms of a and b?

2. *(3 p)*
In a certain region of space, a particle is described by the wave function $\psi(x) = Cxe^{-bx}$, where C and b are real constants. By substituting this into the Schrödinger equation, find the potential energy in this regime and also find the energy of the particle.
(*Hint:* Your solution must give an energy that is a constant everywhere in this region, independent of x.)

3. *(3 p)*
Consider a finite well for the case that the energy E is greater than the well depth V_0. Show that in this case the Schrödinger equation for the region outside the well can be written as

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x), \tag{1}$$

with

$$k = \left[\frac{2m(E - V_0)}{\hbar^2}\right]^{1/2}, \tag{2}$$

where k is a real number. What is the general form of the solution of Eq. (1).