The Dalitz Plot as a Tool in Particle Physics

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Abstract: Some basic kinematical relations and Dalitz plot observables are given and the Dalitz plot as a descriptive tool to extract physics information from event distributions is discussed.

1 Introduction

In reactions with a two–body exit channel the important observables to extract the physics are the energy dependent total cross section σ (Q) and the differential cross section $d\sigma/d\cos\Theta^*$ depending on the angular correlation (Θ^*) between entrance and exit directions in the CM system. Observables are always of the type: number of events per some phase space interval and per number of beam particles. The cross section shows resonance masses M_R and widths Γ_R. The angular distribution $d\sigma/d\cos\Theta^*$ depends on the square of the wave function and thus contains information on the spin/parity J^P content of the reaction. For a three body exit channel there are much more distributions possible and one has to choose the best ones to extract physics information.

The Dalitz plot is the best way to show all observables in a three–body reaction $1 + 2 \to i + j + k$ which depend on exit coordinates alone. The three particles $i+j+k$ form three two–body subsystems $(i+j)$, $(j+k)$ and $(k+i)$ which we can consider as quasiparticles. The quantum numbers and resonance parameters of these quasiparticles m_{ij}, Γ_{ij}, and J^P_{ij} can be seen in the event distribution in the Dalitz plot. The Dalitz plot does not show any correlations with coordinates of the entrance.

2 Basic kinematics

For any reaction like $1+2 \to i+j$ in any system the conservation of energy and momentum holds. Both together are simply written for a two particle reaction in four–vector notation: $\wp_1 + \wp_2 = \wp_i + \wp_j$ with $\wp_i = (\mathrm{E}_i, \vec{\mathrm{p}}_i)$, where

$E_i^2 = m_i^2 + \vec{p}_i^{\,2}$. In different frames of reference the Lorentz transformation conserves one quantity, namely the total available energy in CM $M_{12} = \sqrt{s} = M_{ij}$ (also called effective mass or the length of the four–vector) where $s = (\wp_i + \wp_j)^2$. In figure 1 a graphical presentation of the two–body kinematics is given which follows from these conservation laws.

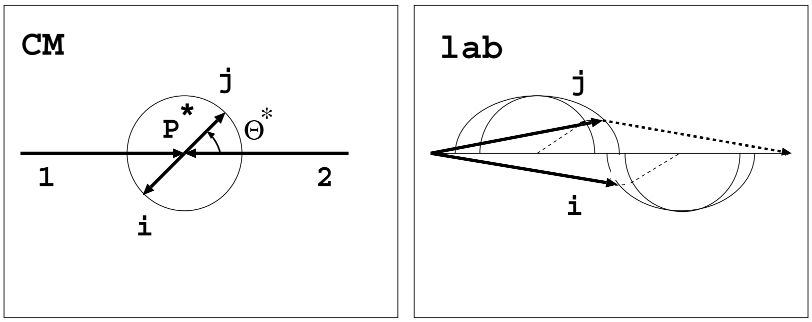

Figure 1: Two–body kinematics in the center of mass and laboratory system.

An important quantity for the determination of CM quantities from lab quantities is the CM momentum (in the exit channel). From the relations above the CM momenta as a function of s and the masses (m_1 and m_2) or (m_i and m_k) are given by:

$$|p_k^*| = |p_l^*| = \sqrt{[(s^2 - 2s(m_k^2 + m_l^2) + (m_k^2 - m_l^2)^2]/(4s)}. \qquad (1)$$

With a target m_2 at rest the scattering angle in the center of mass system $\cos\Theta^*$ is given by

$$\cos\Theta_{1i}^* = \frac{\vec{p}_1 \cdot \vec{p}_i^*}{|\vec{p}_1| \cdot |\vec{p}_i^*|} = \frac{(\vec{p}_i + \vec{p}_j) \cdot \vec{p}_i^*}{|\vec{p}_i + \vec{p}_j| \cdot |\vec{p}_i^*|} \qquad (2)$$

Information about the physics is given in the angular distribution of reaction events shown over $\cos\Theta_{1i}^*$. The contribution of different partial waves to the cross section as well as the J^P of possible resonances in the ij–system are visible in this angular distribution. The mass and width of resonances in the ij–system can be extracted from the excitation function.

A three particle exit channel $1 + 2 \rightarrow i + j + k$ contains three two–body subsystems (ij), (jk), and (ki). They are treated as two–body systems. If (jk) is the subsystem then the partial waves in the (jk)–subsystem (quasi-particle) can be extracted in complete analogy to the two–body case shown above. Now the emission angle of j in the $(j+k)$ CM system is denoted by **, where $\vec{\mathrm{p}}_j^{**} + \vec{\mathrm{p}}_k^{**} = 0$ relative to the "beam direction" which is now along $\left(\vec{\mathrm{p}}_j^* + \vec{\mathrm{p}}_k^*\right)$ in the three–body CM system. The reference axis for angular distributions is now not at all the beam axis $(\vec{\mathrm{p}}_1)$ but the negative direction of i or the direction of the (jk)–subsystem in the three–body CM system, respectively $(\vec{\mathrm{p}}_j^* + \vec{\mathrm{p}}_k^* = -\vec{\mathrm{p}}_i^*)$. One has to be careful in selecting the correct directional correlations. While for example the absolute CM momentum in the (jk)–system is a Lorentz invariant quantity, the CM $\cos\Theta_{ij}^{**}$ is not equal to $\cos\Theta = ((\vec{\mathrm{p}}_j + \vec{\mathrm{p}}_k)\cdot\vec{\mathrm{p}}_j^*)\,/\,(|\vec{\mathrm{p}}_j + \vec{\mathrm{p}}_k|\cdot|\vec{\mathrm{p}}_j^*|)$ created with the lab momenta $\vec{\mathrm{p}}_j$ and $\vec{\mathrm{p}}_k$. With this choice there is no collinear reaction partner to M_{ij}. Such an angular distribution would be nonsense. For a three–body reaction one gets only in the three–body CM system the correct collinear reaction partner, namely m_i with its four vector $(\mathrm{E}_i^*, \vec{\mathrm{p}}_i^*)$.

In the three–body system the counting sequence ijk is important. It defines a certain helicity and going to ikj changes some signs of $\cos\Theta^*$.

3 Dalitz plot coordinates

For a three–body exit channel like $1 + 2 \to i + j + k$ the invariant total energy squared s is given by:

$$\begin{aligned} s = (\wp_1 + \wp_2)^2 &= (\wp_i + \wp_j + \wp_k)^2 \\ &= \wp_i^2 + \wp_j^2 + \wp_k^2 + 2\wp_i\wp_j + 2\wp_j\wp_k + 2\wp_k\wp_i \\ &= (\wp_i + \wp_j)^2 + (\wp_j + \wp_k)^2 + (\wp_k + \wp_i)^2 - \wp_i^2 - \wp_j^2 - \wp_k^2 \end{aligned}$$

which results in:

$$s + \mathrm{m}_i^2 + \mathrm{m}_j^2 + \mathrm{m}_k^2 = \mathrm{M}_{ij}^2 + \mathrm{M}_{jk}^2 + \mathrm{M}_{ki}^2 \qquad (3)$$

with the invariant squared masses $\mathrm{M}_{mn}^2 = (\wp_m + \wp_n)^2$. These invariant squared masses M_{mn}^2 are mostly used as the coordinates of the Dalitz plot. In a geometrical representation the Dalitz plot is a plane in the 3–dimensional $(\mathrm{M}_{ij}^2, \mathrm{M}_{jk}^2, \mathrm{M}_{ki}^2)$ space which is orthogonal to the space diagonal (its direction is $\sqrt{1/3}(1,1,1)$). Each point in the Dalitz plot is given in a vector representation by a scalar product with the space diagonal $(1,1,1)(\mathrm{M}_{ij}^2, \mathrm{M}_{jk}^2, \mathrm{M}_{ki}^2)$. For a constant s two of the three coordinates are sufficient for an unambiguous presentation of the Dalitz plot observables (eq. 3).

Alternative to the invariant squared masses the total energies of the particles can be written at the Dalitz plot axes. Using the four–vector relations in the overall CM system:

$$\wp^*_{jk} = (\wp^*_j + \wp^*_k) = (\wp^*_{entrance} - \wp^*_i) \; ; \; \mathrm{M}^2_{jk} = [(\sqrt{s}, \vec{0}) - (\mathrm{E}^*_i, \vec{\mathrm{p}}^*_i)]^2$$

we find a linear relation between M^2_{jk} and E^*_i given by:

$$\mathrm{M}^2_{jk} = s + \mathrm{m}^2_i - 2\sqrt{s}\mathrm{E}^*_i$$

(and cyclic exchange).

4 Physics in the Dalitz plot

The Dalitz plot allows in a very instructive way to extract physics from the event distribution.

The cross section for a certain reaction can be decomposed in a phase space factor and a reaction matrix element. The phase space element $d\rho$ is given by:

$$d\rho = d\mathrm{E}_i \, d\mathrm{E}_j = \frac{1}{4s} \, d\mathrm{M}^2_{ij} \, d\mathrm{M}^2_{ki} \tag{4}$$

These are surface elements of the Dalitz plot. For a pure S–wave reaction with an isotropic emission of the reaction products these phase space elements have a constant occupation density (same number of events per surface). The integration over the phase space elements is just the available total phase space, i.e. the surface of the Dalitz plot. The projection of the Dalitz plot (surface between its boundary curve) onto the M^2_{ij} or E^*_k axes gives the shape of the mass/energy spectra as expected if only phase space behaviour exists (eq. 2). At low energies the Dalitz plot surface is proportional to the square Q^2 of the excess energy $\mathrm{Q} = \sqrt{s} - \mathrm{m}_i - \mathrm{m}_j - \mathrm{m}_k$. Each point in the Dalitz plot is connected with the 3 two–body subsystems and their internal relative momenta. The physics, for example final state interactions and resonance effects depend on the CM momenta in the two–body subsystems. Therefore each point in the Dalitz plot gets in the simplest picture weight factors g_{nm} from each of the three subsystems, and the cross section represented as an integral over the phase space is modified by these weight factors.

$$\sigma \sim \int d\rho \, g_{ij}(\mathrm{p}^{**}_{ij}) \, g_{jk}(\mathrm{p}^{**}_{jk}) \, g_{ki}(\mathrm{p}^{**}_{ki}) \tag{5}$$

A special feature of the invariant squared mass (or E^*_n) spectra is the linear correlation with the cosine of the relevant angle in the considered subsystem. For a fixed invariant squared mass like M_{ij}^2 the distribution of events on the line in the Dalitz plot corresponds to the $\cos\theta_j^*$ of particle j in the (jk)–subsystem relative to the direction of the (jk)–system. The situation is illustrated in figure 2.

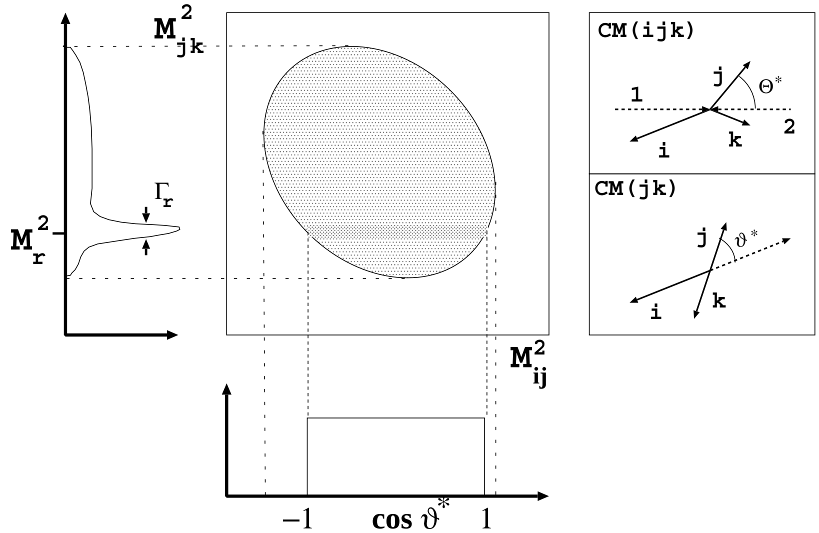

Figure 2: Dalitz plot for a three–body exit channel i, j, k with a resonance in the (jk)–subsystem and the influence of the typical decay angular distribution over the events in the slice of events around $M_{jk}^2 = M_R^2$.

A resonance in the (jk)–system is observed on the M_{jk}^2 axis as an enhancement of events in the Dalitz plot for a fixed mass, and on the M_{ij}^2 axis directly the $\cos\theta_j^*$ distribution in the (jk) CM system is given from which the resonance spin can be derived. In general a resonance can be present in the different subsystems which complicates the situation. The observed angular distribution in one M_{ij}^2 slice can be (strongly) distorted by effects in the other subsystems. Crossing resonances for example can only be disentangled by a Dalitz plot analysis and not in projected M_{ij}^2 spectra alone. Helpful are Dalitz plots at different s values. They change in size and the crossing effects shift and can be disentangled.

The three–body exit observables for a reaction are an incoherent sum over Dalitz plots for different J^P entrance values. In principle these different Dalitz plots for each J^P entrance value may be separable (e.g. by using polarized beams and targets in the entrance). Within the differential distributions for a given J^P entrance interference terms will show up between various amplitude contributions of m_{ij} subsystems and between amplitudes of m_{jk} and m_{ki} subsystems. Therefore the Dalitz plot occupation can have a very complex structure but within an appropriate analysis the individual components can be extracted as was shown by the Crystal Barrel collaboration at LEAR. Details on the partial wave analysis can be found for instance in [1, 2, 3].

References

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