Assignment X: Applications of the Metropolis Algorithm

Due 11/14/2016

1. Lattice Path Integration

(a) Solve the ground state probability for the 1D Harmonic Oscillator via Feynman path integration using the Metropolis algorithm following the implementation suggested by Landau, Paez, and Bordeianu, Computational Physics, 2nd ed., Chapter 28.1.3.

(b) Follow the assessment and exploration of Chapter 28.1.4 and

1. examine some of the actual space-time paths used in the simulation. Compare those paths to the classical trajectory
2. For a more continuous picture of the wave function, make the x lattice spacing smaller; for a more precise value of the wave function at any particular lattice site, sample more points (run longer) and use a smaller time step ε.
3. Explore the effect of making $\hbar$ larger, and thus permitting greater fluctuations around the classical trajectory. Do this by decreasing the value of the exponent in the Boltzmann factor. Determine if this makes the calculation more or less robust in its ability to find the classical trajectory.
4. modify the your code and test the wave function computation for the gravitational potential

$$\begin{aligned} V(x) &= mg|x| \\ x(t) &= x_0 + v_0 t + \frac{1}{2} g t^2. \end{aligned} \tag{1}$$

You may want to set the initial positions to be close to the classical trajectory to ensure convergence.