Assignment VII: Randomness

Due 10/24/2016

1. Random Sequences

(a) Write a simple program to generate random numbers using the linear congruent method, defined by

$$r_i \equiv (ar_{i-1} + c) mod M. \tag{1}$$

The built-in function mod is called with 2 arguments: mod (a,M).

(b) For pedagogical purposes, try the unwise choice: $(a, c, M, r_i) = (57, 1, 256, 10)$. Determine the *period*, i.e.

(c) Take your pedagogical sequence of random numbers and look for correlations by observing clustering on a plot of successive pairs $(x_i, y_i) = (r_{2i-1}, r_{2i}), i = 1, 2 \cdots$. Do *not* connect the plots with lines.

(d) Test the built-in random number generator provided by the *SUN* compiler, *ran* and *drand* for visual correlations by plotting the same pairs as in (c).

(e) Test the Sobol sequence pseudo-random numbers for visual correlations by plotting the same pairs as in (c). Use the program *stest.f90* with call to *sobseqn.f90.*

2. Checks on Random Sequences

(a) Test your own random-number generator and the two built-in random number generators of the *SUN* compiler for uniformity for $k = 1, 3, 7$ and $N = 100, 10^4, 10^5$. in each case print out

$$\frac{1}{N}\sum_{i=1}^{N} x_i^k \tag{2}$$

and

$$\sqrt{N}\left|\frac{1}{N}\sum_{i=1}^{N} x_i^k - \frac{1}{k+1}\right| \tag{3}$$

and check that the latter is of order 1.

(b) Perform the same tests for the Sobol Sequence. Interpret your results, having in mind, that the Sobol Sequence is a pseudo-random sequence.

3. Simulation of Radioactive Decay

This problem simulates the radioactive decay of two nuclei (X and Y) using sampling through random numbers. At $t = 0$ there are $N_X(0)$ nuclei of type X, which can decay radioactively. At a given time t we are left with $N_X(t)$ nuclei. With a transition rate ω_X, which is the probability that the system will make a transition to another state during a time step of one second, we get the following differential equation,

$$dN_X(t) = -\omega_X N_X(t) dt, \tag{4}$$

with the solution

$$N_X(t) = N_X(0)\, e^{-\omega_X t}. \tag{5}$$

The mean life time of the nucleus X is

$$\tau = \frac{1}{\omega_X}. \tag{6}$$

If the nucleus X decays to Y, which can also decay, one gets the following coupled equations,

$$\begin{aligned} \frac{dN_X(t)}{dt} &= -\omega_X N_X(t) \\ \frac{dN_Y(t)}{dt} &= -\omega_Y N_Y(t) + \omega_X N_X(t). \end{aligned} \tag{7}$$

We assume that at $t = 0$ we have $N_Y(0) = 0$. In the beginning, t here will be an increase of N_Y nuclei, however, they will decay thereafter.

For this specific problem let the nucleus ^{210}Bi represent X. It decays through β-decay to ^{210}Po, which is the Y nucleus in our case. ^{210}Po decays through the emission of an α-particle to ^{206}Pb, which is a stable nucleus. ^{210}Bi has a mean lifetime of 7.2 days, while ^{210}Po has a mean lifetime of 200 days.

(a) Make a code which solves the above equations using sampling through random numbers. Consider e.g. what is a reasonable choice of time step Δt. Run your code for $N_X(0) = 10$, 100, and 1000. Plot the results for $N_X(t)$ and $N_y(t)$.

(b) Run adapt your differential equation code from the project to this problem and do the same runs. Take the case $N_X(0) = 10$, 100, and 1000 and plot the for each case the solution obtained in **(a)** together with the result obtained from the differential equations. Comment on this comparison.

(c) When ^{210}Po decays, it produces α-particles. At what time does the production of α-particles reach a maximum? Use your code of **(a)** to find the this maximum in case of $N_X(0) = 100$ and 1000.