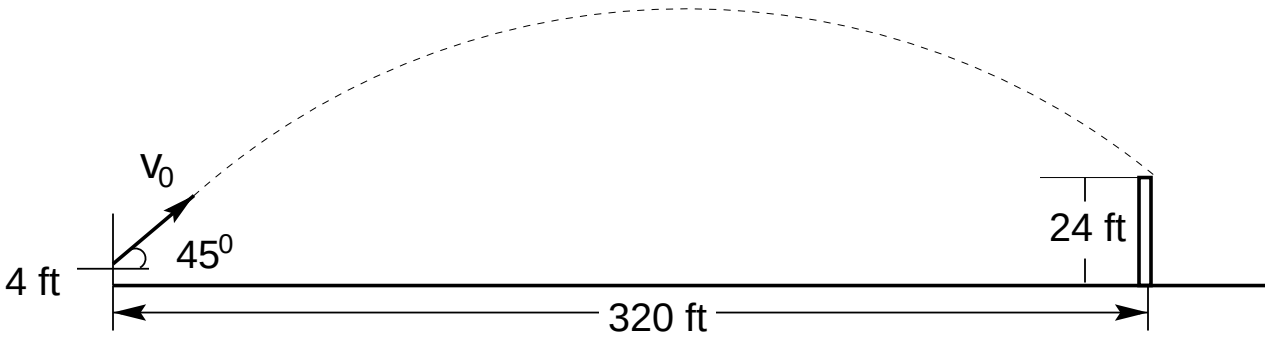
Ex : A batter hits a pitched ball (whose center is **4.0 ft** above home plate) at an angle of **45°** with a range of **350 ft**.

The ball will be a home run if it clears a **24 ft** high fence that is **320 ft** from home plate. Will the ball clear the fence? If so, by how much?



GOAL : Calculate the height of the ball at $\Delta x = 320$ ft.

1). From the data in the first sentence, we can calculate v_0. To proceed, divide motion into x and y.

$$\Delta x = v_0 \cos\theta t, \qquad \Delta y = v_0 \sin\theta t - \frac{1}{2} g t^2$$

When the ball lands: $\Delta x = 350$ ft $\Delta y =$ -4 ft

$$\begin{aligned} \Delta y &= v_0 \sin\theta \left(\frac{\Delta x}{v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{\Delta x}{v_0 \cos\theta}\right)^2 \\ &= \Delta x \tan\theta - \frac{1}{2} \frac{g(\Delta x)^2}{v_0^2 \cos^2\theta} \end{aligned}$$

Now solve for v_0 : $$v_0 = \sqrt{\frac{g(\Delta x)^2}{2(\Delta x \tan\theta - \Delta y)\cos^2\theta}}$$

$$v_0 = \sqrt{\frac{(32 \text{ ft/s}^2)(350 \text{ ft})^2}{2(350 \text{ ft} \cdot \tan 45° + 4 \text{ ft})\cos^2 45°}} = 105 \text{ ft/s}$$

2). Now given v_0, find t to reach the fence at $\Delta x = 320$ ft.

$$\Delta x = v_0 \cos\theta t, \qquad t = \frac{320 \text{ ft}}{105 \text{ ft/s} \cdot \cos 45^\circ} = 4.31 \ s$$

3). Finally, find Δy at this time.

$$\begin{aligned} \Delta y &= v_0 \sin\theta t - \frac{1}{2} g t^2 \\ &= (105 \text{ ft/s})(\sin 45^\circ)(4.31 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(4.31 \text{ s})^2 = 22.78 \text{ ft} \end{aligned}$$

As the ball started out 4 ft above the ground, it is at $y = 26.78$ ft at the wall, and thus it clears by:

$$\Delta y = 26.78 \text{ ft} - 24 \text{ ft} = 2.78 \text{ ft}$$