Assignment II: Errors in Computing, Recursion Relations, and Roots

Due 9/12/2015

There are few simple but golden rules to create programs that build a useful code library. Thus some **hints for good coding**:

- Write small modules (routines/functions); avoid big functions that do everything. However, don't make them too small.
- Write lots of error traps, even for things that are 'obvious'.
- Document as you go along. For each function write a header that includes
  1. Main purpose of function
  2. names and brief explanation of input variables, if any
  3. names and brief explanation of output variables, if any
  4. functions called by this function
  5. called by which function

Please follow those rules in your codes not only for this semester, but throughout your career.

1. Summing Series
Problem **2.2** from Morten Hjorth-Jensens Lecture notes Ch. 2. Please follow the steps suggested in the problem and provide the log-log plot with your analysis of your computation.

2. Continued Fractions
Problem **2.9** from Morten Hjorth-Jensens Lecture notes Ch. 2

3. Bessel-functions via Recursion

(a) Write a program to calculate $j_l(x)$ that will give 'good' values for the first 25 l values for $x = 0.1, 1.0, 10.$. ('Good' here means a relative error of $\approx 10^{-6}(10^{-14})$ for single (double) precision). [Pick one!]

(b) Try with both, upward and downward recursion, but do not try too hard for upward recursion. [Hint: Using single precision shows the error effects more quickly.]

(c) Give results of the downward recursion for different, large values of the starting l, showing the convergence and stability of your results.

(d) Compare the upward and downward recursion methods, printing out l, $j_l^{(up)}$, and $j_l^{(down)}$, and the relative difference

$$\frac{|j_l^{(up)} - j_l^{(down)}|}{|j_l^{(up)}| + |j_l^{(down)}|} \tag{1}$$

(e) The errors in the upward recursion depend on x, and for certain values of x, both up and down recursions give similar answers. Explain the reason for this and what it tells about your program.

4. Solving Nonlinear Equations

Find the positive roots of the function

$$f(x) = x^2 - 7x - \ln x \tag{2}$$

using

1. the bisection method
2. the Hybrid/Newton-Raphson methods based on the Secant

to 5 significant figures. Discuss and document the rate of convergence of both methods and their robustness. For a good estimate of the accuracy of you answer, calculate in each iteration the absolute relative error $|\varepsilon_i|$ (in %) given by

$$|\varepsilon_i| = \left|\frac{x_i - x_{i-1}}{x_i}\right| \times 100 \tag{3}$$

and show this in a table that lists the number of the iteration i, x_{i-1}, x_i, x_{i+1}, and $|\varepsilon_i|$. When running your code, consider if you need prior knowledge of the approximate position of the zero.