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THE MESON THEORY OF NUCLEAR FORCES AND NUCLEAR MATTER

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ABSTRACT

The meson theory of nuclear forces is reviewed both historically and pedagogically. A comprehensive and consistent meson-exchange model for the nucleon-nucleon (NN) interaction is developed. The various meson-exchange contributions to the nuclear force are discussed and their power in quantitatively explaining NN scattering data demonstrated. One-boson-exchange (OBE) approximations to this full meson theory are derived for practical as well as historical reasons. The nature and quality of this useful and traditional approximation is examined. The full meson-exchange model as well as the simplified OBE potential are then applied to nuclear matter with particular stress laid on Brueckner theory. Past (failed) attempts to explain the empirical nuclear matter properties are surveyed. The relativistic Dirac-Brueckner approach to nuclear matter is motivated and explained. The predictive power of this new approach is demonstrated by the results for the nuclear matter saturation. The exceptional quantitative nature of the relativistic meson theory for both the free NN-interaction and nuclear matter is considered a strong motivation for further applications to other fields of nuclear physics, as e.g. nucleon-nucleus scattering and the structure of finite nuclei.

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PREFACE I

The general purpose of these lectures is to serve as an introduction. The specific audience addressed are graduate students in theoretical as well as experimental nuclear physics — in accordance with the goal of this workshop. However, every teacher and researcher in the field of nuclear and intermediate energy physics not specialized to the particular topics of these lectures may find useful information in these notes.

The special purpose of these lectures is manifold. In line with the introductory character I will first undertake a brief excursion into history and then give a pedagogical introduction which is supposed to provide the reader the qualitative and physical understanding of the meson theory of the nuclear force. These chapters can be understood with little familiarity in the field. In addition, my intention is to lead the reader up to the latest status in the two topics addressed in this article, namely the nucleon-nucleon (NN) interaction and nuclear matter from a consistent meson theoretic point of view. In principle, a detailed knowledge of the subtleties of field theoretic perturbation theory is required for our advanced theory of the nuclear force. However, by banning most of the formalism into appendices and through the suggestive character of the Feynmann diagram language the essential points of this serious and detailed description of the NN interaction will also be understandable to the non-expert. References are provided for those who want to go beyond the discussion presented here. Concerning nuclear matter, we will also start with a short introduction and a brief review of the theoretical attempts of the past for which we will in addition quote some appropriate literature. More thoroughly we will describe the modern relativistic approach to nuclear matter and its excellent results.

Finally, these lectures want to point into the future and at the same time also serve practical purposes. Therefore, a representation of the nuclear force in the simple and (for numerical calculations) handy relativistic one-boson-exchange (OBE) approximation in momentum space with all necessary formulae and parameters will be given. This furnishes the future researcher with the necessary equipment needed to start his/her own work in relativistic nuclear structure physics. Future goals in research in this field are suggested and the appropriate background literature is given.

In conjunction with practical aspects, also a non-relativistic configuration space OBE potential will be provided. In spite of the approximations necessary to derive this simplest version of a meson theoretic NN potential, it may be useful for simple qualitative and quantitative comparisons with the recent and increasing attempts, in which the nuclear force is derived from the quark model. The advantage of the non-relativistic approximation is that the contributions of each meson-exchange can be represented in the conventional terms of a central, spin-spin, spin-orbit and tensor force. This facilitates certain physical discussions and the comparison with results from alternative approaches to a microscopic understanding of the nuclear force. This latter aspect is particularly relevant with regard to the second part of this workshop.

PREFACE II

These lectures are embedded in a workshop on relativistic dynamics and quark-nuclear physics. Therefore, it appears appropriate to indicate the relationship of these lectures to the other material being presented in the course of these two weeks.

The mean field theory (MFT), which is covered in the lectures by Chuck Horowitz, is historically one of the first (comprehensive) attempts of a relativistic theory for the nuclear many body problem. One of its great merits is that it first demonstrated the importance of relativistic saturation effects. However, as a first attempt one cannot expect it to be perfect. Therefore, there are naturally some significant drawbacks. One is that MFT has no connection to the free NN interaction. The parameters of the model are fitted directly to the empirical nuclear matter properties; consequently their values are such that free NN-scattering is far beyond a quantitative description. Further, MFT takes only the nucleon, a ω-meson and a fictitious σ-boson as relevant degrees of freedom into account. From a serious and comprehensive theory for the NN-interaction one knows, however, that the π- and ρ-meson are of fundamental importance. Moreover, a realistic description of the intermediate range attraction by the 2π-exchange (which makes the fictitious σ-boson obsolete) requires definitely virtual Δ-isobar excitation in intermediate states. This adds another important degree of freedom to nuclear physics. In these lectures we will gradually develop a quantitative theory for the NN interaction which takes all these essential degrees of freedom into account. After that, we will turn to nuclear matter in a relativistic framework which starts from the free NN interaction. The "Dirac-Brueckner" approach, as this framework came to be known, was created in the spirit of MFT (adding another historical credit to that theory), but at the same time overcomes the fundamental problems and deficiencies in which MFT was trapped. In fact, we will finally explain the empirical saturation properties of nuclear matter quantitatively in terms of the free NN interaction, i.e. in a parameter-free way.

From the relativistic analysis of nucleon-nucleus scattering (lectures by Bunny Clark and Jim Shepard) one has strong empirical evidence for large common scalar and vector fields in the nuclear medium, which are, so far, assumed just phenomenologically. For an understanding of the origin of these fields by microscopic derivation from the free nuclear force, we will present the framework.

John Dubach indicated in his lectures about "Electronuclear Reactions and Meson Exchange Currents (MEC)" that all MEC calculations up to date are rather incomplete and contain serious inconsistencies with regard to the nuclear force. The comprehensive meson-exchange model to be presented in these lectures will provide the basis for consistent and "complete" MEC calculations of the future.

Finally, there is of course a most intimate relationship between these lectures and the general topic of the second part of this workshop, namely quark-nuclear physics. In fact, the latter topic provides in a certain sense the theoretical foundation for a(n) (effective) meson theory being understood as an appropriate approximation to low energy QCD. As this aspect is of fundamental importance, we will handle it specially in the following introduction.

1. INTRODUCTION

Nowadays it is widely accepted that quantum chromodynamics (QCD)[1] is the fundamental theory of strong interactions. On that basis, the nucleon-nucleon (NN) interaction has to be considered as completely determined by the underlying quark-gluon dynamics. However, due to the mathematical problems raised by the non-perturbative character of QCD in the low energy regime, we are far away from a quantitative understanding of the NN force from this point of view.

Closely related and of even broader relevance is the problem of the confinement of hadrons. Here, the intractability of low energy QCD* is usually circumvented by the ad hoc introduction of (picking up one of the most popular euphemisms in modern physics) "QCD inspired" models, e.g. bag or potential models. There are naturally large uncertainties in the details of these models. For example, in the context of bag models, a crucial question is the size of the confinement radius R. Should R turn out to be small ($R \lesssim 0.5$ fm) as e.g. in the little bag,[3] there would be enough space for conventional hadrons like nucleons, mesons and isobars to represent the essential degrees of freedom for a wide range of nuclear physics phenomena, and meson exchange would be a valid picture. In that case, the appropriate procedure is to construct the nuclear force from meson-nucleon and meson-isobar vertices, these being understood as effective descriptions of complicated n-quark reactions. Hadron masses, coupling constants and vertex-form factors, which are the physical parameters of such a meson theory, are then left to be ultimately explained by QCD.[4,5]

Genuinely new quark-gluon-processes, which cannot even effectively be taken into account by meson-exchange, may occur only for overlapping hadrons. Therefore, their role also depends decisively on the hadron size. As discussed, for small radii ($R \lesssim 0.5$ fm) they should have a negligible influence provided the consideration is restricted to phenomena involving comparatively low energies and momentum transfer, as e.g. NN scattering up to a laboratory energy of about 300 MeV and nuclear binding energies. In this case, the introduction of meson-nucleon (-isobar) vertex form factors would be a sufficient accounting of the inner structure of the hadrons and its consequences.

The situation may be substantially different if the bag radius is large ($R \gtrsim 1$ fm) as e.g. is the case in the MIT-bag[6] or the cloudy bag model.[7] In a very naive interpretation these radii

*Presently the only promising treatment of low energy QCD is lattice gauge theory.[2] However, because of computational restrictions on present day computers, lattice QCD is not yet a practical tool for every day nuclear physics.

may tolerate just the pion, leave no space for the heavier scalar and vector bosons, and instead require the inclusion of genuine quark-glon exchanges as dominant contributions. It is worth mentioning that there has been very little success in understanding low energy nuclear forces in such a picture.

For instance, in the work of Maltman and Isgur[8] an attempt is made to describe the deuteron from six quarks with chromodynamics. However, in the end it turns out that the empirical deuteron properties (i.e. the binding energy, the quadrupole and magnetic moment and the asymptotic D/S state ratio) can only be explained when an artificial pion tail is attached, which has nothing to do with the original ansatz of that model. In fact, this pion tail provides almost the total contribution to all deuteron properties. Therefore, this work essentially fails to confirm that the deuteron properties can be deduced from a naive quark-gluon picture.

An appropriate case for the test of large "quarkish" contributions to the nuclear force would be NN scattering, especially in P-waves (L=1), since in addition to the general short-range repulsion the spin-orbit (LS) force comes into play here in a decisive way. It is known, however, that one-gluon-exchange creates too little LS force to account for the empirical NN scattering data.[9-11] The tensor force also turns out to be far too small.[10,11] The pion tail could be invoked for help again, yet, if the large bag radii are applied consistently they cut down the pionic tensor force too much. Moreover, all known models which attempt to derive the nuclear force from the quark model create little or no intermediate range attraction, so that they obtain either only repulsion[12] or, to repair for this defect, artificially add the attraction of scalar boson exchange,[13,14] which, however, contradicts the basic assumptions of those models.

The common source for these problems and contradictions may be that the idea of a bag is interpreted too naively in these models, namely merely by classical geometrical considerations. The fact that it costs ≈1 GeV (the string tension) to separate colored sources by 1 fm may be more relevant to considerations in nuclear physics than the bag radius.[15] In other words, in low energy nuclear physics, even with overlapping bags, the colour singlet exchange may very well continue to be the predominant process.

So far we have discussed only quark models (complemented by perturbative QCD). An alternative description of low energy phenomena is offered by the Skyrmion model. Following a proposal by t'Hooft,[16] QCD is generalized from SU(3) to an $SU(N_c)$ gauge group. In this generalization, $1/N_c$ is the coupling constant. If one assumes confinement, then, in the large N_c limit, QCD is supposedly equivalent to a local field theory of mesons. Further, it can be shown that in this theory the baryons arise as soliton solutions of the meson field equations,[17] an idea which was already suggested by Skyrme[18] about twenty years ago. In this

"Skyrme model" the exact size of the bag radius plays a subordinate role and can in fact be rather small.[19,20] This feature is most beautifully described by the magical picture of the "Cheshire Cat"[21] (here, the bag wall) which tends to fade away when examined closely, leaving behind only its grin (here, the confinement in principal).[22] Vector bosons can be introduced into the Skyrmion model in a natural way.[23,24] More details about this alternative description of low energy phenomena are given in a special lecture.[25]

Thus, from the point of view of the models discussed, which in some sense claim to approximate QCD, it appears that an effective meson theory may very well be the appropriate representation for the NN interaction in the domain of nuclear physics.

In addition, there is traditionally strong phenomenological evidence for a meson theory* of nuclear forces. In fact, meson-exchange is presently the only quantitative model for the NN interaction. Such an accurate representation of the nuclear force is for example needed as starting point for the large field of nuclear structure physics. This adds a further argument in favour of pursuing a meson theoretic description of nuclear physics — an argument which is quite different from those given before, as it is purely "practical." Yet, it is as compelling as the need for a quantitative nuclear structure theory.

We conclude these introductory remarks with the hope that the reader has been left some idea of the various ways in which meson theory is relevant to present day nuclear physics.

Nevertheless, we will briefly turn to the past now.

2. HISTORICAL EXCURSION

We are now going to undertake a short excursion into the history of nuclear forces, with special emphasis on meson theory. It will mean that we shall pass, essentially, through four phases of theoretical developments. The first leads from the very early ansatzes for the nuclear force until the discovery of the pion in 1947/48, and, due to the lack of empirical evidence in those days, is bound to be of highly speculative character. The following period is naturally, prevailingly, devoted to the pion. The third period starts with the discovery of the vector bosons in the early

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*We should note that nowadays the term "meson theory" is strictly speaking incorrect, since a "meson theory" in the fundamental sense of the word does not exist. (QCD may turn out to be a theory.) However, for historical reasons, this term is well established — due to the fact that originally it was really believed to be a theory — and we will continue using it. More precisely, one should speak of something like an "effective meson model."

1960's and, therefore, deals with models applying several different bosons. Another important topic of that period is dispersion theory, developed alternatively to field theory, since the latter had fallen somewhat in disgrace for some time. The finale starts in the early 1970's and is marked by the development of absolutely accurate meson theoretic potentials, essentially, along two lines: dispersion relations and field theory. Both turn out to be quite successful, bringing history — at least in the framework of meson theory — to a good ending.

2.1 The Speculative Start

Quite naturally, the theory of nuclear forces begins in 1932 right after the discovery of the neutron by Chadwick[26] suggesting a special force between protons and neutrons to explain nuclear binding. This same year, Heisenberg[27] publishes his first ansatz for the nuclear force, in which he already introduces the isospin formalism. Majorana[28] follows shortly after. Proton-proton scattering experiments develop rapidly during the 1930's up to 1 MeV laboratory energy.[29] They soon indicate that an additional force, besides the electrostatic Coulomb force, must exist also between two protons, leading finally to the hypothesis of the "charge independence" of nuclear forces.[30]

The first fundamental idea for a deeper origin of the nuclear force appears as soon as 1935. Yukawa[31] suggests that a particle with an "intermediate" mass (compared to electron and proton), therefore called "meson", could be responsible for the interaction energy between proton and neutron. The short (finite) range nature of the nuclear force is already well known at this time from the saturation properties of finite nuclei; the massive character of the exchanged particle predicted by Yukawa provides the force with such a finite range. In the first consideration[31] a charged scalar boson in classical field theory is assumed to act between proton and neutron only, in accordance with Heisenberg's[27] first assumptions. Soon Yukawa himself reconsiders his proposal in the framework of quantized field theory,[32] and further extends his idea with a group of collaborators.[33]

What nobody can possibly foresee at this time: the stage for a half-century struggle of hope and desperation is set. The known forces in these days are just the Coulomb and the gravitational force both having a very simple form. Naturally, one expects first something equally simple for the nuclear potential, e.g. just one central-force Yukawa: $Ce^{-\mu r}/r$. However, even just phenomenologically, the nuclear force shall finally turn out to be much more complicated due to its spin dependence. In addition, field theory — the framework within which the nuclear force is to be derived — shortly runs into fundamental mathematical problems with itself and with its application. Both will act soon as a seemingly never-ending source of oppressing problems and startling

surprises, which together with some great discoveries and successes gives the history of nuclear forces the touch of a detective novel.

It first starts encouragingly: In 1937 a "meson" is found in cosmic ray, the muon.[34] It is interpreted (as we know now, incorrectly) as the particle predicted by Yukawa, particularly since its mass (≈106 MeV) appears to be roughly right, according to what is known about the approximate range of the nuclear force. This discovery arouses considerable interest in Yukawa's idea; and because of that, the misinterpretation has a lucky side. So, Kemmer[35] feels inspired to suggest a rich variety of possible meson fields including pseudoscalar, axial-vector and tensor (for an explanation of these terms see Sec. 3), after Proca,[36] in 1936, has already considered vector fields. Also a "symmetric theory" (the ancient term for a theory with iso-vector bosons, i.e. bosons of three charge states: +, −, neutral) is proposed by Kemmer[37] and Bhabha[38] to account for the known hypothesis of charge independence.[30] This suggestion is made in spite of the experimental fact that at this time only charged "mesons" (μ^+, μ^-) can be found. These (in lowest order) cannot be exchanged between like nucleons, and therefore seriously violate charge independence. (Some authors express the hope that the two meson exchange may balance that.) Moreover, in 1939, Kemmer[39] discusses an entirely new, higher dimensional meson field equation, which has recently been revived in the context of meson-nucleus scattering.[40]

Wick[41] gives a concrete picture of how to relate the mass of a particle, m, to the range, R, of the force caused by its exchange, based on Heisenberg's uncertainty relation:

$$\Delta E \cdot \Delta t \approx \hbar \qquad (2.1)$$

With $\Delta E = mc^2$, the energy required to create the mass of the particle* (since the spontaneous process of "virtual" particle creation violates energy conservation), and the particle moving with the speed of light, c, allowing it the time $\Delta t = R/c$ to "stay in the air", one obtains for the range:

$$R \approx \hbar/mc \qquad (2.2)$$

which is identical to the Compton wave length of that particle. (We shall see later, Sec. 5.2, that this range estimate is not to be taken too literally, as, in practice, it underestimates the range substantially.)

*We neglect the kinetic energy of the exchanged particle assuming that its momentum $kc \ll mc^2$.

The experimental discovery of the quadrupolemoment of the deuteron by Kellog and coworkers[42] in 1939 gives rise to theoretical considerations of increasing sophistication. It is derived that vector fields create a tensor force leading to a quadrupolemoment in the deuteron, but with the wrong sign compared to experiment. This problem is soon overcome by also including pseudoscalar fields. In these "mixed meson theories", in which vector and pseudoscalar fields are assumed simultaneously (Møller and Rosenfeld[43]: with meson of equal mass; Schwinger[44]: with the vector meson heavier than the pseudoscalar), the problematic r^{-3} singularity in the tensor force can be removed. With the Schwinger force, about 1/3 of the empirical quadrupolemoment can be reproduced.[45] Concerning the singularity problem, Bethe,[46] in 1940, suggests the use of a "cut-off" for small r, which can be interpreted as assuming extended meson sources (i.e. extended nucleons). This idea and the problems it raises, especially with regard to the relativistic invariance of the theory, are examined in length, apart from others, by Pauli. An account of this question, as well as many other topical issues in meson theory of this time (e.g. the "strong coupling theory"), can be found in his lectures given at the MIT in fall 1944.[47] Most interestingly, Pauli concludes, from the fact that the pseudoscalar "symmetric" theory predicts the right sign for the deuteron quadrupolmoment, that this is most likely the correct theory — long before the pion is found and its spin and parity determined. Also quite early it is recognized by some physicists that vector and scalar fields create a spin-orbit force (Breit[48] (1937/38), Rosenfeld[49]). Empirical evidence for this is seen in the spectra of light nuclei. E.g. Rosenfeld* in 1948: "The occurrence of a rather large spin-orbit coupling in ^{5}He may be regarded as an indication of the existence of mesons of spin one."

In 1948/49 A.E.S. Green[51] takes up again the Kemmer idea[37] of a rich variety of meson fields. He shows that the problem of short-range singularities can also be overcome when generalized meson fields with higher derivatives in the Lagrangian are used. Part of his work is rejected from publication for being to "speculative."[52]

For further details about this first, "speculative" period of meson theory (i.e. before the discovery of the pion), we refer the interested reader to the above-mentioned lectures by Pauli,[47] and the book by Wentzel[53] which contains an informative chapter on this topic. An extremely thorough account of the considerations in these days, including many contemporary details, is given in the book by Rosenfeld about nuclear forces,[50] published in 1948.

Experiment finishes this period: In 1947 Conversi, Pancini and Piccioni[54] show that the μ does not interact strongly with

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* Ref. 50, p.368.

nuclei, and therefore is, in fact, not a meson — it is a lepton. The same year, a meson with a mass of about 140 MeV, which does interact strongly, the "pion", is found in cosmic ray by Occhialini and collaborators,[55] and shortly after in the Berkeley cyclotron laboratory.[56] The final and conclusive confirmation of the reality of mesons is given in 1949 by the Swedish Academy of Science by awarding the Nobel prize to Yukawa.

2.2 The pion battle

Quite understandably, the new reality of a strongly interacting meson motivates vigorous theoretical efforts to describe the nuclear force, now, by the pion only. This will become the program of the following decade — the 1950's. Naturally, it starts with high expectations and great enthusiasm (but will end, however, in deep disappointment). The success of renormalization has just put field theory on firm grounds. The pion appears to be the particle for the strong interaction in analogy to the role of the photon in quantum electrodynamics (QED). Considering the great quantitative successes of QED, it is hard to set a limit on the expectations for strong interaction theory.

The work of Japanese physicists deserves our special attention for this time, as — in the tradition of Yukawa — they probably did the most in this field and at the same time may have also been ignored the most outside their country. In 1951, Taketani and associates[57] (TNS) present their historic suggestion to subdivide the nuclear force into three regions. The far-sighted character of this suggestion is best indicated by the fact that this subdivision is still in use today. TNS speak of a "classical" (long range, $r \gtrsim 2$ fm),* a "dynamical" (intermediate range, 1 fm $\lesssim r \lesssim 2$ fm) and a "phenomenological" or "core" (short range, $r \lesssim 1$ fm) region. In the classical region the longest range part of the potential, namely, the one-pion-exchange (OPE) is dominant (the pion having the smallest mass of all mesons or multi-meson configurations). In the dynamical region the two-pion-exchange (TPE) becomes important. Finally, in the core region everything can contribute: in particular, multi-pion exchange, heavy mesons, and (from today's point of view) quark-gluon exchange. This classification has been of utmost practical importance because it allows a step-by-step exploration of the two-nucleon interaction. Thus it is possible, to first calculate the longer range parts of the potential, and correlate the results with experiments sensitive to just that region. (We will do this in Sec. 4.) In this way, the whole problem — with all its oppressive complexity — does not have to be faced at once. By

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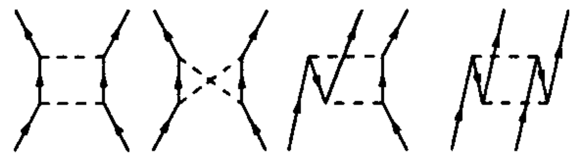
*r denotes the distance between (the centres of) two nucleons.

the way, in the light of TNS e.g. the use of extended sources ("cut-offs") does not appear so forbidden anymore!

In the decade under consideration, the one-pion exchange becomes experimentally well established as the long range part of the nuclear force.* The evidence comes from the analysis of NN scattering data and the deuteron. Speaking in dispersion theoretic terms (see Sec. 2.3) — the scattering amplitude in the non-physical region of the complex cos θ plane (θ being the scattering angle in the CM system) has two symmetrically situated poles associated with contribution of one-pion intermediate states. The experimental data is extrapolated to these poles to yield the pion nucleon coupling constant. This procedure can be applied both near 0° and 180° for the scattering angle θ. The πN coupling constant obtained in this way agrees with that known from πN scattering.[59] An equivalent procedure can be applied in the framework of the (partial wave) phase-shift analysis, where for sufficiently high orbital angular momentum L (corresponding to large distances) OPE alone is assumed. It turns out that the χ^2 for the phase-shift solutions is a minimum for the correct pion mass and coupling constant (as known from πN scattering).[60,61] In the case of the deuteron the evidence for the one-pion-exchange comes from the quadrupolemoment which is almost entirely explained by OPE.[62,63,64] It is also realized that the asymptotic D/S state, sometimes called η, is determined by far from OPE.[64,62] However, reliable measurements of η will not appear before 1980. In fact, nowadays, η may be considered as the most precise and compelling evidence for the reality of the pion in the nuclear force.[65]

As the OPE contribution to the nuclear force combines all the pleasant features a physicist may wish from a theory — such as easy to evaluate and most satisfactory to explain data — so does the TPE evolve in an opposite way. It is painful to evaluate and for a long time it has not even been doing well in correlating data. The calculations are not only extremely complicated and tedious, but, in addition, they are beset, for a long time, by a number of serious ambiguities, which lead to quantitatively rather different results causing serious controversies. The many efforts of pion theoretical potentials of the 1950's are usually divided into two groups: The Taketani-Machida-Onuma (TMO)[66] and the Brueckner-Watson (BW)[67] type. In the former case an S-matrix is evaluated directly from meson field theory, from which in turn a potential is derived. In contrast, the BW method is based on an expansion in the particle number and derives a potential directly. The main differences between the two approaches are that the box-diagram (Fig. 2.1(a)) and the pair terms (Fig. 2.1(c) and (d)) are

*For a survey on the related Japanese work see Ref. 58, especially p.32 therein.



(a) (b) (c) (d)

FIG. 2.1. Some two-pion-exchange contributions to the NN interaction. (a) is called a box diagram, (b) a crossed box. (c) and (d) contain "pair terms." There are also pair term diagrams with crossed pion exchange, which we have not shown here. Full lines denote nucleons, dashed pions. The underlying time axis is vertical, pointing upwards into the future.

always included in TMO whereas BW excludes the box from the beginning and can also at will leave out the pair terms. For this "pair suppression" there is evidence from πN scattering in S-waves where the pair term leads to (by about one order of magnitude) too large a scattering length. Therefore it has been suggested that the suppression of virtual pair terms might be a general feature of meson theory.[68] In addition, BW find an almost exact cancellation of the one-pair (Fig. 2.1(c)) and the two-pair (Fig. 2.1(d)) contribution to the NN interaction. A further source of discrepancies in the pion theories of the 1950's are ambiguities in the subtraction of the iterated OPE necessary to extract a potential.[69] Apart from the general uncertainty in the results, the spin-orbit force derived from TPE turns out to be too weak (by one order of magnitude) than experimentally needed.[70]

The balance of this decade is given in a quote by Goldberger[71] from 1960: "There are few problems in nuclear theoretical physics which have attracted more attention than that of trying to determine the fundamental interaction between two nucleons. It is also true that scarcely ever has the world of physics owed so little to so many. In general, in surveying the field, one is oppressed by the unbelievable confusion and conflict that exists. It is hard to believe that many of the authors are talking about the same problem or, in fact, that they know what the problem is."

Already in 1953, Bethe — in an article for the Scientific American[72] — estimates that in the preceding quarter century more man hours of work have been devoted to the NN problem than to any other scientific question in the history of mankind. In fact, a second quarter century will follow in which about one order of magnitude more man hours are invested in this problem.

The interested reader will find an excellent review of this period in the book by Moravcsik,[73] the review article by

Phillips[73] and a more comprehensive account in Bethe and de Hoffmann.[74] The enormous work by the Japanese physicists is summarized in two Supplements.[58,75]

Fortunately, there is also another line of research on the NN interaction during the 1950's; and this line, developing in almost complete independence from the theoretical efforts discussed so far, is more successful. The goals are much more modest — it is the attempt to give a simple phenomenological description of the NN potential.

Data from NN scattering up to about 400 MeV accumulate rapidly during this period and the phase-shift analyses improve both through better theoretical constraints and an increasing data base.* Gradually, sufficient empirical information on the NN interaction up to 300 MeV laboratory energy (and especially around that latter energy[77]) is available for a phenomenological potential description.

The basic aim of a potential description of the two-nucleon interaction is twofold. One is to provide an economical summary of the data for comparison with potential-like results from theory (e.g. meson theory). The other aim of a phenomenological potential is to serve as an input for nuclear structure calculations.

From general invariance considerations, already carried out in the 1940's,[78] the most general form of a non-relativistic potential can be deduced. It consists of a central, spin-spin, tensor, spin-orbit and quadratic spin-orbit term.** The first attempt to fit the NN scattering data with just the first three terms fails.[79] Another attempt, which includes in addition the spin-orbit force, is quite successful.[80] It is the first (semi-) quantitative NN-potential ever constructed — known by the names of Gammel and Thaler[80] and published in 1957. This potential uses a hard core at small distances suggested by the trend of the $^1S_0$ phase-shift to turn negative for laboratory energies $\gtrsim$ 250 MeV. (A hard core was first considered by Jastrow[81] in 1951 to account for the seemingly isotropic differential cross section in pp scattering at 340 MeV.) The spin-orbit force is demanded by the triplet P-phase-shifts. Its relatively short-range character is evident from the fact that F-phase shifts do not show much spin-orbit splitting anymore. The important role of the spin-orbit force in nuclei was already forseen in the late 1930's (Breit[48]) and assumed in the late 1940's (Rosenfeld,† Mayer and Jensen[82]).

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*A review with particular stress on the experimental aspect is found in the book on the NN interaction by Wilson.[76]

**For more details see Ref. 78 and Moravcsik, Ref. 73, p.46.

†Ref. 50, p.368.

Another early semi-quantitative potential is that of Signell, Zinn and Marshak.[83]

A few years later improved phenomenological hard-core potentials follow, notably the Hamada-Johnston[84] and the Yale[85] potentials. Both use all five terms for a potential mentioned above. In contrast to Gammel-Thaler, these latter potentials also fit the deuteron accurately since they include the OPE potential. The "Reid" hard- and soft-core potentials[86] appear in the late 1960's. The soft-core version becomes one of the most applied potentials in nuclear physics of the 1970's. Phenomenological potentials, as we have them discussed here, use in general on the order of 50 parameters to fit the data.

2.3 The OBEP and "Dispersive" Phase

Now, we take up again, the description of the meson theoretic development of the nuclear force. It is now 1960. This time is essentially characterized by two points: the failure of the pion field-theoretic program and the accumulation of rich phenomenological knowledge about the NN force — particularly concerning a short-range repulsion and a spin-orbit force. It is obvious that this has to revive the old idea of vector meson exchange (which has both the features just mentioned) discussed already in the late 30's (Breit[48]) and in the 40's (Rosenfeld[49,50]). As vector boson exchange is not renormalizable, it had been discarded for a number of years. Nambu,[87] Breit,[88] Sakurai[89] and Frazer and Fulco[90] reconsider the possibility of vector bosons with regard to the NN interaction as well as in conjunction with the electromagnetic form factor of the nucleon. Their presentiment is soon confirmed: in 1961 the ω-meson is discovered[91]; the ρ follows shortly after.[92] Both are spin one bosons, the ρ being a 2π- and the ω a 3π-resonance with masses of approximately 770–780 MeV.

The discovery of heavy mesons leads to the one-boson-exchange (OBE) model for the NN interaction, which breaks the dead-lock situation in meson theory. The OBE model is based on the old idea that the nuclear force is meson mediated. However, it tries to take advantage of the observations in high energy physics that two or more particles (e.g. pions) as a group like to behave most of the time as if they were forming a single particle with a definite mass and definite intrinsic quantum numbers (i.e. they are "correlated" or even form a resonance). It is hoped and, in fact, assumed in the OBE model, that the uncorrelated many-pion exchange (i.e. with no interaction in between the pions "in the air") may be negligible (apart from iterative contributions which are generated by the unitarizing equation). (In contrast, the calculations of the 1950's were concerned only with the uncorrelated plural-pion exchange.) This basic assumption is certainly too extreme and we will see how and why the OBE-model can make up for that.

There is also a very pragmatic reason for this model. As we see from the work of the 1950's, the explicit evaluation of multipion exchanges is very difficult and ambiguous. In contrast, the calculation of one-particle exchanges is relatively straightforward.

All known OBE models include a large contribution from one or two isoscalar scalar bosons with their mass in the range of 400 to 800 MeV, to provide the indispensable intermediate range attraction for the nuclear force. These particles are supposed to represent 2π-S-wave resonances. Indeed, during the 1960's and early 70's, the Particle Data Group listed in their meson table an alleged isoscalar scalar 2π-S-wave resonance (called σ or ϵ) in the above mass range.[93] This low mass ϵ has disappeared from the tables since 1976[94] in favour of an $\epsilon(1200)$ (which cannot be used by OBE potentials to provide intermediate range attraction and, therefore, is useless within that model). Thus, on this point of the OBE model, confirmation does not happen. However, as the model is very successful, it seems to indicate that in the case of the NN-interaction even uncorrelated 2π-exchange may be well simulated by a boson of definite mass and zero width. Still, from a more fundamental point of view this is not satisfactory and therefore we will come back to this point later.

Some of the first OBE potentials (OBEP) are those of Bryan and coworkers,[95] Wong and collaborators,[96] Japanese theorists,[97,98] McKean[99] and A.E.S. Green and associates,[100] who could pick up the threads of earlier work.[51,52] One of the great successes of all these models is that they need in the order of only 10 parameters to fit the NN data (in contrast to phenomenological potentials which require about 50). These parameters are meson-nucleon coupling constants and cutoff parameters.

The improvement of the OBE model continues into the 1970's. In fact, OBEPs which accurately describe all NN data up to about 300 MeV and the deuteron are not provided until the middle 1970's. Such potentials, represented in configuration space (r-space), are constructed by the Nijmwegen group[101] and Sprung and coworkers[102] — the latter being, however, semiphenomenological. The OBE concept further improves by considering three-dimensional relativistic reductions of the Bethe-Salpeter[103] equation and by working in momentum space to avoid the drastic approximations necessary to obtain analytic r-space expressions. The most accurate work along this line is done by the Bonn group.[104-106] Tjon and collaborators[107] finally solve the four-dimensional Bethe-Salpeter equation in the ladder approximation constructing an OBEP in that framework.

The work up to 1971 is well summarized by Moravcsik[108] with a complete list of the bibliography. Good reviews on this subject are the contribution by S. Ogawa et al. in the Supplement Ref. 75 and the Proceedings of the First International Conference on the Nucleon-Nucleon Interaction held in Gainsville, Florida, in 1967

(Ref. 109), which also reflects some of the enthusiasm of the early OBEP-years.

I mentioned before that quite apart from the quantitative success of the OBE model in fitting the empirical NN data, such models cannot be conceptually accepted to form a complete theory, as it is hard to believe that the uncorrelated plural-particle exchange should be totally negligible. The longest range component of such exchanges, and therefore the most important of that kind, is the two-pion-exchange (TPE). How to take into account the TPE more accurately, or even "completely," is the other main topic — besides the OBE model — of the 1960's.

Of course, this topic is not new. It was one main goal of the 1950's, and it failed at that time. In retrospect, and with the new knowledge which accumulated during the 1960's, we now understand the reason for this failure: the old program did not consider correlations between the exchanged pions or pion-resonances, which, as the OBE model has demonstrated, play a crucial role in the two-nucleon force.

Naturally, the new goal must be to include "all" correlated and uncorrelated multi-particle exchanges; first, for two pions.

There are two conceptually very different ways to actually calculate these contributions: by field theory and by dispersion relations. It is psychologically quite understandable, that after the failure of the field theoretic program in the previous decade, there is not much motivation to try this again. Therefore, during this period, there is only little work along this line. Fortunately, field theory is not dead forever; in fact, it will revive intensely later on.

A completely alternative approach to multi-particle exchange is now pursued, which has become known as "dispersion relations." Apart from the disappointment and the doubt about field theoretic techniques there is also a second positive motivation for this new approach: it can take correlated and uncorrelated multi-particle exchange into account within the same framework.

Since in the actual work on the NN interaction presented later in these lectures, the dispersion theoretic approach is never used (we will present a field theoretic model), however, results gained from dispersion relations are sometimes quoted, so let us here, very briefly, outline what this approach is about. For an excellent introduction see Moravcsik,* and for more comprehensive explanations see Mandelstam[110] or Chew.[111]

As this new approach is born out of a frustration with field theory — with its formidable problems of renormalization, convergence, selection of diagrams (e.g. pair terms or not) and how to develop a potential concept — dispersion theory attempts to avoid these shortcomings from the beginning trying to deal with

*Ref. 73, p.104.

physically observable quantities only. Lagrangians, Hamiltonians and potentials do not occur anymore. Instead, the new theory deals directly with reaction amplitudes. In fact, one of its great advantages (especially with regard to the NN interaction) is, that it can relate different measurable reactions to each other. In this way it provides the framework for a consistency check between such different processes as NN, πN and nucleon-electron scattering (nucleon electromagnetic form factors).*

The principal framework of dispersion relations is based on three fundamental assumptions: causality, unitarity and crossing symmetry. From the first the analyticity of the reaction amplitude is concluded. The third allows us to relate processes which differ from each other only by the interchange of some incoming and outgoing particles of the reaction. One-particle exchange appears as a pole in the reaction amplitude. In this way, dispersion theory gives empirical evidence for the reality of meson-exchange. (We discussed this for the pion before.)

To calculate the 2π-exchange contribution to the NN interaction with the help of dispersion theory, one proceeds as described schematically in Fig. 2.2. The total "diagram," (a), is divided into two halves, (b), each of which, when considered in the t-channel (i.e. looking horizontally from left to right into the diagram (c)), represents an amplitude $N\bar{N} \to 2\pi$. This amplitude is constructed from empirical input from πN scattering, (d),** and ππ scattering (shaded circle in (e)).

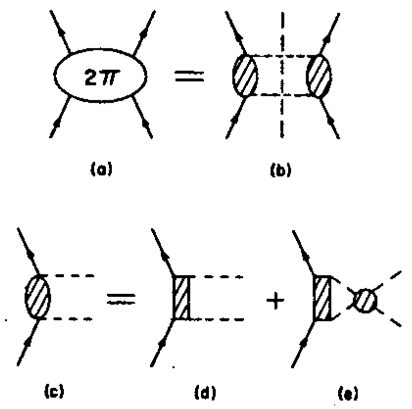

FIG. 2.2. 2π-exchange contribution to the NN interaction as viewed by dispersion theory and explained in the text. (Full lines represent nucleons, dashed lines are pions.)

Work along this line starts as early as 1960 (e.g. Amati et al.[112]), and soon after many groups are actively engaged in this field. (For a comprehensive review of the past literature see Moravcsik.[108]) Nevertheless it takes until the end of the decade for quantitative results (see e.g. Signell and Durso,† Furuichi,†† Binstock,[113] Chemtob et al.,[114] Vinh Mau et al.[115]). All these results indicate that the 2π-exchange is able to provide the intermediate range attraction of the nuclear force. Still, these efforts are far from constructing a full quantitative NN potential.

*See e.g. Signell and Durso, Ref. 109, p.635 and earlier references therein.

**The shaded rectangle in (d) stands for the nucleon and all possible πN resonances (e.g. Δ(1232)).

Thus finishes the OBEP and "dispersive" period of the 1960's.

2.4 The finale

The finale takes place essentially in the 1970's and early 1980's. It is concerned with the ultimate goal of finally providing an absolutely quantitative nuclear force, with a derivation based on something which can be (almost) called a theory. (Phenomenological potentials deliberately and OBEPs for reasons given in Sec. 2.3 do not fulfil this requirement.)

Again, the work proceeds along two lines: dispersion theory and field theory. Both lines turn out to be finally very successfull. Therefore, in the end there will be two accurate NN potentials with a sound theoretical basis: The Paris and the Bonn potential.

However, let us proceed in chronological order. First, the final disperion theoretic efforts.

In continuation of the work of Chemtob et al.,[114] which was mentioned at the end of the last subsection, the Stony Brook Group[116] constructs a potential, in which the dispersion theoretic result for the 2π-exchange is complemented by one-π and one-ω exchange. For short distances the eikonal form factor is used.[117] The fit to the phase-shifts of NN scattering is semi-quantitative. The Paris group publishes a potential that same year.[118] In this case the short range part of the potential ($r \lesssim 1$ fm) is treated purely phenomenologically. For the 2π-exchange contribution derived by dispersion relations both groups agree quantitatively. Further refinements and a convenient representation of the potential are left to the Paris group, which publishes their final version in 1980.[119] The potential is expanded in Yukawa functions of multiples of the pion mass.

A brief review about the dispersion theoretic approach to the NN interaction is given by Vinh Mau.[120] Detailed explanations can be found in the book by Brown and Jackson.[121]

Let us now turn to the field theoretic line. After a decade of prevailing abstinance, for reasons explained earlier, the field theoretic approach is revived by the work of Lomon and Partovi.[122] They evaluate the 2π-exchange Feynmann diagrams with nucleons and

†Ref. 109, p.635

††Ref. 75, p.190

represent the result in the framework of the relativistic three-dimensional reduction of the Bethe-Salpeter equation[103] proposed by Blankenbecler and Sugar.[123] It is a non-static approach to the 2π-exchange. By using this well-defined framework the ambiguity is absent of how to construct and subtract the iterated one-pion-exchange, when defining a potential. So, in 1970 Lomon and Partovi finally showed how to do the 1950's program correctly. Their quantitative result is neither close to BW[67] nor to TMO.[66] In the following years Lomon and coworker extend their calculations by also introducing a σ-boson.[124]

For a "complete" field theoretic model for the 2π-exchange more diagrams than those in Ref. 122 and 124 must to be included. From dispersion theory one knows that isobars in intermediate states lead to substantial contributions (see Signell and Durso,* Sugawara and von Hippel,[125] Tamagaki**). A field theoretic model has to take that into account. Further, 3π and 4π exchange should be considered. The program aiming for such a comprehensive and "complete" field theoretic model for the NN interaction is pursued by the Bonn group. Over a number of years this group evaluates step by step the 2π-exchange diagrams[126-129] and finally also the relevant 3π and 4π exchanges.[127,130,131] The complete set of diagrams gives in a quantitative description of all known NN data (up to about 300 MeV) and the deuteron with very high precision.[132,133] (More about this in Sec. 4.)

There are several advantages to a field theoretic model. First, it determines the off-shell behaviour of the potential in a well-defined way. As dispersion theory deals with reaction amplitudes, which are always on-shell, the off-shell behaviour remains undetermined in such an approach and is left to guess-work or using the simplest ansatz (e.g. Yukawa's). The "complete" set of diagrams, which a field theoretical model provides, is a necessary basis for the consideration of related processes, e.g. meson-exchange current corrections to electromagnetic properties. Moreover, when applying the nuclear force in the many-body problem, medium effects alter the nuclear force. They can be evaluated on the basis of a field theoretic model. The same is true for many-body forces. (More about this in Part II of these lectures.)

Herewith finishes our excursion into history. It started with Heisenberg[27] and Yukawa[32] and took us through half a century. It is a story of excitement, error, depression and especially of hard, hard work which, however, finally has led to a successful end.

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*Ref. 109, p.635

**Ref. 109, p.629

After history has given us a general background, let us now get to know the details of the meson theory of the NN interaction. This will be done in the next two sections.

3. A PEDAGOGICAL INTRODUCTION

In this section we want to look more closely into meson-exchange. Our goal is to understand the principle features meson-exchange can predict for the NN problem. With "understand" we mean here really understand from the point of view of theory, and that is, we want to directly and explicitly derive things starting from first principles. This sounds very ambitious, but we will see that this can be done quite easily (for the lowest order contributions, to which we will restrict ourselves in this section).

As theory is no end onto itself, we have to compare the results of our theoretical derivations with experiment, to see if anything is predicted which agrees with reality. For this purpose, we want to recall first, what is known empirically about the NN interaction. In this section the discussion can be restricted to essentially qualitative considerations. The logic behind this is, that we had better make sure, first, that our theory offers the right qualitative features, before going to a quantitative approach. (The latter will be done in Sec. 4.) Moreover, the stress in this section is on physical understanding; a qualitative consideration can serve that purpose much better than the chaos of precise numbers and curves (of which we will get enough, anyhow, in Sec. 4).

3.1 Facts About the Nuclear Force

Of course, we cannot mention all the facts. For a comprehensive description of the NN interaction the reader is referred to the literature. The first two books devoted exclusively to the two-nucleon problem are still today excellent introductions into this field (Moravcsik,[73] Wilson[76]). An excellent introductory review has been written by Signell.[134] We further recommend the book by Brown and Jackson.[121] Conventional nuclear physics textbooks also have in general a useful introductory chapter on the two-nucleon problem, see e.g. Segrè.[135]

Here, we will pick only the most remarkable and outstanding empirical properties of the nuclear force. These are essentially five, and we will go through them one by one indicating also same evidence for each of them.

1. Nuclear forces are of short range (finite range). "Short" is meant here in comparison with the Coulomb or gravitational force. Early evidence for this was seen from the saturation

properties of nuclei. When going from the A=4 nucleus (A=number of nucleons in a nucleus), namely helium, upwards to higher A nuclei, one realizes that the binding energy per nucleon in a nucleus is about constant. The density remains also roughly the same as the radius of heavy nuclei is proportional to $A^{1/3}$. If the nuclear force were of large range, like e.g. the Coulomb force, the potential energy per particle would increase with A and so would the density. On the other hand, for light nuclei ($A \lesssim 4$) the binding energy per particle does grow. The deuteron is bound by 2.2 MeV, ^3H by 8.5 MeV. It is best, to analyze this in terms of energy per "bond." Thus, the binding energy per bond is about 2 MeV in the two-nucleon system and 3 MeV for the triton. In ^4He we have ≈4.5 MeV per bond (28 MeV total binding). The conclusion is, when nucleons are pulled closer to each other by more bonds (due to more particles) also the energy per bond increases (up to saturation). From this Wigner,[136] in 1933, concluded that the nuclear force has to be of short range (namely shorter than the deuteron diameter of about 4 fm and about equal to the radius of the alpha particle of about 1.7 fm).

2. The Nuclear Force is Attractive in its Intermediate Range. "Intermediate" is meant here with respect to the total range of the nuclear force, which we now subdivide according to TNS[57] in short, intermediate and long range. The proof for the attractive character of the nuclear force (at least, in a certain range) is given by the fact of nuclear binding. The range of this attraction can be obtained — now, more precisely than in point 1 — by considering the central density of heavy nuclei, which is known from electron scattering experiments from nuclei. This density (= nuclear matter density) is about 0.17 fm^{-3} providing for each nucleon a volume with a radius of about 1 fm. Thus, the average distance between the centers of two nucleons in the interior of a nucleus is about 2 fm. This should be about the effective range of the attraction. Further evidence for the partially attractive character of the nuclear force comes from scattering. The $^1S_0$-phase-shift is positive (equivalent to attraction) for a laboratory energy (E_{LAB}) $\lesssim$ 250 MeV, see Fig. 3.1. (For an explanation of the term of phase-shifts and how to obtain them from scattering data and, further, for the "spectroscopic" notation used for defining partial wave NN states, as e.g. "$^1S_0$," see Messiah,* Moravcsik** or Wilson***.)

*Ref. 138, p.385

**Ref. 73, p.68

***Ref. 76, p.110

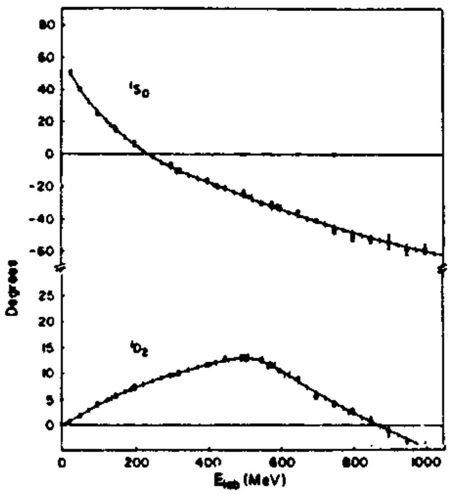

FIG. 3.1. Empirical $^1S_0$ and $^1D_2$ phase-shifts from NN scattering. The curves are an energy-dependent phase-shift analysis. The error bars represent results from an energy-independent analysis. (From Arndt et al.[137])

3. The Nuclear Force has a Repulsive Core. Such an assumption would help explaining the saturation of nuclear forces and the constant nuclear density. But this argument is not compelling proof for a repulsive core as saturation can also be generated in other ways (e.g. by "exchange" forces[27] or by Pauli and relativistic effects (see Part II)). Historically, as we pointed out in Sec. 2.2, a repulsive core was first suggested by Jastrow[81] in explaining the isotropy of proton-proton (pp) differential cross sections at E_{LAB} = 340 MeV. However, the best argument is the behaviour of the $^1S_0$ and $^1D_2$ phase-shifts as a function of increasing energy. The latter stays positive (equivalent to attraction) up to about 800 MeV (see Fig. 3.1) whereas the $^1S_0$-phase-shift turns negative (equivalent to repulsion) around 250 MeV. As an S-state (orbital angular momentum L=0, no centrifugal barrier) feels the innermost region of the force, whereas in a D-state (L=2) the nucleons are kept apart through the centrifugal barrier, one may conclude that a repulsion of short range is indicated. The rule of thumb, that relates the range of a potential, R, to the highest orbital angular momentum state, L_{MAX}, which is still affected by that range, is given by the semi-classical formula*:

$$L_{MAX} = R \cdot p \qquad (3.1)$$

*From now on we use natural units:
$\hbar = c = 1$; conversion factor: $\hbar \cdot c = 197.3$ MeV·fm

where p, the momentum of a nucleon in the centre of mass (CM) frame of the NN system, is related to E_{LAB} by:

$$E_{LAB} = \frac{2p^2}{M} \tag{3.2}$$

with M the mass of the nucleon. For E_{LAB} = 250 MeV, the turning point in the $^1S_0$ phase-shift, we have $p \approx 1.7\ fm^{-1}$. From this we obtain with $L_{MAX} \lesssim 1$:

$$R \lesssim 0.6\ fm \tag{3.3}$$

for the radius of the repulsive core.

4. There is a Tensor Force. The most striking evidence for this is seen in the deuteron: the quadrupolemoment,[42] the magnetic moment (which requires a D-state contribution) and the asymptotic D/S ratio.[65] The fact that for $^3S_1$ a bound state exists and for $^1S_0$ does not, is also an indication (the tensor force operator is zero in singlet (S=0) states; in triplet states it provides attraction, especially, through its second order contribution). Further evidence is given by the non-vanishing ε_J "mixing parameters" obtained from a phase-shift analysis (Fig. 3.2). This parameter is proportional to the matrix element for a transition from L = J - 1 to L' = J + 1. Of all operators, which can occur in the most general form of a non-relativistic

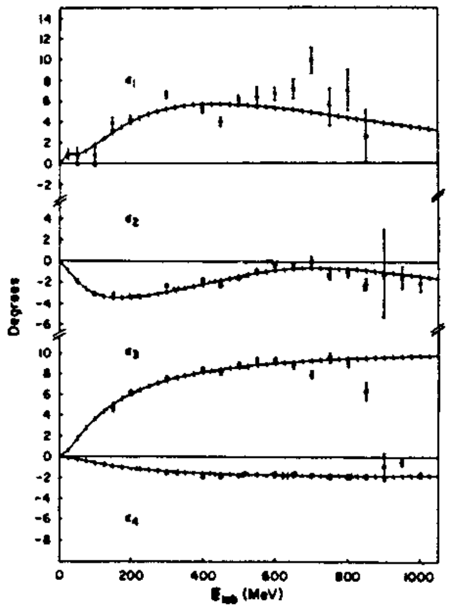

FIG. 3.2. Empirical ε_J mixing parameters for J=0 to 4. Notation as in Fig. 3.1. (From Arndt et al.[137])

potential,[78,*] only the tensor operator has non-vanishing matrix elements for this kind of transition.

5. There is a Spin-Orbit Force. This was first observed in the single particle model for nuclei,[48,50,82] though this refers to an "effective" nuclear force in the nucleus, which is not identical with the "free" NN interaction we are discussing here. However, these two forces should be related; in fact in principal the effective force could be derived from the "free" within a many-body theory (see e.g. Part II of these lectures). Clear evidence came from the first reliable phase-shift analysis of NN scattering;[77] the triplet P-waves can only be explained by assuming a strong spin-orbit force,[80,61] see Fig. 3.3. (In singlet states, $\underset{\sim}{S}$=0, and S-states, L=0, there is no spin-orbit force, as the $\underset{\sim}{L}\cdot\underset{\sim}{S}$ operator has vanishing matrix elements.)

Speaking in more general terms, point 4 and 5 are dealing with two special cases of spin-dependences in the nuclear force. In fact, there is more of it, namely, in addition, there exists a spin-spin force. However, the empirical evidence for this in the free NN interaction is not so outstanding as for the tensor and

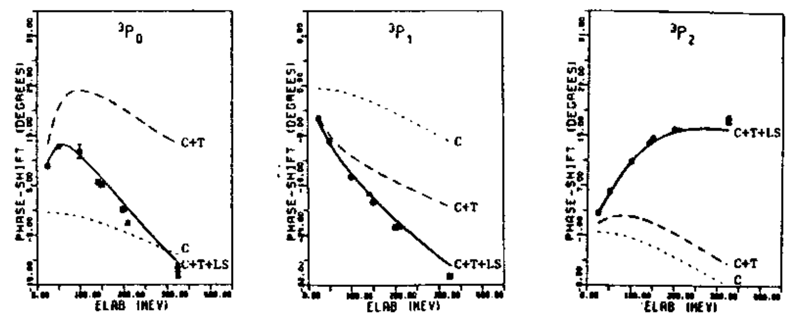

FIG. 3.3 Theoretical phase-shifts for triplet P-states calculated with (full line) and without (dashed line) a spin-orbit force. In addition, the phase-shifts as predicted by the central force (= central + central spin-spin) only are shown (dotted). The difference between the dotted and dashed line is due to the tensor force. It is clearly seen that a variation of the tensor force could not make up for the discrepancies, which occur between theory and experiment, when the LS force is omitted. The error bars represent the results from energy-independent phase-shift analyses.

*See also Ref. 73, p.46.

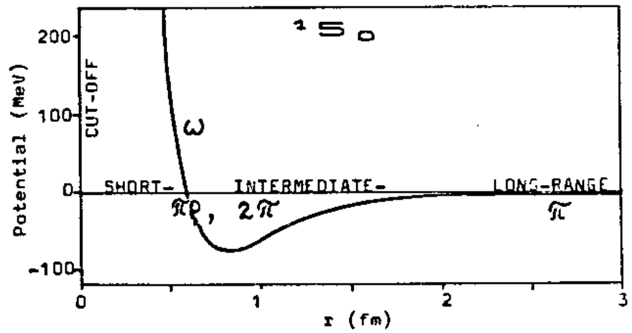

FIG. 3.4. r-dependence of the central force in the $^1S_0$-state. The greek letters refer to meson-exchange contributions to the nuclear force being explained in Sec. 4.

spin-orbit force. Therefore, we have not listed it here separately.

From our discussion of the empirical features of the nuclear force, a r-dependence of the central force like that plotted in Fig. 3.4 is to be expected.

This finished our summary of the empirical facts about the nuclear force. With these facts in the back of our mind, we can now turn to first attempts of a theoretical explanation.

3.2 The Historic Idea

In the 1930's the best established and most striking (and for a long time also the only reliably known) feature of the nuclear force was its short-range nature[136] by which it is distinguished from all other forces well-known at that time, namely Coulomb and gravitation. That is why, the first theoretical attempts concentrated on how to derive a force of finite range from some more fundamental idea. Yukawa[31] did this in 1935 by constructing a strict analogy to quantum-electro-dynamics (QED). His first consideration[31] was carried out in the framework of classical field theory, and we will repeat it here.

In QED a field of particles with zero mass, the photons, is assumed fulfilling a field equation, which is — in static approximation (and considering the fourth component of the field only) — the Laplace equation of classical electrodynamics:

$$-\Delta V(\vec{x}) = e\delta(\vec{x}) \tag{3.4}$$

($\Delta \equiv \nabla^2$, Laplace-Operator)*
The solution is:

$$V(r) = \frac{e}{4\pi}\frac{1}{r}\,; \qquad r \equiv |\vec{x}| \tag{3.5}$$

the familiar Coulomb potential.

In analogy, in meson theory a field of particles with non-zero mass, μ, the mesons, is assumed, fulfilling a field equation, which is the Klein-Gordon equation:

$$(\Box + \mu^2)\phi(x) = g\bar{\psi}(x)\psi(x) \tag{3.6}$$

Assuming the nucleons, represented in Eq. (3.6) by their fields $\psi(x)$, as infinitely heavy and fixed at the origin, we obtain the static approximation of this equation:

$$(-\Delta + \mu^2)\phi(\vec{x}) = g\delta(\vec{x}) \tag{3.7}$$

The solution is:

$$\phi(r) = \frac{g}{4\pi}\frac{e^{-\mu r}}{r} \tag{3.8}$$

the "Yukawa" potential. Through the exponential, which is a direct consequence of the massive character of the particles, this potential has the desired finite range. For zero-mass, μ=0, we are back to the Coulomb potential.

This consideration was the birth of particle physics.

Simple and traditional estimates of the range, R, identify it with

$$R = \frac{1}{\mu} \tag{3.9}$$

From this a range of 1.4 fm is predicted for the pion, which is certainly too small; in fact, at that range the pion just starts to become dominant. This argument and further results displayed in Sec. 5.2 indicate that the conventional range estimate, Eq. (3.9), is generally too small. The essential reasons for this are that the coupling constants are large and that the final nuclear potential is a result of strong interferences of large contributions. In practice an "empirical" factor of 3 to 4 should be applied to $1/\mu$ in Eq. (3.9).

*For notation see Ref. 138, 139.

3.3 Quantized Field Theory, Perturbation Theory and Feynman Diagrams

As we saw, the very first, "historic" consideration was done in the framework of classical field theory. Of course, in a correct treatment one has to use quantized field theory. This has been developed first for QED. Dealing with interacting fields, perturbation theory is applied, which is reasonable for a coupling constant of 1/137. The contributions from perturbation theory are most conveniently represented by Feynman diagrams.[139,140]

In the case of nucleons and mesons ("meson theory") the coupling constants are in the order of 10. Therefore, it must appear questionable if a perturbation expansion is appropriate. Nevertheless, it is customary to use perturbation theory and, consequently, to consider the various possible contributions in the language of Feynman diagrams. (An exception is the old "strong coupling theory,"[47] which, however, on the other hand has the disadvantage of being based on classical field theory.) A justification for the use of perturbation theory can be given in terms of the Taketani program[57] (compare Sec. 2.2). Contributions of increasing order, which may finally be divergent, are of shorter and shorter range. Therefore, for the long and intermediate range, there is definitely only a finite number of contributions. Thus, one may have confidence in the predictions for these ranges. In the very short range part of the force, due to the quark-structure of hadrons, meson-exchange cannot be taken seriously, anyhow. For that reason, in most meson theories, one allows for a partly phenomenological treatment of the short distances by the introduction of vertex form factors, which, in a certain sense, take the extended structure of hadrons effectively into account. Fortunately, since the nuclear force is repulsive at short distances, the phenomenology of the very short range is "masked" behind a repulsive wall. Thus, one expects that, at least, for energies typical for nuclear physics, the uncertain part of the nuclear force at very short distances and the special way, in which it may be treated in a particular model, is insignificant.

For the reasons given, we follow here the conventional treatment and consider meson-exchange in perturbation theory; that is, more practically speaking, we will be dealing with Feynman diagrams. Obviously, we shall start with the lowest order: the one-boson-exchange contribution to NN scattering. (In fact, this is the only order we will deal with in this section.) The respective Feynman diagram is depicted in Fig. 3.5. Since we are working in the center of mass (CM) system of the two interacting nucleons the momenta of the two incoming particles are $\vec{q}$ and $-\vec{q}$, the outgoing $\vec{q}'$ and $-\vec{q}'$. We consider the diagram as Born contribution to a real scattering process. Thus, the nucleons are "on their mass shell" (real physical nucleons), i.e.

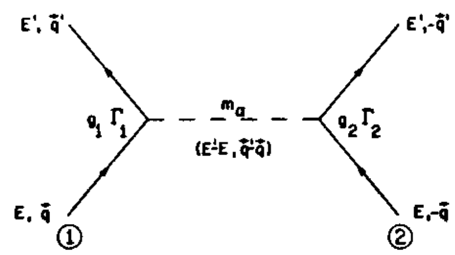

FIG. 3.5. Feynman diagram for the one-boson-exchange contribution to NN scattering considered in the CM frame. Full lines denote nucleons, the dashed line a boson with mass m_α. The underlying time axis is vertical, pointing upwards into the future.

$$E = \sqrt{M^2 + \vec{q}^2}$$

$$E' = \sqrt{M^2 + \vec{q}'^2} \; ; \qquad M \equiv \text{nucleon mass} \tag{3.10}$$

Further, the process takes place "on the energy shell," i.e. energy is conserved; consequently the energy of the nucleons before, E, and after, E', the scattering process must be the same:

$$E' = E \tag{3.11}$$

According to the "Feynman rules"[139] the depiction, Fig. 3.5, is converted into the formula:

$$\frac{g_1 \bar{u}_1(\vec{q}')\Gamma_1 u_1(\vec{q}) P_\alpha g_2 \bar{u}_2(-\vec{q}')\Gamma_2 u_2(-\vec{q})}{q^2 - m_\alpha^2} \tag{3.12}$$

where the left half of the numerator represents the left part of the diagram; for the right respectively.

An outgoing nucleon is represented by a Dirac spinor,* e.g. for particle 1:

$$u_1(\vec{q}) = \sqrt{\frac{E+M}{2E}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1 \cdot \vec{q}}{E+M} \end{pmatrix} \tag{3.13}$$

(Here and in the following we suppress spin-indices and spin functions.) An outgoing nucleon is represented by an adjoint Dirac spinor; again for particle 1:

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*For notation and underived material see Ref. 139.

$$\bar{u}_1(\vec{q}\,') = u_1^\dagger(\vec{q}\,')\gamma^0$$
$$= \sqrt{\frac{E'+M}{2E'}}\left(1, \frac{\vec{\sigma}_1\cdot\vec{q}\,'}{E'+M}\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$
$$= \sqrt{\frac{E'+M}{2E'}}\left(1, -\frac{\vec{\sigma}_1\cdot\vec{q}\,'}{E'+M}\right) \tag{3.14}$$

The normalization of the Dirac spinors eq. (3.13), (3.14) is:

$$u^\dagger(\vec{q})u(\vec{q}) = 1 \tag{3.15}$$

For reasons of simplicity and consistency with later chapters of these lectures, we have chosen this normalization. As a consequence Eq. (3.12) represents already the T-matrix (of NN scattering in Born approximation), and not the invariant amplitude $\mathfrak{M}$. (The relation between them is in our case: $T(\vec{q}\,',\vec{q}) = M^2/E^2\,\mathfrak{M}(\vec{q}\,',\vec{q})$). The T-matrix is an adequate starting point for deriving a nonrelativistic potential, which is our objective in this section (for qualitative and pedagogical considerations). For a more accurate and relativistic treatment, which is required and used in the case of reliable and quantitative work, see Sec. 4 and 5. The dashed (meson) line in Fig. 3.5 represents the propagator which in Eq. (3.12) appears as

$$\frac{P_\alpha}{q^2-m_\alpha^2} \tag{3.16}$$

where $q^2 = (E'-E)^2 - (\vec{q}\,'-\vec{q})^2 = -(\vec{q}\,'-\vec{q})^2$, (using Eq. (3.11)) is the square of the four-momentum transferred by the meson. Thus, we have for the propagator:

$$\frac{P_\alpha}{-(\vec{q}\,'-q)^2 - m_\alpha^2} \tag{3.17}$$

In simple cases the numerator of the propagator, denoted here by P_α, is just 1, so that we can forget about it. For vector boson exchange, however, it is:

$$P_V = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_\alpha{}^2} \tag{3.18}$$

Since the vector bosons couple to a conserved nucleon current the second term will become zero in the actual calculations. Thus, we can use for vector bosons exchange:

$$P_V = -g_{\mu\nu} \tag{3.19}$$

The last pieces which are still unexplained in Fig. 3.5 and Eq. (3.12) are the "vertices" $g_1\Gamma_1$ and $g_2\Gamma_2$. These are obtained from the interaction Lagrangian (densities).*

In fact, logically we should have begun with the interaction Lagrangians, as they are the starting point for the development of the field theoretic perturbation theory, the lowest order result of which (for NN and excluding renormalization) is our Feynman diagram Fig. 3.5. In any case, the respective interaction Lagrangians for Fig. 3.5 are:

$$\mathcal{L}_i = g_i\bar{\psi}\Gamma_i\psi\phi_\alpha; \qquad i=1,2 \tag{3.20}$$

where $(\bar{\psi})\psi$ is the (adjoint) nucleon Dirac field and ϕ_α the meson field operator. Comparison of Eq. (3.20) with (3.12) shows in an obvious way, how to obtain the vertex from a Lagrangian in a simple case. So far, the Γ_i have been just unexplained general symbols.

Now, where do we get the explicit Lagrangians from? Here, we are allowed to use our fantasy. In fact, everything is allowed for $\mathcal{L}$, as long as we make sure, that it is hermitean and obeys certain symmetries; e.g. it should be a Lorentz scalar. In general, however, the fields themselves and/or at most, their first derivatives are used to "build" an interaction Lagrangian.** We have to know the transformation properties of the boson field, ϕ_α, since this has to be "counterbalanced" by a suitable choice for $\bar{\psi}\Gamma_i\psi$ such that the whole $\mathcal{L}$ fulfills the required symmetries. How this works in practice, we will see right now, through the example of some simple, but important cases.

3.4 Various Boson Fields and their Role in NN

In this section we want to go systematically through some of the simplest boson fields and their simplest couplings. In each case we shall consider the one-boson-exchange diagram and derive from it explicitly what it predicts for the NN interaction.

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*We will leave out the term "densities" in what follows, though strictly speaking we will always be dealing with Lagrangian densities here.

**We do not discuss here questions of renormalizability; we are dealing with "effective" Lagrangians, anyhow, see Introduction.

3.4.1 The pseudoscalar (ps) field

Pseudoscalar means that the field, ϕ_{ps}, switches sign in the case of either a space or a time reflection. Particles with negative intrinsic parity, e.g. the π and η, have this property.* To "counterbalance" this we have to find an expression $\bar{\psi}\Gamma\psi$ (compare Eq. (3.20)) which has the same property as ϕ_{ps}, to obtain a scalar for the whole expression for the interaction Lagrangian. The simplest case with this property is** $\bar{\psi}\gamma_5\psi$. Thus

$$\mathcal{L}_{ps} = g_{ps}\bar{\psi}i\gamma_5\psi\phi_{ps} \tag{3.21}$$

(The i is needed for the hermicity, as γ_0 and γ_5 anti-commute.) The one-boson-exchange contribution, Fig. 3.5, for this interaction is according to Eq. (3.12) and (3.16):

$$g_{ps}^2 \frac{\bar{u}_1(\vec{q}\,')i\gamma_5 u_1(\vec{q})\bar{u}_2(-\vec{q}\,')i\gamma_5 u_2(-\vec{q})}{-(\vec{q}\,'-\vec{q})^2 - m_{ps}^2} \tag{3.22}$$

We will evaluate the left half of the numerator explicitly here (using Eq. (3.13) and (3.14)):

$$\bar{u}_1(\vec{q}\,')i\gamma_5 u_1(\vec{q}) = i\sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left(1, -\frac{\vec{\sigma}_1\cdot\vec{q}\,'}{E'+M}\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ \frac{\vec{\sigma}_1\cdot\vec{q}}{E+M}\end{pmatrix}$$

$$= i\sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left(\frac{\vec{\sigma}_1\cdot\vec{q}}{E+M} - \frac{\vec{\sigma}_1\cdot\vec{q}\,'}{E'+M}\right)$$

$$= \frac{i}{2E}\,\vec{\sigma}_1\cdot(\vec{q}-\vec{q}\,') \tag{3.23}$$

where in the last step we applied Eq. (3.11) ("on-shell"). Repeating the analogous calculation for the right half of the numerator of Eq. (3.22) and putting everything together, we obtain for the whole diagram the following "momentum space potential":

*For more about this aspect and the experimental evidence see e.g. Segrè, Ref. 135, p.757.

**See Ref. 139, p.26.

$$\hat{V}_{ps}(\vec{k}) = -\frac{g_{ps}^2}{4M^2}\frac{\vec{\sigma}_1\cdot\vec{k}\;\vec{\sigma}_2\cdot\vec{k}}{\vec{k}^2+m_{ps}^2} \tag{3.24}$$

where $\vec{k} \equiv \vec{q}\,' - \vec{q}$ and the approximation, E=M, is assumed. Rewriting this as

$$\hat{V}_{ps}(\vec{k}) = -\frac{1}{12M^2}\frac{g_{ps}^2}{\vec{k}^2+m_{ps}^2}\left\{\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{k})\right\} \tag{3.24a}$$

with

$$S_{12}(\hat{k}) \equiv 3\vec{\sigma}_1\cdot\hat{k}\,\vec{\sigma}_2\cdot\hat{k} - \vec{\sigma}_1\cdot\vec{\sigma}_2; \qquad \hat{k} \equiv \frac{\vec{k}}{|\vec{k}|} \tag{3.24b}$$

it becomes obvious that we have created a spin-spin and a tensor force. We Fourier transform into coordinate space:

$$V_{ps}(\vec{k}) = \frac{1}{(2\pi)^3}\int d^3k\; e^{-i\vec{k}\cdot\vec{x}}\; V_{ps}(\vec{k})$$

$$= +\frac{1}{(2\pi)^3}\frac{g_{ps}^2}{4M^2}(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})\int d^3k e^{i\vec{k}\cdot\vec{x}}\frac{1}{\vec{k}^2+m_{ps}^2}$$

$$= \frac{g_{ps}^2}{4\pi}\frac{1}{4M^2}(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})\frac{e^{-m_{ps}r}}{r}; \qquad r \equiv |\vec{x}| \tag{3.25}$$

where the integral has been solved easily by applying Cauchy's integral formula, and $\vec{\nabla}$ is the differential operator with respect to $\vec{x}$. The differentiation yields

$$(\vec{\sigma}_1\cdot\vec{\nabla})(\vec{\sigma}_2\cdot\vec{\nabla})\frac{e^{-m_{ps}r}}{r} = \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\;\nabla^2\frac{e^{-m_{ps}r}}{r} + S_{12}(\hat{r})\frac{m_{ps}^2}{3}$$

$$\times\left(1 + \frac{3}{m_{ps}r} + \frac{3}{(m_{ps}r)^2}\right)\frac{e^{-m_{ps}r}}{r} \tag{3.26}$$

and $$\vec{\nabla}^2\frac{e^{-m_{ps}r}}{r} = m_{ps}^2\frac{e^{-m_{ps}r}}{r} - 4\pi\,\delta(\vec{x}) \tag{3.27}$$

(compare Eq. (3.7), static Klein-Gordon equation). In the following we leave out the $\delta(\vec{x})$ function term, as it drops out as soon as extended source (cut-offs) are used which is customary in meson theory and physically reasonable. Thus, our final result for the coordinate space potential is:

$$V_{ps}(\vec{x}) = \frac{g_{ps}^2}{4\pi}\frac{m_{ps}^2}{12M^2}\left\{\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{x})\left(1 + \frac{3}{m_{ps}r} + \frac{3}{(m_{ps}r)^2}\right)\right\}\frac{e^{-m_{ps}r}}{r} \tag{3.28}$$

with

$$S_{12}(\hat{x}) \equiv 3\vec{\sigma}_1\cdot\hat{x}\,\vec{\sigma}_2\cdot\hat{x} - \vec{\sigma}_1\cdot\vec{\sigma}_2; \qquad \hat{x} = \frac{\vec{x}}{r} \tag{3.29}$$

which is called the tensor operator. Its matrix elements are zero for singlet (S=0) states and its expectation value vanishes for S-states (L=0, spherical states). The important property is that it has non-vanishing matrix elements for triplet states of non-diagonal L.* The final balance is: a weak spin dependent central force and a strong tensor force.

The best known pseudoscalar field is the pion. There exist three charge states of the pion: +, -, neutral or with other words, its isospin is one; it is an isovector particle. In such a case the Lagrangian Eq. (3.21) is slightly extended:

$$\mathcal{L}_{ps} = g_{ps}\bar{\psi}i\gamma_5\vec{\tau}\psi\cdot\vec{\phi}_{ps} \tag{3.30}$$

where the three components of $\vec{\phi}_{ps}$ are operators in isospin space, as there are now three charged states. $\vec{\tau}$ is the usual isospin operator for isospin 1/2 particles, here the nucleons. $\vec{\tau}\cdot\vec{\phi}_{ps}$ is an invariant in isospin space. By that, the charge-independence of the interaction is guaranteed.** As a consequence, for isovector particle exchange, the Feynman diagram Eq. (3.22) and the potentials derived, Eq. (3.24) and (3.28), obtain a factor $\vec{\tau}_1\cdot\vec{\tau}_2$.

In summary let us stress the important points. We started with a boson field for which we assumed that it was pseudoscalar (equivalent to a particle with negative intrinsic parity which is observed in nature, e.g. for π, η). Consequently, we had to use the γ_5-coupling (as the simplest possibility to comply with certain indispensible symmetries). A small calculation of a few

*For more details see e.g. Ref. 138, Vol. II, p.579.

**For more details see Ref. 139, p.222.

lines then leads directly to a tensor force. In this way it is easily understood that, starting from first principles, the pion creates a tensor force.

Note also that the γ_5-coupling projects small components of the Dirac spinors onto large components, Eq. (3.23). Therefore, it is, in its analytical structure, a "weak" coupling. The reason, why the pion, nevertheless, is non-negligible, is the small mass of the pion, which strengthens the potential (note, that the meson mass squared appears in the denominator of the Feynman diagram, Eq. (3.12)). In fact, the simple rule of thumb, to roughly compare the strength of two OBE contributions of the same kind, is to consider:

$$\frac{g_\alpha^2}{m_\alpha^2} \tag{3.31}$$

From this argument it is now obvious that a heavy ps-particle lead to very small contributions. Examples are the η (m_η=549 MeV) and the η' ($m_{\eta'}$=958 MeV).

We mentioned before that for a certain field, in general, several (in principal infinitely many) couplings are possible. So, for a ps-field a derivative coupling is also commonly considered, the pseudovector (pv) coupling:

$$\mathcal{L}_{pv} = \frac{f_{ps}}{m_{ps}}\bar{\psi}\gamma_5\gamma^\mu\psi\partial_\mu\phi_{ps} \tag{3.32}$$

The resulting left vertex is

$$\Gamma_{pv} = -\frac{f_{ps}}{m_{ps}}i\gamma_5\gamma^\mu(q'-q)_\mu \tag{3.33}$$

($i\partial_\mu$ is the momentum operator; $(q'-q)_\mu$ the four-momentum of the exchanged meson.)

Application in the Feynman diagram, Eq. (3.12), leads, in the numerator, to expressions like $\gamma^\mu q_\mu u_1(\vec{q})$ and $\bar{u}_1(\vec{q}\,')\gamma^\mu \vec{q}\,'_\mu$. The Dirac equation* allows to simplify these:

$$\gamma^\mu q_\mu u_1(\vec{q}) = Mu_1(\vec{q}) \tag{3.34}$$

$$\bar{u}_1(\vec{q}\,')\gamma^\mu q'_\mu = M\bar{u}_1(\vec{q}\,') \tag{3.35}$$

*See Ref. 139, p.283

With these replacements the upper left part of the Feynman diagram yields:*

$$f_{ps}\frac{2M}{m_{ps}}\,u_1(\vec{q}\,')i\gamma_5 u_1(\vec{q}) \tag{3.36}$$

When compared to Eq. (3.22) it turns out that this is exactly the same result as for ps coupling, provided we relate the coupling constants as

$$g_{ps} = f_{ps}\frac{2M}{m_{ps}} \tag{3.37}$$

In this consideration, as in all of this section, the nucleons are on their mass shell. In such a case the Dirac equations, Eq. (3.34), (3.35), apply, and we saw that, then, the ps and pv couplings are equivalent. Off-shell this is in general not true, compare Sec. 5.1

As ps and pv couplings are equivalent on-shell, we can derive our non-relativistic form of OPE also by starting from the pv coupling Eq. (3.32) and proceed as follows; let us consider only the important part of the vertex Eq. (3.33) (abbreviating $k \equiv q' - q$):

$$\begin{aligned}\tilde{\Gamma}_{pv} &= \gamma_5\gamma^\mu k_\mu \\ &= \gamma_5\gamma^0 k_0 + \gamma_5\gamma^i k_i; \qquad i=1 \div 3 \\ &= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}k_i \\ &= \begin{pmatrix}\vec{\sigma}\cdot k & 0\\ 0 & -\vec{\sigma}\cdot\vec{k}\end{pmatrix}\end{aligned} \tag{3.38}$$

where we use $k_0 = E' - E = 0$ (Compare Eq. (3.11)).

Non-relativistic approximation means assuming $|\vec{q}|,\ |\vec{q}\,'| \ll M$ and therefore neglecting the small (lower) components in the Dirac spinor Eq. (3.13); i.e.

$$u_1(\vec{q}) = \begin{pmatrix}1\\0\end{pmatrix}; \qquad \bar{u}_1(\vec{q}\,') = (1,0) \tag{3.39}$$

Sandwiching the vertex Eq. (3.38) with these "Dirac" spinors and recollecting the constant factors yields:

*Note the γ_5 and γ^μ anticommute.

$$\bar{u}_1(\vec{q}\,')\Gamma_{pv}u(\vec{q}) = -i\,\frac{f_{ps}}{m_{ps}}\,\vec{\sigma}\cdot\vec{k} \tag{3.40}$$

a (non-relativistic) pseudoscalar in three-dimensional space. Repeating the same consideration for the right vertex (note that the momentum carries an opposite sign on the right) and taking Eq. (3.37) into account leads again to the momentum space OPE Eq. (3.24). In this way the non-relativistic character of the derivation is more obvious.

3.4.2 The Scalar (s) Field

It has the simplest interaction Lagrangian one can possibly think of (for meson nuclear coupling):

$$\mathcal{L}_s = g_s\bar{\psi}\psi\phi_s \tag{3.41}$$

The one-scalar-boson exchange contribution is:

$$g_s^2\,\frac{\bar{u}_1(\vec{q}\,')u_1(\vec{q})\bar{u}_2(-\vec{q}\,')u_2(-\vec{q})}{-(\vec{q}\,'-\vec{q})^2 - m_s^2} \tag{3.42}$$

Again, we evaluate explicitly the left half of the numerator:

$$\begin{aligned}\bar{u}_1(\vec{q}\,')u_1(\vec{q}) &= \sqrt{\frac{(E'+M)(E+M)}{4E'E}}\begin{pmatrix}1, & \dfrac{-\vec{\sigma}_1\cdot\vec{q}\,'}{E'+M}\end{pmatrix}\begin{pmatrix}1\\ \dfrac{\vec{\sigma}_1\cdot\vec{q}}{E+M}\end{pmatrix} \\ &= \sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left\{1 - \frac{\vec{\sigma}_1\cdot\vec{q}\,'\vec{\sigma}_1\cdot\vec{q}}{(E'+M)(E+M)}\right\} \\ &= \sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left\{1 - \frac{\vec{q}\,'\cdot\vec{q} + i\vec{\sigma}_1\cdot(\vec{q}\,'\times\vec{q})}{(E'+M)(E+M)}\right\}\end{aligned} \tag{3.43}$$

where we used the well-known vector identity:
$\vec{\sigma}\cdot\vec{a}\vec{\sigma}\cdot\vec{b} = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$. Defining: $\vec{k} \equiv \vec{q}\,' - \vec{q}$ and $\vec{p} \equiv 1/2(\vec{q}\,'+\vec{q})$ we obtain now for the expression Eq. (3.43):

$$\sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left\{1 - \frac{\vec{p}^{\,2} - \frac{1}{4}\vec{k}^2 + i\vec{\sigma}_1\cdot(\vec{k}\times\vec{p})}{(E'+M)(E+M)}\right\} \tag{3.44}$$

Leaving out terms of second and higher order in momentum, setting $E' \approx E \approx M$ and repeating the analogous calculation on the right, the final result for the full diagram Eq. (3.42) is:

$$\hat{V}_s(\vec{k}) = -\frac{g_s^2}{\vec{k}^2+m^2}\left\{1+\frac{\frac{1}{2}(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(-i)(\vec{k}\times\vec{p})}{2M^2}\right\} \tag{3.45}$$

The first term on the right hand side is a strong attractive central force, the second a spin-orbit force. This becomes even more obvious after a Fourier transformation into r-space.

For the total spin we introduce $\vec{S} \equiv 1/2(\vec{\sigma}_1+\vec{\sigma}_2)$ and in preparing the Fourier transform we consider:

$$\frac{1}{(2\pi)^3}\int d^3k\ \vec{S}\cdot(-i)(\vec{k}\times\vec{p})\ \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2+m_s^2}$$

$$= -\vec{S}\cdot(\vec{\nabla}\times\vec{p})\ \frac{1}{(2\pi)^3}\int d^3k\ \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2+m_s^2}$$

$$= -\vec{S}\cdot(\vec{\nabla}\times\vec{p})\ \frac{e^{-m_s r}}{4\pi r}$$

$$= \vec{S}\cdot\vec{L}\ m_s^2\left\{\frac{1}{m_s r}+\frac{1}{(m_s r)^2}\right\}\frac{e^{-m_s r}}{4\pi r} \tag{3.46}$$

with $\vec{L} \equiv \vec{x}\times\vec{p}$.

Thus, for the full r-space scalar exchange potential one obtains:

$$V_s(\vec{x}) = -\frac{g_s^2}{4\pi}\left\{1+\vec{L}\cdot\vec{S}\ \frac{m_s^2}{2M^2}\ \frac{1}{m_s r}+\frac{1}{(m_s r)^2}\right\}\frac{e^{-m_s r}}{r} \tag{3.47}$$

Let us repeat, the scalar meson-exchange causes a strong attractive central force and a spin orbit force. From the explicit derivation we realize that the strong central force is due to the fact that the scalar coupling projects large components of the Dirac spinors on large components. The negative over-all sign is a consequence of having a second order in the coupling constant. The spin-orbit force can be traced back to the small components of the Dirac spinors. Therefore, it is a genuine relativistic effect.

Comparing with the ps coupling potential, Eq. (3.28), we see that the latter is a factor

$$\frac{m_{ps}^2}{4M^2} \tag{3.48}$$

smaller than the scalar case considered in this subsection. For the pion, the mass of which is about 1/7 M, this results into a factor 1/200. Note, however, that this result is only valid for a range in which the Yukawa is about one.

3.4.3 The Vector (v) Field

A vector boson has spin one, like a photon, and is represented by a four vector field. To form a Lorentz scalar one can couple it to another four vector, e.g. the nucleon Dirac current in analogy to the coupling of a photon to an electron:

$$\mathcal{L}_v = g_v\bar{\psi}\gamma_\mu\psi\phi_v^\mu \tag{3.49}$$

The evaluation of the one-(vector-)boson exchange is, by now, straightforward for us:

$$g_v^2\ \frac{\bar{u}_1(\vec{q}')\gamma_\mu u_1(\vec{q})(-g^{\mu\nu})\bar{u}_2(-\vec{q}')\gamma_\nu u_2(-\vec{q})}{-(\vec{q}'-\vec{q})^2-m_v^2} \tag{3.50}$$

We will consider the γ_0 term only:

$$\bar{u}(\vec{q}')\gamma_0 u(\vec{q}) = \sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left(1,\ \frac{-\vec{\sigma}_1\cdot\vec{q}'}{E'+M}\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ \frac{\vec{\sigma}_1\cdot\vec{q}}{E+M}\end{pmatrix}$$

$$= \sqrt{\frac{(E'+M)(E+M)}{4E'E}}\left\{1+\frac{\vec{\sigma}_1\cdot\vec{q}'\vec{\sigma}_2\cdot\vec{q}}{(E'+M)(E+M)}\right\} \tag{3.51}$$

The further calculation is pretty much the same as in the scalar case and therefore not repeated here.

$$\hat{V}_v(\vec{k}) = \frac{g_v^2}{\vec{k}^2+m_v^2}\left\{1-3\ \frac{\vec{S}\cdot(-i)(\vec{k}\times\vec{p})}{2M^2}\right\} \tag{3.52}$$

From the γ^0 term one obtains only a factor $-1/2M^2$ for the spin-orbit term as in the scalar case, however, the inclusion of $\vec{\gamma}$ leads to the factor $-3/2M^2$. Coordinate space potential:

$$V_v(\vec{x}) = \frac{g_v^2}{4\pi}\left\{1-\vec{L}\cdot\vec{S}\ \frac{3m_v^2}{2M^2}\left(\frac{1}{m_v r}+\frac{1}{(m_v r)^2}\right)\right\}\frac{e^{-m_v r}}{r} \tag{3.53}$$

We find a strong repulsive central force and a spin-orbit force which has the same sign as in the scalar case, but is by a factor of 3 stronger (if $m_s = m_v$). For the coupling discussed the ω-meson is the most famous example. The repulsion is plausible by analogy with the one-photon-exchange between like charges which also causes repulsion. The "charge" in the case of nucleons is the baryon number, which 1 for nucleons and (-1) for antinucleons.

The detailed reasons for the strong central force and the spin-orbit force are principally the same as in the scalar case.

In analogy to the magnetic dipole coupling of the photon to the nucleon explaining the anomalous magnetic moments of the nucleon, one can in the case of the vector bosons also consider a so-called tensor coupling (not to be confused with the non-relativistic tensor force, S_{12}):

$$\mathcal{L}_t = \frac{f_v}{2M} \bar{\psi}\sigma_{\mu\nu}\psi\partial^\mu\phi_v^\nu \tag{3.54}$$

with

$$\sigma_{\mu\nu} \equiv \frac{i}{2}\left[\gamma_\mu,\gamma_\nu\right] \tag{3.55}$$

The vertex is:

$$\Gamma_{t,\nu} = -i\,\frac{f_v}{2M}\,\sigma_{\mu\nu}(q'-q)^\mu \tag{3.56}$$

The non-relativistic reduction of this coupling proceeds similiar as in the pv case considered at the end of subsection 3.4.1. Let us cast the vertex (without constant factors) into a more transparent form (note that it is a four vector):

$$\begin{aligned}\tilde{\Gamma}_t^\nu &= \sigma^{\mu\nu}k_\mu \\ &= \left(\sigma^{io}k_i,\ \sigma^{ij}k_i\right); \qquad i,j,\ell = 1 \rightarrow 3 \\ &= \left(\sigma^{io}k_i,\ \epsilon^{ij\ell}\begin{pmatrix}\sigma^\ell & 0\\ 0 & \sigma^\ell\end{pmatrix} k_i\right) \\ &= \left(-i\begin{pmatrix}0 & \sigma^i\\ \sigma^i & 0\end{pmatrix} k_i,\ -\begin{pmatrix}(\vec{\sigma}\times\vec{k}) & 0\\ 0 & (\vec{\sigma}\times\vec{k})\end{pmatrix}\right)\end{aligned} \tag{3.57}$$

(See the Appendix A of Ref. 139 for the details of the identities used in this derivation; also $k_o = E' - E = 0$ (on-shell) is used.) Sandwiching this vertex with the non-relativistic "Dirac spinors," Eq. (3.39), and recollecting the constant factors yields:

$$\bar{u}_1(\vec{q}\,')\Gamma_t^\nu u_1(\vec{q}) = +\,i\,\frac{f_v}{2M}\,\vec{\sigma}\times\vec{k} \tag{3.58}$$

Note that this coupling is of the "spin-transverse" type as compared to the "spin-longitudinal" $\vec{\sigma}\cdot\vec{k}$ coupling found for ps. For the full OBE diagram one finally obtains in the approximation considered here:

$$\hat{V}_t(\vec{k}) = \frac{-f_v^2}{4M^2}\,\frac{(\vec{\sigma}_1\times\vec{k})\cdot(\vec{\sigma}_2\times\vec{k})}{\vec{k}^2+m_v^2} \tag{3.59}$$

Since

$$(\vec{\sigma}_1\times\hat{k})\cdot(\vec{\sigma}_2\times\hat{k}) = \vec{\sigma}_1\cdot\vec{\sigma}_2 - \vec{\sigma}_1\cdot\hat{k}\,\vec{\sigma}_2\cdot\hat{k} \tag{3.60}$$

the Fourier transformation into configuration space can be done with the help of the formulae we use in the case of ps-coupling, compare Eq. (3.25)-(3.27). The result is:

$$V_t(\vec{x}) = \frac{f_v^2}{4\pi}\,\frac{m_v^2}{12M^2}\left\{2\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{x})\left(1 + \frac{3}{m_v r} + \frac{3}{(m_v r)^2}\right)\right\}\frac{e^{-m_v r}}{r} \tag{3.61}$$

It is important to stress that the tensor force obtained has the opposite sign compared to the ps case. (Therefore, the ρ-meson, which is a representative of the tensor coupling Eq. (3.54), damps the strong tensor force of the pion at short distances. Also, the ρ, like the π, is an isovector particle, so that a factor $\vec{\tau}_1\cdot\vec{\tau}_2$ should be attached to Eq. (3.61)).

It is worth mentioning that the relationship between vector mesons and the photon is, in fact, much more intimate as just by analogy.

In the "vector dominance" model for the electro-magnetic form factor for the nucleon it is assumed that the photon prevailingly couples to the nucleon by "going through" a vector boson as "mediator." As a consequence of this the strong interaction tensor couplings of the vector bosons and the anomalous moments of the nucleon are related. In fact, thinking strictly within that model the latter gives the evidence for a strong tensor coupling of vector bosons and the nucleon. For more details about this interesting topic see Ref. 141.

3.5 Summary of the Qualitative Features of Boson-Exchange in NN

The main features we deduced systematically in the previous subsections are summarized in Table 3.1.

We should finish this section by going back to its beginning. We listed and explained five important properties of the nuclear force which are strongly indicated (or even proven) by experimental finding. When we compare these five points now with Table 1, we see that in fact for each empirical property of the nuclear force one can think of at least one boson field which would offer an explanation. With other words, boson theory tenders all the qualitative features necessary to account for the empirical facts known about the nuclear force.

The fact that just the three fields discussed in this section seem to provide "all you need," is, or course, tempting. When you give in to the temptation, you have the one-boson-exchange (OBE) model. In this model one does not care too much about the question whether each boson applied would really exist. What one essentially tries to do is to exploit the favourable features of the simple one-particle-exchange as much as possible. The amazing quantitative success, which can be achieved within that simple approach, provides a posteriori the justification. This is about the philosophy of the old OBE model. However, from a more

TABLE 3.1

Various meson-nucleon couplings and their consequences for the NN interaction as deduced from the OBE contribution

Coupling	Type of Forces	
ps	spin-spin (weak)	tensor (strong)
s	attractive central (strong)	spin-orbit
v	repulsive central (strong)	spin-orbit (same sign as s)
t	spin-spin (weak)	tensor (sign opposite to ps)

Abbreviations: pseudoscalar (ps), scalar (s), vector (v), tensor (t).

Note: the signs assumed in this table for s and v refer to isoscalar bosons of that type.

fundamental point of view such a concept is not satisfactory. Therefore, we will pursue in the next section a more serious and systematic line. Still, when that line has been carried through to a successful end, we will raise the question, if a simple parametrization of the full result is possible. This will, in fact, lead us then to the OBE model, however, now, on a sound foundation. Thus, the approximations to be made and their consequences can be checked carefully and a physical interpretation of the simplified terms representing the OBE model can be given. Such OBE potentials (OBEP) will be discussed in Sec. 5. In Sec. 5.2, in which we present a coordinate-space OBEP, the contributions of the different bosons to central, tensor and spin-orbit force will be demonstrated quantitatively.

4. A SERIOUS MESON-EXCHANGE MODEL FOR THE NN-INTERACTION

After all these preliminaries of the various kinds (history, qualitative considerations etc.) we have a good physical background and (hopefully) also a strong motivation to finally dive into precise theoretical and quantitative work. That is, we want to derive, now, the NN interaction systematically step by step from meson theory and compare our results accurately with the experimental data. It is our intension to work on a basis, which is well founded by our knowledge and experience from meson and nuclear physics, and to try for an approach that is "complete" insofar as it includes all (mesonic) processes which by any reasonable consideration are relevant to the NN problem. In addition, the derivation is aimed to be reliable and unambiguous from the field theoretic point of view.

Let us state in more detail what we mean by some of these points. First and most obviously, we want to use existing mesons only (and not allow for fictitious bosons). Secondly, we will also include plural meson exchange. It will be important to take uncorrelated and correlated multi-particle exchange into account. Thirdly, according to the mass-range relation we will start at long range and then step by step include all relevant diagrams with an exchanged mass up to about the cut-off mass used in the meson-nucleon vertex functions. The use of such "cutoffs" is natural in the light of the discussion given in the Introduction. They suppress meson-exchange for small distances. In fact, originally such form factors were introduced in meson theory in a purely ad hoc way supplying a sufficient fall-off for high momenta, necessary to get a solution of the scattering equation. Nowadays, due to the quark structure of hadrons, the form factor is an in principle founded concept, being related to the hadron size. Anticipating part of our results, we obtain cut-off masses in the range of 1.2-1.5 GeV. Consequently we include diagrams up to a total exchanged mass of about 1 GeV. Finally, we will be concerned with the subtleties of field theory. Certainly, we will not use the static approximation and will take meson retardation (recoil effects) into account completely.

There are many reasons why we pursue this careful and comprehensive approach to the NN interaction. First there is a more fundamental or "intrinsic" reason: namely, one wants to know if and to which extent meson theory alone is able to provide a quantitative model for the NN interaction, the resulting vertices to be determined by QCD. Further, the field theoretic approach used, provides an unambiguously defined off-shell behaviour for the nuclear force, which is important e.g. for Bremsstrahlung and in the nuclear many body problem. The underlying formalism and the set of diagrams contributing to the NN interaction is a sound basis for the consistent determination of meson-exchange current contributions to the electromagnetic properties of nuclei (e.g. the deuteron, ^{3}He, etc.). Moreover, the explicit field theoretic description of the contributions to the NN interaction allow for a reliable evaluation of the medium effects on the nuclear force when applied in the nuclear many body problem (more about this in Part II of these lectures). Finally, the model is an excellent basis for the consideration of charge-independence and charge-symmetry breaking of the nuclear force due to the mass differences between the charge states of mesons, nucleons and isobars.[142] Also, the real part of the $N\bar{N}$ interaction (implied by the NN interaction due to G-parity) can be determined in an unambiguous way. Several of the aspects mentioned here will become more clear in the end of this section when the model is displayed explicitly.

4.1 "Formalities"

The model we are going to develop in this section is based on field theory. It is therefore appropriate to note a few details on how we will treat the field theoretic perturbation theory explicitly. For a discussion of the issue, if perturbation theory is at all adequate, see Sec. 3.3 and Sec. 4.5.

Our general scheme is to start from a field theoretic Hamiltonian H and to treat nucleons, isobars (essentially Δ) and mesons on an equal footing. Consequently the Hamiltonian does not contain a NN potential, but NN-meson and NΔ-meson vertices. These are gained from the interaction Lagrangians which we list here:

$$\mathcal{L}_{pv} = \frac{f_{ps}}{m_{ps}} \bar{\psi}\gamma_5\gamma^\mu\psi\partial_\mu\phi_{ps}$$

$$\mathcal{L}_s = g_s\bar{\psi}\psi\phi_s \qquad \text{(4.1 Cont'd)}$$

$$\mathcal{L}_v = g_v\bar{\psi}\gamma_\alpha\psi\phi_v^\alpha + \frac{f_v}{4M}\bar{\psi}\sigma_{\mu\nu}\psi(\partial^\mu\phi_v^\nu-\partial^\nu\phi_v^\mu)$$

$$\mathcal{L}_{N\Delta\pi} = \frac{f_{N\Delta\pi}}{m_\pi}\bar{\vec{\psi}}\vec{T}\psi_\mu\partial^\mu\vec{\phi}_\pi + \text{h.c.}$$

$$\mathcal{L}_{N\Delta\rho} = i\frac{f_{N\Delta\rho}}{m_\rho}\bar{\psi}\gamma_5\gamma_\mu\vec{T}\psi_\nu(\partial^\mu\vec{\phi}_\rho^\nu-\partial^\nu\vec{\phi}_\rho^\mu) + \text{h.c.} \qquad (4.1)$$

(In the first lines the well-known isospin dependence which has to be incorporated for isospin one mesons is suppressed; "h.c." stands for "hermitean conjugate.") ψ_μ denotes the field operator for the Δ-isobar (Rarita-Schwinger spinor*). More explanations can be found in Ref. 127 and 133.

We will treat H in time-ordered perturbation theory.[53,145] Since this "old fashioned" perturbation theory corresponds to standard many body theory, it will allow us to go from the two-body problem to the many body problem in a well-defined way. By that it provides the adequate framework to take medium effects consistently into account.[146] Also, in the time-ordered formalism, meson retardation (the recoil effect) is treated correctly. It turns out that fourth-order diagrams** can differ by about a factor of two, depending if meson retardation is taken into account or not. This clearly indicates the importance of recoil effects in the evaluation of e.g. the 2π-exchange.[127]

We shall leave out negative-energy intermediate states in fourth and higher order contributions. There are several arguments for this "pair suppression" (apart from the "historical" ones which we mentioned already in Sec. 2.2): It has been shown by Zuilhof and Tjon[147] in a covariant calculation using the Bethe-Salpeter equation in the ladder approximation that the contributions from anti-nucleon intermediate states are small provided pseudovector coupling is used for the NNπ vertex, which, however, is suggested as an effective coupling because of chiral invariance.[148] Furthermore, according to quark-model arguments the nucleon-antinucleon ($N\bar{N}$) vertex is considerably suppressed compared to the NN vertex.[149] The general covariance of the theory is destroyed by leaving out anti-particles. However, in NN, relativistic aspects like meson retardation are much more important than those more formal ones, as covariance.

In time-ordered perturbation theory all possible time-orderings of a diagram have to be taken into account separately.

*See e.g. Ref. 143, p.288; and Ref. 144.

**This refers to the order in the coupling constant.

For brevity, however, we will not always display them all. Therefore, the diagrams depicted subsequently have to be considered as an abbreviated notation, just standing for all possible time-ordering (except for Z-graphs).

The non-iterative (irreducible) diagrams contributing to the nuclear force form the kernel ("driving force") of an integral equation which determines the T-matrix of NN scattering from which the phase-shifts are obtained. With other words the irreducible contributions are iterated in the calculation to any degree. More details about the formalism can be found in Ref. 133.

4.2 The Mesons

The latest status of mesons with masses below 1350 MeV is given in Table 4.1. We list non-strange mesons only. The reason is that strange particle exchange would convert nucleons into strange baryons, since strangeness is conserved in strong interaction; therefore it is excluded from our considerations of NN.

In building now a meson theory of the NN interaction, the way not to proceed is to slam all 14 mesons listed in the table together and pray that everything works out fine. Even if it would work out, we would not know why. As we want to have a detailed physical understanding for what is going on, we will have to proceed in a more careful and reasonable way. First, we should scan the meson list, Table 4.1, critically.

Some mesons are most probably essentially $s\bar{s}$ states (s= strange quark). In the quark picture, they can couple to the nucleon by gluon exchange only (without accompanying quark exchange), which is a suppressed process ("Zweig forbidden"[151]). These arguments apply to the ϕ, which is $s\bar{s}$ to 99.9%,[152] and at least in part to the S^*, which, however, may also be a $K\bar{K}$ state. For these reasons the ϕ and S^* are omitted from further considerations of the NN problem. For other mesons there are arguments, why they contribute little to the nuclear force. E.g. the η and especially the η' are heavy pseudoscalar particles which couple by γ_5; for that reason their contribution is very small (compare Sec. 3.4.1). The δ-meson with a mass of 983 MeV has only a weak contribution because of its relatively heavy mass. Note, that the meson propagator (for zero momentum transfer) is proportional to $1/m_\alpha^2$, with m_α the meson mass. Thus, the size of a one-boson-exchange contribution goes generally down with the square of the meson mass. Therefore, the "nest" of mesons in the area of 1200 to 1300 MeV will contribute only little. In addition, the coupling constants of those mesons are rather moderate.* However, the main argument why the very heavy mesons are not very relevant in NN is related to the $\omega(783)$. The one-ω-exchange is

*See Ref. 143, p.335.

TABLE 4.1

NON-STRANGE MESONS
WITH MASSES LESS THAN 1350 MeV AND THEIR PROPERTIES*

Name	J^P	I^G	Mass (MeV)	Full Width (MeV)	Dominant Decay Mode
$\pi^\pm$	0^-	1^-	139.57	0	$\mu^\pm\nu$
π^0	0^-	1^-	134.96	0	$\gamma\gamma$
η	0^-	0^+	548.8	0.001	$\gamma\gamma$, $3\pi^0$
ρ	1^-	1^+	769	154	2π
ω	1^-	0^-	782.6	9.9	3π
η'	0^-	0^+	957.6	0.3	$\eta\pi\pi$
S^*	0^+	0^+	975	33	2π, $K\bar{K}$
δ	0^+	1^-	983	54	$\eta\pi$, $K\bar{K}$
ϕ	1^-	0^-	1020	4	K^+K^-
B	1^+	1^+	1234	150	$\omega\pi$
f	2^+	0^+	1274	178	2π
A_1	1^+	1^-	1275	315	$\rho\pi$
D	1^+	0^+	1283	26	$\eta\pi\pi$, 4π
ϵ	0^+	0^+	1300	200-600	2π
A_2	2^+	1^-	1318	110	$\rho\pi$

J≡Spin, P≡Parity, I≡Isospin, G≡G-Parity

repulsive (see Sec. 3.4.3) and has a large coupling constant ($g_\omega^2/4\pi \approx 10$-$20$). By that it masks the contributions of all heavier mesons, which are, according to the mass-range relation, of shorter range.

An additional argument for leaving out the very heavy mesons refers to the fact that the meson-nucleon form factor is in the range of 1.2 to 1.5 GeV; it does not make sense to take meson-exchange serious in a range, to which phenomenological ad hoc modifications are applied anyhow and in which the extended structure of the hadrons must come into play.

*from Ref. 150.

The final balance of the critical scan of the long list of mesons is that essentially only 3 mesons survive: π, ρ and ω. The η and δ are sometimes included in boson-exchange models, however, only for purposes of fine tuning when fitting the NN data.

From Sec. 3 we know that with these three survivor mesons we obtain the tensor force, the short-range repulsion and the LS force.* So, already on the one-boson-exchange level several essential features of the nuclear force are explained (by existing mesons). Still, one important property is missing: the intermediate range attraction. An isoscalar scalar meson with an intermediate mass does not exist. However, as we discussed before, we have to consider plural meson exchange anyhow, and we will see how that provides us with the missing parts.

In the calculation of multi-meson exchange we will proceed systematically, i.e. in the spirit of the TNS[57] program (compare Sec. 2.2). The one-pion-exchange and its properties we discussed already in length. Thus, the next step is to consider the exchange of two pions to which we will turn now.

4.3 The 2π-exchange

Our model for the 2π-exchange contribution to the NN interaction is shown in Fig. 4.1. The essential features of the model are that is takes the effects from nucleon resonances (isobars) and direct ππ interaction into account, which are both phenomena well-known from πN scattering and other related processes. The low-lying (J=3/2, I=3/2) Δ-resonance with a mass of 1232 MeV is of particular

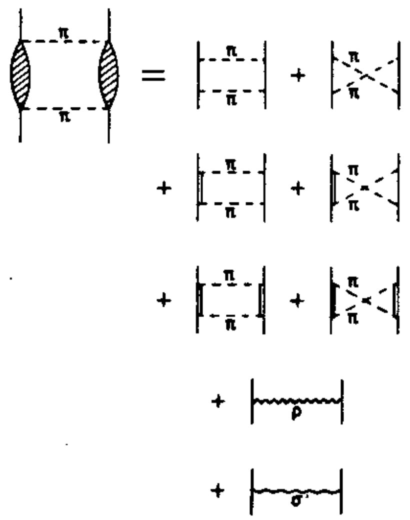


FIG. 4.1. Our model for the 2π-exchange contribution to the NN-interaction. A full line represents a nucleon and a double line a Δ(1232)-isobar. Further explanations are to be found in the text.

*The character of a particle and therefore also the character of its coupling to the nucleon is obtained from the second column in Table 4.1 (J^P): 0^+ ↔ scalar; 0^- ↔ pseudoscalar; 1^- ↔ vector; 1^+ ↔ axial vector.

importance. The effect of other resonances have been considered, too, in the literature and found to be negligible; e.g. the $P_{11}(1440)$ is considered in Ref. 153 and the $F_{15}(1688)$ in Ref. 154. Therefore, we confine ourselves to the Δ(1232) in our model.

The various crossed box diagrams, which have been left out in most field theoretic models, since they are hard to evaluate, have to be taken into account, as they are non-negligible and help to provide an isoscalar character for the 2π-exchange contribution (at least in high partial waves). An almost isoscalar character is suggested by results from dispersion theory.*

The six upper diagrams in Fig. 4.1 represent the uncorrelated 2π exchange. As mentioned before there may also be strong correlations between the two pions when "in the air". How to picture this is indicated in Fig. 4.2. If the two interacting pions are in relative P-wave, a resonance occurs: the ρ-meson. On the other hand, the strong ππ interaction in relative S-wave does not lead to a resonance. However, Durso et al.[153] have shown that the three diagrams on the left hand side of Fig. 4.2 may well be approximated by the exchange of a scalar isoscalar boson with a broad mass distribution, which we will denote by σ'. In the quoted work the ππ-S-wave interaction is determined by a fit to the empirical ππ-S-wave phase shifts. It is then applied in the NN process as indicated in Fig. 4.2. In this way a definite mass (namely, 660 MeV), full width (525 MeV) and a coupling constant of $g^2_{\sigma'}/4\pi$ = 13±3 is obtained for σ'. We use the quoted mass distribution and a coupling constant of 10.

The alternative way to derive the 2π-exchange contribution to the NN-interaction is through dispersion theory, in which empirical information of πN- and ππ-scattering is used in order to evaluate the amplitude $N\bar{N} \to \pi\pi$.[120,**] We perform a quantitative comparison with the results from this latter approach in higher partial waves of NN-scattering (equivalent to long and intermediate range) in which OPE and 2π are the only contributions

FIG. 4.2. Correlated 2π-exchange contributions considered in the work of Durso et al.[153] The circled S symbolizes the ππ-S-wave interaction adjusted to empirical ππ-S-wave scattering. Further notation as in Fig. 4.1.

*For a more thorough discussion of this aspect see Ref. 155.

**See also Sec. 2.3

apart from a small one-ω-exchange contribution. We further compare our model with the empirical phase shifts of NN-scattering. Both comparisons are given in Fig. 4.3.

Figure 4.3 demonstrates a close agreement of our 2π-exchange model with the empirical NN-phase-shifts as well as with the results from dispersion theory. This double agreement confirms that our model is physically most reasonable.

After this successful check of our 2π-exchange model in high partial waves, we now proceed to states of lower angular momentum (equivalent to shorter ranges), which will clearly exhibit the need for the inclusion of additional processes. Nevertheless, let us first consider the results obtained only from the contributions we have taken into account so far.* In Fig. 4.4 the dashed line (ππ 1.3) demonstrates that the 2π-exchange is too attractive in most cases. As these partial waves are sensitive to short range effects, we vary the vertex form factor (cutoff) of the pion and show the effect in that figure. (This variation has no effect in the higher partial waves considered in Fig. 4.3 before.) It is seen that the attraction can be reduced in this way (see e.g. $^1S_0$ curve "ππ 0.9"), however, a simultaneous fit of the $^1P_1$ and the $^3P_1$-phase-shift can never be achieved in this way, as a stronger cutoff lowers the $^3P_1$ and raises the $^1P_1$, although both phase-shifts had to be lowered for a closer agreement with the empirical phase-shift data. Also a consistent fit of $^1S_0$ and $^3P_1$ is obviously impossible: When the $^1S_0$-phase-shift is correct (namely for Λ =0.9 GeV), the $^3P_1$ is still considerably too high.

Our conclusion states that a model consisting of one-meson- and 2π-exchange only is unable to describe the empirical NN-data if the concept of meson-nucleon vertex form factors is applied in a consistent way, and no other short-range contributions or phenomenological modifications are allowed.

*Note, that in low partial waves we iterate all contributions in the integral equation for the T-matrix; in this case the iterative 2π-exchange diagram (upper left in Fig. 4.1) is left out in the kernel.

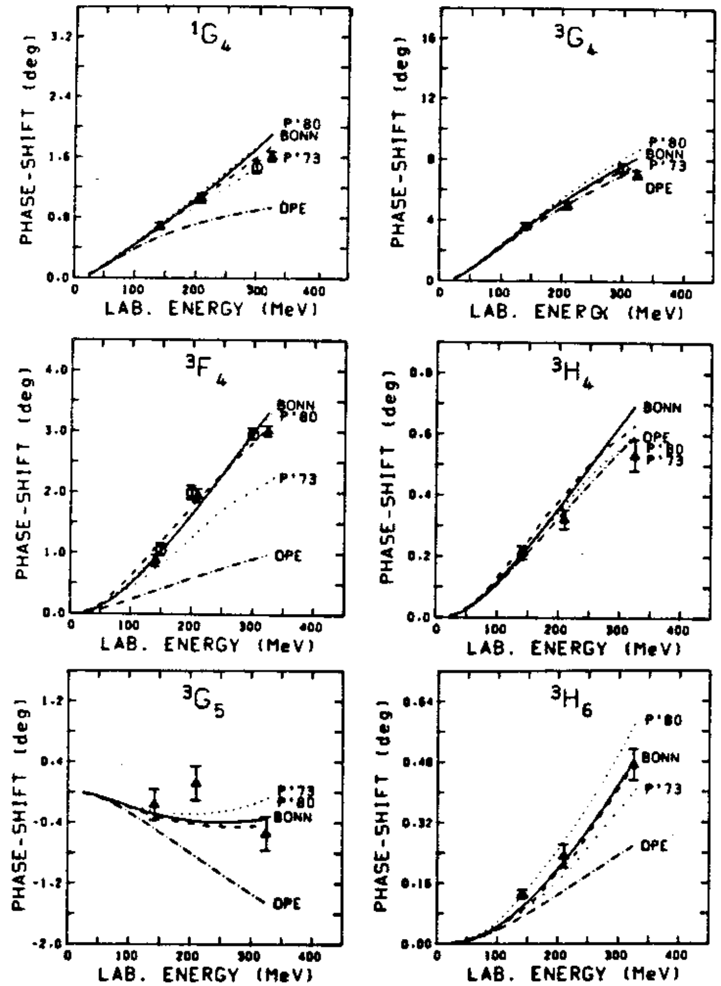


FIG. 4.3. Some higher angular momentum partial wave phase-shifts of NN-scattering. The full line labeled BONN contains all contributions from our model for the 2π exchange shown in Fig. 4.1 plus OPE and one-ω-exchange ($g_\omega^2/4\pi$=5.7). The dotted lines represent the results from dispersion theory, P'73 is taken from Ref. 156 and P'80 from Ref. 119. The dashed line is the energy-dependent analysis of Arndt et al.[137] The energy-independent analyses (error bars) are taken from Ref. 137 (octagon) and Ref. 157 (triangle). The OPE contribution is displayed in dash-dot. Note that all phase-shifts in this figure are evaluated in Born approximation as the effect of the iteration of the kernel is negligible in these high partial waves.

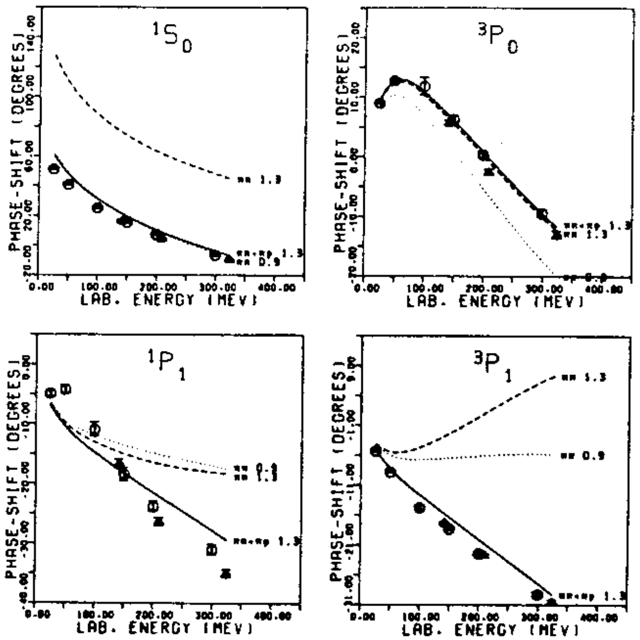


FIG. 4.4. 2π-exchange contributions to the phase-shfits of four different low angular momentum partial waves with two different choices of the cutoff-mass, Λ_π, regularizing the pion-vertex, namely Λ_π=1.3 GeV (denoted by ππ 1.3) and Λ_π=0.9 GeV (ππ 0.9). For the former case the effect of the additional πρ-contribution (compare Sec. 4.4) is also demonstrated (ππ+πρ 1.3). In addition all curves contain the one-meson-exchange of π, ω and δ.

4.4 The πρ-exchange

The failure of the model, developed so far, in low partial waves is not a disaster. In systematically and stepwise building up the nuclear force from meson-exchange, starting from long range and proceeding gradually to shorter ranges, we have up till now included only one- and two-pion exchanges "completely" (and the ω which is a 3π resonance). The next step is to take 3π-exchanges (apart form the ω) systematically into account.

One remarkable contribution of that kind is the exchange of π and ρ. We know that on the one-meson-exchange level π and ρ play the roles of opponents bearing opposite signs for the tensor force. For that same reason, it is to be expected that the πρ-contribution will in general have the opposite sign to 2π.

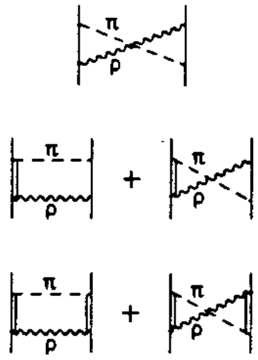


FIG. 4.5. πρ-contributions to the NN-interaction considered in this work.

Figure 4.5 displays the contributions in analogy to the diagrams of uncorrelated 2π-exchange. (Note, that only irreducible diagrams are included in the kernel as the iterative ones are generated automatically in the T-matrix equation; all stretched box diagrams are also taken into account, though they are not shown in Fig. 4.5.)

The effect of all πρ-diagrams is demonstrated in Fig. 4.4 curve "ππ+πρ 1.3" in comparison to curve "ππ 1.3." Clearly, this contribution is physically of utmost importance in quantitatively describing the low partial waves of NN scattering, or with other words for the short range part of the interaction.

The fact that the πρ-contributions are of relevance has been pointed out already in the work of Durso et al.[155] In that work, however, the πρ-contribution was suggested as an instrument to reduced the ω-coupling, which is always rather large in OBE-models compared to it SU(3) prediction:*

$$\frac{g_\omega^2}{4\pi} = 9\,\frac{g_\rho^2}{4\pi} \approx 5 \qquad (4.2)$$

Our findings are that this suggestion is not realistic for two reasons: First, the πρ-contribution sometimes varies tremendously from state to state (compare e.g. the contribution in $^3P_0$ and $^3P_1$ of Fig. 4.4) which is not the case with the ω; second, the over-attraction of the 2π-exchange in low partial waves is such that in addition to a rather large ω-coupling further repulsion is needed. Therefore, the effect of the πρ-contribution is to function as a counterpart of the 2π-exchange contributions and not to partially provide the general short range repulsion of the NN-interaction

*See e.g. Ref. 121, p.236.

which, namely, should be about equally strong in all partial waves.

At this stage of the development of our model it is instructive to ask the question what the fictitious scalar isoscalar σ-boson of the former one-boson-exchange (OBE) models stood for. The old belief that together with the ρ it replaced the sum of all correlated and uncorrelated irreducible 2π-exchange contributions is true only for high partial waves (where the πρ-contributions are negligible because of their short-range); in low partial waves the 2π-exchange contribution apart from the ρ appears by no means to be isoscalar scalar, whereas after its smoothening out by the πρ-exchanges an approximation by a one-σ-exchange appears possible, as demonstrated in Fig. 4.6.

With other words, the 2π-exchange provides also rather short-ranged contributions which are adequately counterbalanced only by the πρ-exchanges.

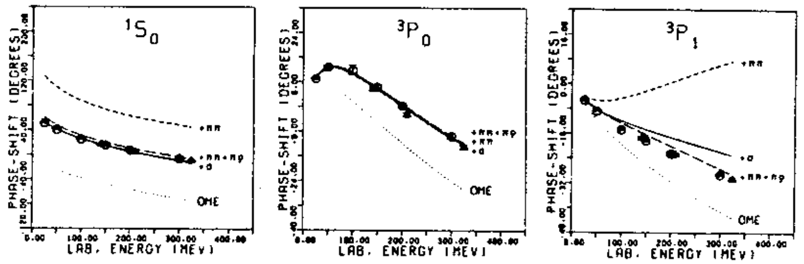


FIG. 4.6. The approximation of the 2π-exchange plus πρ-exchange by a σ-boson. The dotted curve contains all iterated one-meson-exchange (OME) contributions (i.e. π,ρ,ω,δ). The other contributions are always added on top of OME. "ππ" denotes the full 2π-exchange from our model. The other notation is obvious.

4.5 Final Results for the NN-System

In addition to the contributions discussed so far we also include further irreducible 3π- and 4π-exchanges in an approximate way. They turn out to be of little importance as they cancel each other to a large extent. This also indicates a kind of convergence with an increasing number of pion exchanges, if the diagrams are grouped in an appropriate way. For more details concerning this point see Ref. 133.

In Table 4.2 we give the meson parameters used in the final description of the NN data. We compare them to information from other sources and generally observe a close agreement. Note that not all parameters are used as fit parameters. The NNπ coupling

TABLE 4.2

MESON PARAMETERS APPLIED IN OUR MODEL AND FROM OTHER SOURCES

vertex	meson-mass m_α(MeV)	$\frac{g_\alpha^2(t=m_\alpha^2)}{4\pi}$	$g_\alpha^2(t=0)/4\pi$; (f_V/g_V)	coupling constants from other sources or comments	cutoff-mass Λ_α(GeV)	n_α
NNπ	138.03	14.4[a]	14.08	14.28±0.18 πN-scattering, Ref. 158 14.52±0.40 pp forward dispersion relation, Ref. 159 NN-phase-shift analyses: 14.25 Ref. 157 (Bugg) 14.5 Ref. 137 (Arndt)	1.3	1
ρ	769.0	0.84	0.41;(6.1)	0.55±0.06 (6.1±0.6) fit to $N\bar{N}$→ππ partial waves, Ref. 160	1.4	1
ω	782.6	20	10.6	12.0 Ref. 161 8.1±1.5 Ref. 162	1.5	1
δ	983	2.82	1.62	-	2.0	1
σ'	550	5.7	4.57	ππ-S-wave interaction, zero-width fit to the result of Ref. 153	1.7	1
NΔπ	138.03	0.224	0.218	quark model value[b]	1.2	1
ρ	769	20.45	4.86	quark model value[c]	1.4	2

$g_\alpha(t) \equiv g_\alpha^0 \left\{ \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 - t} \right\}^{n_\alpha}$; nucleon mass: M=938.926 MeV; mass of Δ-isobar: 1232 MeV.

$g_\alpha^0 \equiv g_\alpha(t=m_\alpha^2)$

[a] $g_\pi^2 = \left(\frac{2M}{m_\pi}\right)^2 f_\pi^2$; [b] $f_{N\Delta\pi}^2 = \frac{72}{25} f_\pi^2$; [c] $f_{N\Delta\rho}^2 = \frac{72}{25} g_\rho^2 \left(\frac{m_\rho}{2M}\right)^2 (1+f_\rho/g_\rho)^2$

(See Ref. 163 for details of the quark model derivation.)

was taken from empirical source: the $N\Delta\pi$- and $N\Delta\rho$-coupling from the quark model. The essential fit parameters are the pion- and rho-cutoff-masses and the omega- and NN-rho-vector-coupling constants as well as the δ-meson coupling. The excellent results for the NN-system are displayed in Table 4.3 for the deuteron and low energy scattering and in Fig. 4.7 for NN-scattering up to a laboratory energy of 325 MeV. It turns out that the np-phase-shifts and np-observables are described better by our model than by popular (semi-) phenomenological potentials in spite of the fact that our model has only about 1/10 of the number of free parameters of those other models. For a detailed discussion of our results we refer the interested reader to Ref. 133.

TABLE 4.3

DEUTERON AND LOW ENERGY PARAMETERS PREDICTED BY OUR MODEL (THEORY) AND FROM EXPERIMENT (EXPERIMENT).

	Theory	Experiment	
Deuteron:			
binding energy, E_B(MeV)	2.22465	2.224644±0.000046	Ref.143
D-state probability, P_D(%)	4.25	5±2	
quadrupole-moment, Q_D(fm^2)	0.281	0.2860±0.0015	Ref. 143
asymptotic S-state, A_S	0.9046	0.8846±0.0008	Ref. 65
asymptotic D/S-state, η	0.0267	0.0271±0.0004	Ref. 65
root-mean-square radius (fm)	2.0016	1.9635±0.0045	Ref. 143
ΔΔ-probability (%)	0.5		
np low energy scattering			
singlet: a_s(fm)	-23.740	-23.748±0.010	Ref. 143
r_s(fm)	2.766	2.75 ±0.05	
triplet: a_t(fm)	5.427	5.424±0.004	
r_t(fm)	1.755	1.759±0.005	

In the theoretical results quoted here the nucleonic wave function of the deuteron has been normalized to unity for simplicity. In a more refined consideration of the deuteron, Δ- and mesonic components should be separated out. They would, on the one hand, reduce the normalization of the nucleonic wave function and by that the theoretical result of most deuteron quantities cited, but, on the other hand, add meson-current contributions which will at least partly compensate the former effect.

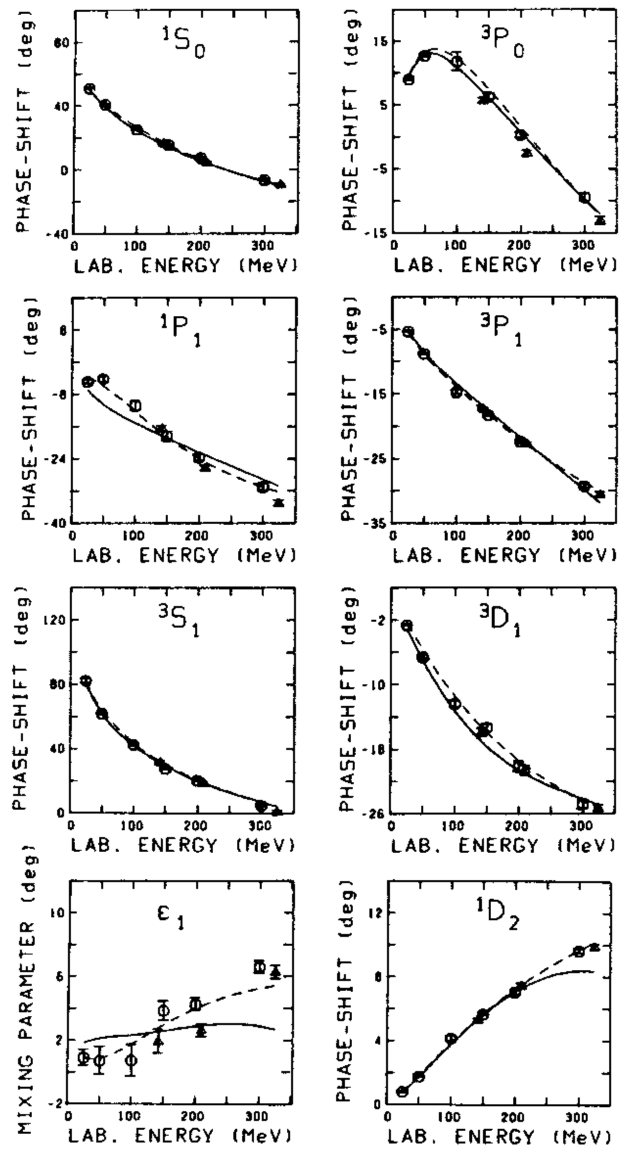


FIG. 4.7. Cont'd

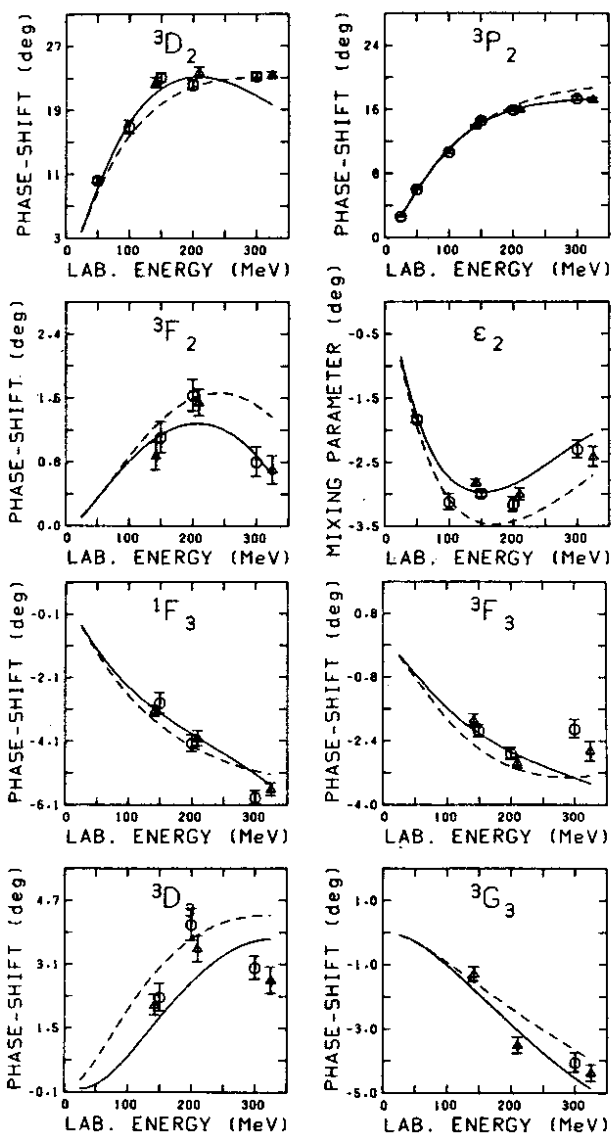


FIG. 4.7. Cont'd

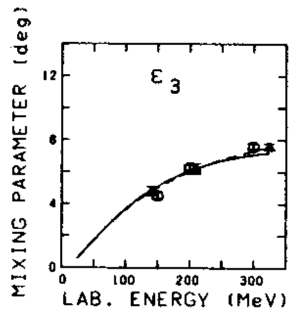


FIG. 4.7. np phase-shifts. The full line represents our predictions. The dashed line refers to the energy-dependent phase-shift analysis of Arndt et al.[137] Error bars denote the energy-dependent analyses of Bugg and coworkers[157] (triangle) and of Arndt et al.[137] (octagon).

5. THE PARAMETRIZATION OF THE NUCLEAR FORCE BY OBE-TERMS

Once one has shown that meson theory, performed consistently works quantitatively, one can think about a simple parametrization of that result, which would make applications in nuclear structure physics more easy. We will present two parametrizations by OBE terms, one relativistic presented in q-space, and one non-relativistic in r-space.

However, one should keep in mind that one has to make sacrifices to obtain a simpler version of the nuclear force. For example, the off-shell behaviour may be altered, the medium effects may be changed. But not in all applications in nuclear structure physics all subtleties of the nuclear force play a role. Therefore, there is not the nuclear force, there are various approximations to the nuclear force, which differ by their degree of sophistication. And depending on the problem under consideration, a more crude or more refined representation of the nuclear force is adequate.

5.1 A Relativistic OBEP in q-space

In this section we will present a relativistic one-boson-exchange potential (OBEP) in momentum space which approximates the full model discussed in Sec. 4. It is particularly suitable for relativistic nuclear structure calculations.

In most applications it is convenient if the potential is energy independent. (The time-ordered formalism in which the full model of Sec. 4 was developed, always results in an energy dependent potential.)

Traditionally relativistic energy independent potentials are obtained within the framework of so-called 3-dimensional reductions of the 4-dimensional Bethe-Salpeter equation,[103] which reads in operator notation:

$$T = \bar{V} + \bar{V}\hat{G}T \tag{5.1}$$

where T is the scattering amplitude, $\bar{V}$ the sum of all connected two-nucleon irreducible diagrams and $\hat{G}$ the relativistic two nucleon propagator.

The solution of Eq. (5.1) raises formidable mathematical and numerical problems.[107] Therefore so-called 3-dimensional relativistic reductions which can be handled more easily have been suggested. They are reviewed and discussed in the work of Woloshyn and Jackson[164] and in Ref. 121. The basic idea is to replace Eq. (5.1) by a set of coupled equations:

$$T = W + WgT \tag{5.2a}$$

$$W = \bar{V} + \bar{V}(\hat{G}-g)W \tag{5.2b}$$

where the propagator g is chosen such that Eq. (5.2a) reduces to a 3-dimensional integral equation. It is a common practice to leave out the second term on the right hand side of Eq. (5.2b) assuming that the old and new propagators are sufficiently close to keep that term small. This is an obvious desire as the inclusion of the full Eq. (5.2b) would spoil the significant simplification which is the whole purpose of the reduction.

Here, we will choose the Thompson equation[165]:

$$T(\vec{q}',\vec{q}) = \bar{V}(\vec{q}',\vec{q}) - \int d^3k\, \bar{V}(\vec{q}',\vec{k})\, \frac{M^2}{E_k^2}\, \frac{\Lambda_+^{(1)}(\vec{k})\, \Lambda_+^{(2)}(-\vec{k})}{2E_k-2E_q-i\epsilon}\, T(\vec{k},\vec{q}) \tag{5.3}$$

with $\pm\vec{q}$ ($\pm\vec{q}'$) the initial (final) momenta of the interacting nucleons in the cm frame, M the nuclear mass, $E_k = \sqrt{M^2 + \vec{k}^2}$, $E_q = \sqrt{M^2 + \vec{q}^2}$, and $\Lambda_+^{(i)}(\vec{k})$ the positive energy projection operators of the i-th nucleon with momentum $\vec{k}$.

The following arguments are in favour of that equation:

(i) In the model calculations of Ref. 164 the Thompson results are the closest to those gained from the full Bethe-Salpeter equation compared to all other 3-dimensional reductions which can be cast into the form Eq. (5.2a) and are discussed in that paper. (Note that Thompson is in fact case "F" of Ref. 164 and not case "D" as stated in that work.)

(ii) Using the Thompson equation and the meson parameters of Tjon[107] we almost exactly reproduce the $^1S_0$ phase shifts which he obtains by solving the full Bethe-Salpeter equation. (This is, in fact, also true when the Blankenbecler-Sugar[123] equation is used.)

(iii) Thompson (like Blankenbecler-Sugar) does not include a retardation-like term in the meson propagator, i.e. for the exchange of a scalar boson the propagator is:

$$\frac{-1}{m_\alpha^2+(\vec{q}'-\vec{q})^2} \tag{5.4}$$

where the notation is explained in Fig. 3.5 and m_α denotes the mass of the exchanged boson. In the former work[104] of our group at Bonn we used to include a retardation-like term in the propagator:

$$\frac{-1}{m_\alpha^2+(\vec{q}'-\vec{q})^2 - (E_{q'}-E_q)^2} \tag{5.5}$$

However, the correct retardation is best considered in time-ordered perturbation theory, see Fig. 5.1, in which we have the following meson propagator:

$$\frac{-1}{\omega_\alpha(\omega_\alpha+E_{q'}+E_q-Z)} \tag{5.6}$$

with $\omega_\alpha \equiv \sqrt{m_\alpha^2 + (\vec{q}'-\vec{q})^2}$ the meson energy and $Z=2E_q$ the starting energy of the two interacting nucleons. (For more details see Ref. 166.) Eq. (5.6) more explicitly:

$$\frac{-1}{\sqrt{m_\alpha^2+(\vec{q}-\vec{q}')^2}\left(\sqrt{m_\alpha^2+(\vec{q}-\vec{q}')^2}+E_{q'}-E_q\right)} = \frac{-1}{m_\alpha^2+(\vec{q}-\vec{q}')^2+(E_{q'}-E_q)\sqrt{m_\alpha^2+(\vec{q}-\vec{q}')^2}} \tag{5.7}$$

As the intermediate momenta $\vec{q}'$ are prevailingly larger than $\vec{q}$ (e.g. in a 4th order diagram) the retardation term (last term on the right in the propagator Eq. (5.7)) is positive in most cases, whereas in Eq. (5.5) it is always negative and therefore wrong. It is just an artifact of that particular 3-dimensional reduction. For the reasons given, a meson propagator without the retardation like term of Eq. (5.5) is to be preferred.

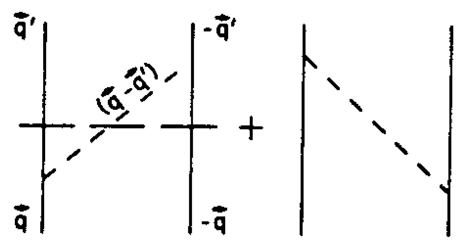


FIG. 5.1. A one-boson-exchange in time-ordered perturbation theory. The long dashed line indicates the states involved in the propagator.

Anticipating some of the nuclear matter formalism which we will introduce in Sec. 8, we want to indicate another fatal feature of the propagator Eq. (5.5). In the Dirac-Brueckner approach to nuclear matter one replaces

$$E_q \rightarrow \tilde{E}_q = \sqrt{\tilde{M}^2 + \vec{q}^2}$$

and

$$E_{q'} \rightarrow \tilde{E}_{q'} = \sqrt{\tilde{M}^2 + \vec{q}'^2} \qquad (5.8)$$

with $\tilde{M} < M$.

This replacement blows up the retardation-like term in Eq. (5.5) and because of its negative sign enhances the propagator. As a consequence the (attractive) second order in V is increased leading to substantially more attraction in nuclear matter. In fact, this increase of attraction is so disastrous that there is no saturation anymore. However, this is all an unphysical effect which only arises by taking all the incidental details of the propagator Eq. (5.5) too seriously.

There is a way to estimate the medium effects on the meson propagator correctly. It is again best considered in time ordered perturbation theory, Fig. 5.1 and Eq. (5.6). The replacement (5.8) applied to Eq. (5.6) weakens the propagator which leads to less attraction in nuclear matter (in contrast to the pseudo-effect discussed in the previous paragraph). This effect has been evaluated quantitatively by our group already some time ago (see Ref. 167).

In a recent paper[168] we also showed that the inclusion of the pion-selfenergy in the propagator more than compensates the medium effect discussed so far. Therefore we will not consider propagator effects here and use a meson propagator without retardation or retardation-like terms.

It should be noted that the arguments (ii) and (iii) given in this section also apply to the Blankenbecler-Sugar equation, which means that it could be used equally well. Therefore the final decision for Thompson was made by purely aesthetic aspects:

With our normalization of the Dirac spinors,

$$u(\vec{q})^\dagger u(\vec{q}) = 1, \qquad (5.9)$$

which is the proper one for nuclear matter, the R-matrix version of the Thompson equation, Eq. (5.3), assumes the simple form:

$$R(\vec{q}',\vec{q}) = V(\vec{q}',\vec{q}) - \mathcal{P} \int d^3k \frac{V(\vec{q}',\vec{k})R(\vec{k},\vec{q})}{2E_k - 2E_q} \qquad (5.10)$$

$\mathcal{P}$ denotes the principal value. This equation has exactly the form of the familiar Lippmann-Schwinger equation with the non-relativistic energies replaced by relativistic ones. For more details see Appendix A.

For the V in Eq. (5.10) we construct a relativistic OBEP with coupling constants and cutoff parameters as close as possible to the full model of Sec. 4. The diagrams of 2π and $\pi\rho$ exchange are replaced by a scalar isoscalar boson with the mass 550 MeV. The parameters of this OBEP are given in Table 5.1. The explicit momentum space formulae can be found in Appendix A. How the phase-shifts compare to the full model can be seen in Fig. 5.2.

TABLE 5.1

PARAMETERS OF A RELATIVISTIC OBEP IN THE FRAMEWORK OF THE THOMPSON EQUATION REPRESENTED IN q-SPACE ("OBEP A")

Meson	J^P	I	$\frac{g_\alpha^2}{4\pi}$	$\frac{f_v}{g_v}$	Meson Mass (MeV)	Λ_α(GeV)
π	0^-	1	14.6^a		138.03	1.3
ρ	1^-	1	0.95	6.1	769	1.3
ω	1^-	0	20.0	0.0	782.6	1.5
δ	0^+	1	4.9973		983	1.5
η	0^-	0	3.0^a		548.8	1.5
σ	0^+	0	7.8749		550	2.0

[a] $g_\alpha^2 = \left(\frac{2M}{m_\alpha}\right)^2 f_\alpha^2$; for π and η the pv coupling is applied.

Nucleon mass: M=938.926 MeV.

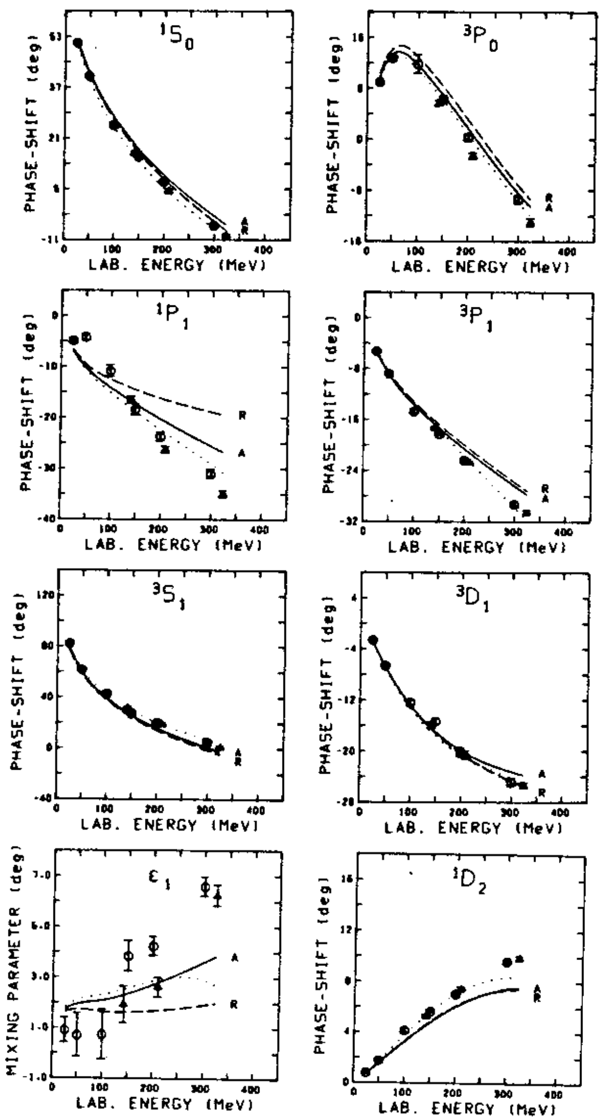


FIG. 5.2. Cont'd

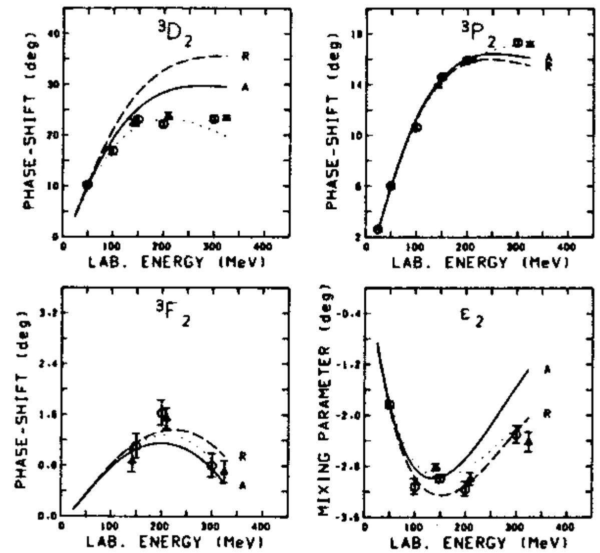


FIG. 5.2. Phase-shift predictions from two OBEPs in comparison to the results from the full model presented in Sec. 4 (dotted line). A (full line): relativistic momentum-space OBEP as discussed in Sec. 5.1 and defined in Table 5.1 and Appendix A. R (dashed line): non-relativistic r-space OBEP as discussed in Sec. 5.2 and defined in Appendix B and Table. 5.2.

5.2 A Non-Relativistic OBEP in r-Space

For purely practical reasons we present in this subsection a non-relativistic OBEP in r-space. Its derivation proceeds along the lines explained in Sec. 3. The advantages of this kind of representation is that one can consider explicitly the contributions to the central, tensor and spin-orbit force. Further, it is illustrative to explicitly see the r-dependence of the various mesonic contributions to the different terms of the potential.

The parameters are given in Table 5.2 and the explicit formulae in Appendix B. The phase-shift fit is compared to other models in Fig. 5.2. Obviously one has to make sacrifices in the quality of the fit when using the OBE approximation to the full meson theory.

The contributions of the various mesons to different parts of the force are plotted in Fig. 5.3-5.5.

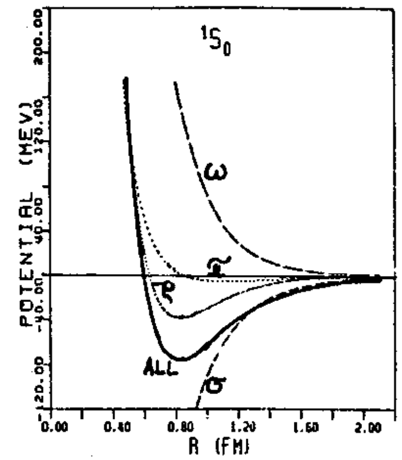
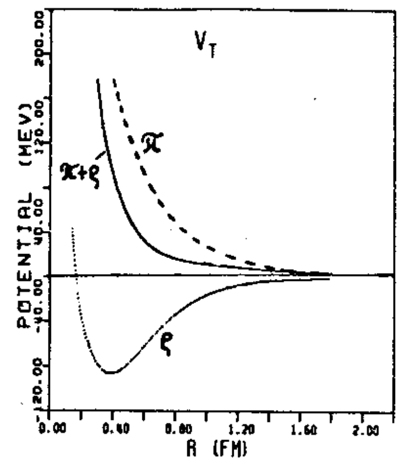


FIG. 5.3. r-space potential in the $^1S_0$ state (full line) and the contributions of the four most important mesons.



Tensor potential for π, ρ, and the sum of both.

TABLE 5.2

PARAMETERS OF A NON-RELATIVISTIC OBEP IN r-SPACE ("OBEP R")

Meson	J^P	I	$\frac{g_\alpha^2}{4\pi}$	$\frac{f_v}{g_v}$	Meson Mass (MeV)	Λ_α(GeV)
π	0^-	1	14.6		138.03	1.3
ρ	1^-	1	0.95	6.1	769.0	1.3
ω	1^-	0	20.0	0.0	782.6	1.5
δ	0^+	1	3.7064		983.0	1.5
η	0^-	0	3.0		548.8	1.5
σ	0^+	0	8.0568		550.0	1.75

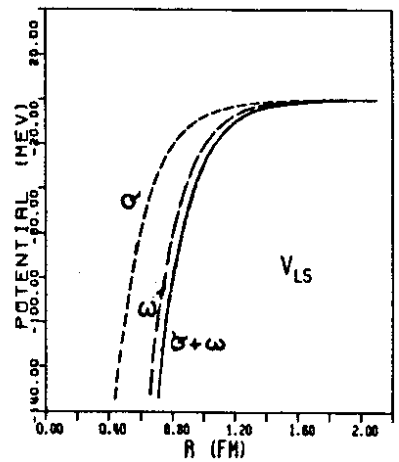


FIG. 5.5. Spin-orbit potential with the contributions from σ and ω.

6. GENERAL REMARKS ABOUT NUCLEAR MATTER

Nuclear matter is defined to be a hypothetical uniform system of nucleons of specified neutron and proton density interacting via the strong force without electromagnetic interactions. Here, we will consider only the case in which the neutron and proton densities are equal (symmetric nuclear matter). Our most precise quantitative information concerning nuclear matter comes from nuclear structure. The energy per nucleon in nuclear matter, E/N, can be deduced from the volumn term in the Bethe-Weizäcker mass formulae.[135] Semi-empirical information on the saturation energy and density of nucleons can be obtained from calculations of the charge distribution of closed shell nuclei.[169-171] Thus, nuclear matter is determined to have a binding energy per particle

$$E/N = -16.0 \pm 0.5 \text{ MeV} \tag{6.1}$$

and a saturation density

$$\rho_o = 0.16 \pm 0.015 \text{ fm}^{-3} \tag{6.2}$$

corresponding to a Fermi momentum*

$$k_F = 1.35 \pm 0.05 \text{ fm}^{-1} \tag{6.3}$$

* $\rho_o = \frac{2}{3\pi^2} k_F^3$

Another important quantity is the compression modulus (sometimes incorrectly called compressibility)

$$\mathcal{K} = k_F^2 \frac{d^2}{dk_F^2} E/N(k_F) \tag{6.4}$$

where the derivatives are to be taken at the equilibrium. It specifies the stiffness of nuclear matter with respect to bulk compression. Empirical information on $\mathcal{K}$ can be deduced from the systematics of the isoscalar monopole vibration, or breathing mode, in nuclei.[172] Thus, one obtains

$$\mathcal{K} \approx 210 \text{ MeV} \tag{6.5}$$

Introductions into nuclear matter theory are to be found in Ref. 173-176.

7. CONVENTIONAL BRUECKNER THEORY OF NUCLEAR MATTER

The basic quantity of Brueckener theory is the reaction matrix, G, satisfying the Brueckener-Bethe-Goldstone integral equation which reads in operator notation:

$$G = V - V \frac{Q}{e} G \tag{7.1}$$

and can be written diagrammatically as shown in Fig. 7.1. V, again, denotes the NN-potential (or more general: the kernel), Q the Pauli projection operator in nuclear matter which prevents nucleons from scattering into occupied intermediate states: 1/e is the two-nucleon propagator in the medium. Equation (7.1) is

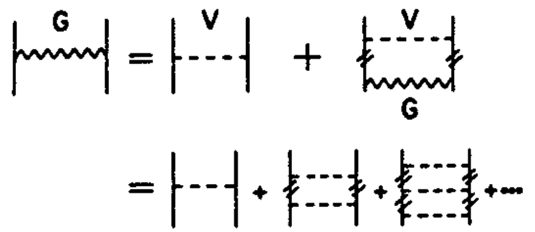


FIG. 7.1. Diagramatic representation of the Brueckner integral equation and the G-matrix. The double slash on intermediate lines indicates the change of the nucleon propagator and the Pauli-blocking in the medium.

defined in strict analogy to scattering the only difference being the Pauli-projector and the change of the nucleon propagator in the nuclear medium.

The goal of Brueckner theory is to evaluate the energy per nucleon in nuclear matter as a function of the density which is in lowest order in G:

$$E/N(k_F) = \frac{1}{N} \langle T \rangle + \frac{1}{2N} \langle G \rangle \tag{7.2}$$

where the first term on the right hand side stands for the average kinetic energy of the nucleons below the Fermi surface defined by the Fermi momentum, k_F, the second term indicates the average potential energy due to all effective two nucleon interactions. The explicit form of Eq. (7.1) is:

$$\langle \vec{k}_3\vec{k}_4|G(k_F,w)|\vec{k}_1\vec{k}_2\rangle = \langle \vec{k}_3\vec{k}_4|V|\vec{k}_1\vec{k}_2\rangle - \sum_{\vec{k}_m,\vec{k}_n} \langle \vec{k}_3\vec{k}_4|V|\vec{k}_m\vec{k}_n\rangle \cdot \frac{Q(k_F,\vec{k}_m,\vec{k}_n)}{\varepsilon(\vec{k}_m)+\varepsilon(\vec{k}_n)-w} \langle \vec{k}_m\vec{k}_n|G(k_F,w)|\vec{k}_1\vec{k}_2\rangle \tag{7.3}$$

where spin and isospin indices as well as the exchange terms, which we do include, have been suppressed. The single-particle energy of a nucleon in nuclear matter is defined by:

$$\varepsilon(\vec{k}_i) = T(\vec{k}_i) + U(\vec{k}_i) \tag{7.4}$$

with

$$U(\vec{k}_i) = \sum_{|k_j|<k_F} \langle \vec{k}_i\vec{k}_j|G(k_F,w=\varepsilon(\vec{k}_i)+\varepsilon(\vec{k}_j))|\vec{k}_i\vec{k}_j-\vec{k}_j\vec{k}_i\rangle \tag{7.5}$$

Equation (7.2) is explicitly:

$$\frac{E}{N}(k_F) = \frac{1}{N} \sum_{|\vec{k}|<k_F} \langle \vec{k}|T|\vec{k}\rangle + \frac{1}{2N} \sum_{|\vec{k}_1|,|\vec{k}_2|<k_F} \langle \vec{k}_1\vec{k}_2|G(k_F,w=\varepsilon(\vec{k}_1)+\varepsilon(\vec{k}_2))|\vec{k}_1\vec{k}_2-\vec{k}_2\vec{k}_1) \tag{7.6}$$

where T denotes the kinetic-energy operator.

In conventional Brueckner theory non-relativistic energies are used, i.e. $T(\vec{k}_i) = \vec{k}_i^{\,2}/2M$ (in analogy to the non-relativistic

Lippmann-Schwinger equation of scattering) and any phenomenological NN-potential (i.e. Reid[86]) is suitable to be used for V.

In the first attempts for a relativistic extension of Brueckner theory (which we will subsequently call: conventional relativistic Brueckner theory) Eq. (7.1) was applied with the relativistic kinematical factors and energies which occured in the analogous scattering equation (e.g. Blankenbecler-Sugar or Thompson, compare Sec. 5.1.) Phenomenological potentials are in general unsuitable for this approach as at least the most popular ones like Reid[86] or Paris,[119] are defined for the non-relativistic Schrödinger equation. Therefore OBEPs as well as more comprehensive meson-exchange models have been used at this stage. As we know, an OBE consists of the sum of single meson-exchanges in which the nucleons are naturally represented by four-component free Dirac spinors, $u(\vec{q})$, i.e. the spinors satisfy the free Dirac equation:

$$(\not{q}-M)u(\vec{q}) = 0 \tag{7.7}$$

Within the scheme of conventional relativistic Brueckner theory unchanged free spinors are taken over into the nuclear matter calculation. Therefore the potential applied to nuclear matter is exactly the same as in NN-scattering and that is why the results from this first attempt of a relativistic approach were not substantially different from those of non-relativistic Brueckner theory.

Figure 7.2 contains a survey of results obtained in the two approaches discussed so far. Only the saturation minima of the saturation curves, $E/N(k_F)$, are given. The squares stand for calculations using a free spectrum for nucleons above the Fermi surface ("gap"). The difference in the results displayed in Fig. 7.2 is mainly due to a different strength of the tensor force contained in the NN-potentials applied. This strength is best measured by the %-D-state, P_D, in the deuteron which the potentials give rise to. For Hamada-Johnston[84] it is 6.97% (the highest point in the plot), for HM2[106] we have P_D=4.32% (the lowest point).

The circles indicate results in which a continuous choice is applied for the single particle spectrum of the nucleons.[177,178] These results are generally 4-5 MeV more attractive compared to gap-calculation and appear to simulate the 3- and 4-body correlation.[179,180] Finally, Δ(1232)-isobars and their special medium effects are included in results symbolized by a small triangle.

It is easily seen that each type of calculation has its own "Coester line" (so-called after Ref. 181). Though improvements with respect to the empirical area can be observed when extensions within the conventional scheme are applied (continuous choice,

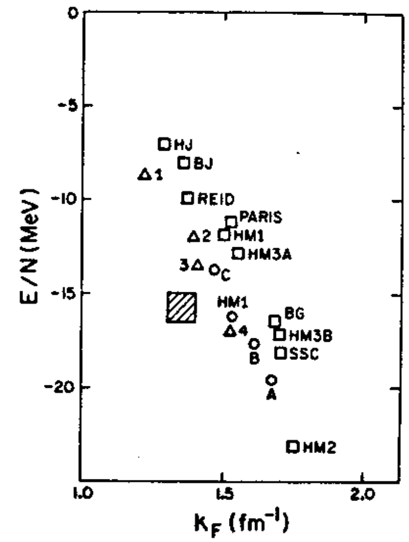


FIG. 7.2. Energy per nucleon in nuclear matter, E/N, as a function of the Fermi momentum k_F. The shaded square represents the empirical nuclear matter saturation. A brief history of results obtained in nuclear matter Brueckner theory is given by the small squares, circles and triangles which indicate the saturation minima of different conventional approaches explained in the text. The abbreviations for the potentials applied are given below: HJ: Hamada-Johston[84]; BJ: Bethe-Johnson[182]; REID: Reid-soft-core-potential[86]; PARIS: Ref. 119; HM1; Holinde-Machleidt(1)[105]; HM3A, HM3B: Ref. 183; BG: Bryan-Gersten[184]; SSC: Sprung-de Tourreil super-soft-core potential C[185]; HM2: Holinde-Machleidt(2)[106]; Δ1,2: Ref. 186; Δ3: Ref. 187; Δ4: Ref. 188.

Δ's), all bands of results clearly miss the empirical range. This summarized a long standing problem in nuclear matter theory.

8. THE DIRAC-BRUECKNER APPROACH TO NUCLEAR MATTER

8.1 The Idea and the Formalism

In this section we will explain a special extension of conventional relativistic Brueckner theory, subsequently called the Dirac-Brueckner approach.

The basic idea of this new approach (first introduced by Shakin and coworkers[189]) is that, as in the mean field theory,[190] one realizes that the nucleons in nuclear matter are exposed to a strong common scalar and vector field and therefore by no means free particles. Consequently free Dirac spinors satisfying the free Dirac Eq. (7.7) cannot be an adequate representation of a nucleon in nuclear matter. Instead, spinors obtained in a Dirac equation containing the strong common potentials should be used:

$$(\not{k}-M-\Sigma)u(\vec{k}) = 0 \qquad (8.1)$$

with

$$\Sigma = A(\vec{k}) + \gamma^0 B(\vec{k}) \qquad (8.2)$$

the selfenergy operator in nuclear matter. $A(\vec{k})$ and $B(\vec{k})$ represent the scalar and vector potential in nuclear matter respectively.

As it turns out that the k-dependence of A and B is very weak (namely, when expanding $A(\vec{k}) = A_0 + A_1(\vec{k}^2/k_F^2)$ and $B(\vec{k}) = B_0 + B_1(\vec{k}^2/k_F^2)$ one gets: $A_1/A_0 \approx B_1/B_0 \approx 0.05$), they can be assumed to be constant to a good approximation.

With $\tilde{M} \equiv M + A$ and $\tilde{E} \equiv \sqrt{\tilde{M}^2+\vec{k}^2}$ the Dirac spinor satisfying Eq. (8.1) is simply:

$$\tilde{u}(\vec{k},s) = \sqrt{\frac{\tilde{E}+\tilde{M}}{2\tilde{E}}}\begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{k}}{\tilde{E}+\tilde{M}} \end{pmatrix}\chi_s \qquad (8.3)$$

where the normalization $\tilde{u}^\dagger\tilde{u}=1$, appropriate for nuclear structure calculations, is applied, as throughout this work.

The selfenergy is given by Eq. (7.5) with

$$\Sigma(\vec{k}) = \bar{\tilde{u}}(\vec{k})\Sigma\,\tilde{u}(\vec{k}) = U(\vec{k}) \qquad (8.4)$$

As Σ depends on the G-matrix (compare Eq. (7.5)) and the G-matrix depends on Σ via the single particle energies and the Dirac spinors defined in Eq. (8.1) a self-consistency is required.

Formally all formulae of conventional Brueckner theory Eq. (7.3)-(7.6), still apply; however, their explicit meaning is now more refined, namely:

$$|\vec{k}\rangle = \tilde{u}(\vec{k})$$

$$\langle\vec{k}| = \bar{\tilde{u}}(\vec{k}) \qquad (8.5)$$

(This means that the potential V in Eq. (7.3) is obtained from that used in free scattering (which is given in Appendix A) by replacing: $M \to \tilde{M}$ and $E \to \tilde{E}$.)

$$T = \vec{\gamma}\cdot\vec{k} + M \qquad (8.6)$$

and a term -M should be added to Eq. (7.6).

$$\varepsilon(\vec{k}) = \langle\vec{k}|T|\vec{k}\rangle + \langle\vec{k}|\Sigma|\vec{k}\rangle$$

$$= \frac{M\tilde{M}+\vec{k}^2}{\tilde{E}} + \frac{\tilde{M}}{\tilde{E}}A + B$$

$$= \tilde{E} + B \qquad (8.7)$$

Assuming A and B constant, they can be determined once $\varepsilon(\vec{k})$ has been evaluated for two different $\vec{k}$. $k=(1/2)k_F$ and $k=k_F$ is an appropriate choice ($k\equiv|\vec{k}|$). The explicit formulae for nuclear matter Brueckner theory are given in more detail in Appendix C.

Special care has to be taken with the ρ-exchange potential being derived from the Lagrangian:

$$\mathcal{L}_{NN\rho} = g_\rho\bar{\psi}\gamma^\mu\psi\phi_\mu + \frac{f_\rho}{4M}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_\mu\phi_\nu-\partial_\nu\phi_\mu) \qquad (8.8)$$

As $f_\rho/4M$ is a coupling constant which could as well be defined by $f'_\rho/4m_\rho$, where m_ρ denotes the mass of the ρ-meson, the M in Eq. (8.8) must not be replaced by $\tilde{M}$. If that was done incorrectly, there would be no saturation in nuclear matter, as we will see later.

8.2 Former Calculations

The first work within the approach outlined in the previous subsection was done by Shakin and coworkers.[189] However, they take the new effect only in first order perturbation theory into account. Single particle energies and wave functions (Dirac spinors) in the medium are not determined selfconsistently, though this is a substantial requirement in Brueckner theory. Further, outdated one-meson-exchange potentials were used and a 3-dimensional relativistic reduction of the Bethe-Salpeter equation which, if applied consistently, in the Dirac approach, results in a disaster in nuclear matter due to an unphysical retardation term in the meson propagators (compare Eq. (5.5) and subsequent discussion). Also, the pseudo-vector coupling of the pion to the nucleon is required for the Dirac approach; however, the potentials applied by the Brooklyn group were constructed and fitted to the NN data using the pseudoscalar coupling. For all these reasons the results are not conclusive.

Horowitz and Serot[191] have recently and independently from us[192] demonstrated how to perform the Brueckner selfconsistency in the Dirac approach both correctly and elegantly. As they intend to show relative effects only, they do not use a quantitative nuclear force in their calculations.

The work quoted so far leaves open the question as to what the predictions of the Dirac-Brueckner approach to nuclear matter are when performed consistently and correctly.

Further, all past work applied only OBEP, in which the fictitious σ-boson causes a large relativistic medium effect. Therefore the critical question to be asked is: Will the relativistic saturation effects survive when the fictitious term is omitted and realistic and explicit 2π- and $\pi\rho$-exchanges introduced instead?

8.3 Results for the Full Meson-Exchange Model

All questions raised in the last subsection can be answered by performing a fully selfconsistent Dirac-Brueckner calculation as outlined in Sec. 8.1 with our full quantitative meson-exchange model introduced in Sec. 4.

This is what we will do in this subsection. To keep the computing time within responsible limits we have to do some minor simplifications. We use box-diagrams only and include the crossed 2π-exchange diagrams with intermediate Δ-isobars effectively by increasing the $N\Delta\pi$-coupling constant from $f^2_{N\Delta\pi}/4\pi = 0.23$ to 0.36. We make sure that the absolute size of the uncorrelated and correlated 2π-exchange is kept in agreement with the original model and that the fit of the NN-phase-shifts is retained.

The result is displayed in Fig. 8.1 by the full line labelled FULL MODEL. The empirical nuclear matter saturation properties are obviously described quantitatively by our result. The compressibility is 250 MeV. This is a very satisfactory result, since in the mean field theory[190] 550 MeV are obtained for $\mathcal{X}$.

For the particular examinations in this subsection we also construct an OBEP using time-ordered perturbation theory, which is applied in the full model, in order to compare the nuclear matter results gained with an OBEP with those from the full meson exchange model. The objective is to find out if the relativistic effect is the same in both cases. As the relativistic effect is also off-shell dependent the same type of perturbation theory has to be applied for both models. Therefore, the OBEP presented in Sec. 5.1 (OBEPA) is not suitable for this direct comparison.

The results with this OBEP in time ordered perturbation theory are also shown in Fig. 8.1 (dash-dot curves). It is seen that the OBEP does indeed have about the same "Dirac effect" as

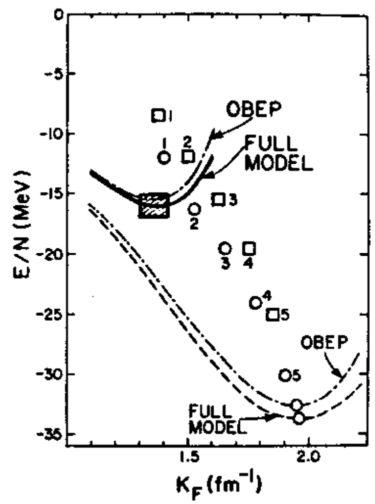


FIG. 8.1. Energy per nucleon, E/N, in nuclear matter versus the Fermi momentum, k_F. The full curve shows the result for the relativistic Dirac-Brueckner calculation using the model described in the text. The dashed curve is obtained with that same model in conventional Brueckner theory. The dash-dot curves refer to the respective calculations using the OBE-approximation to the full model in time-ordered perturbation theory. The squares and circles denote the saturation minima of conventional Brueckner calculations of the past using various different models for the NN-interaction. Squares are used when, for the particle spectrum above the fermi surface, free nucleon energies were used. Circles with the same number as a square refer to the corresponding calculations using a continuous single particle spectrum. The shaded rectangle denotes the empirical range of nuclear matter saturation.

the full model. More details about this comparison are to be found in Ref. 193.

After this result we feel justified to further examine the relativistic effect within the OBE model.

As mentioned before, it is more practical to work with energy independent potentials in nuclear structure. Therefore, we constructed in Sec. 5.1 a relativistic energy-independent OBEP. (The OBEP used in this subsection for reasons of performing an adequate comparison is energy-dependent since it is based on time-ordered perturbation theory.) The OBEP of Sec. 5.1 will be applied to nuclear matter in the next subsection.

8.4 Results with (Energy Independent) Relativistic OBEPs

The results for the Dirac-Brueckner approach to nuclear matter using the relativistic energy-independent OBEP presented in Sec. 5.1 ("OBEPA") are displayed in Fig. 8.2 by the full line labelled "A". The full curves labelled "B" and "C" refer to such calculations using variations of OBEPA with an increased tensor force (by using a larger cutoff mass for the πNN vertex).

The reason why we have performed the calculations with three different OBEPs is that the characteristic results for a certain type of many-body approach is not a point in the energy versus density plot; it is a band (a Coester band). There is not the nuclear force, there are several potentials, which describe the NN data equally well and never-the-less differ, namely, essentially in the strength of the tensor force. This difference leads to characteristic variations in the nuclear matter results, which always have a Coester-band structure.

Therefore, the reasonable question to be asked is if the band characteristic for one theory is oriented such that it would pass through the empirical area. Obviously our new approach provides additional strongly density-dependent repulsion such that the empiricial result can be met.

On the background of the failure of the many desperate attempts of the past to explain the empirical nuclear matter saturation, our result is not trivial.

The role of the various mesons in the relativistic saturation mechanism is demonstrated in Fig. 8.3. The effect of ω, σ and π is about equally strong, the π acting mainly through its second order contribution to the $^3S_1$-state. The ρ and especially the η and δ have very little effect. The consequence of using the wrong treatment of ρ-exchange and of the ps-coupling for the π are also demonstrated in Fig. 8.3. Both lead to unphysical results showing no saturation in nuclear matter.

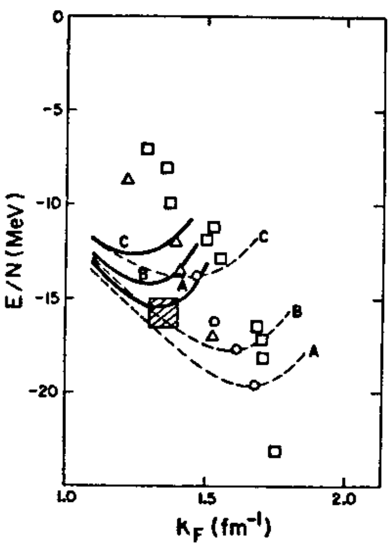


FIG. 8.2. Same as Fig. 7.2, but including the results of this subsection. For curves labeled "A" the OBEP presented in Sec. 5.1 and Table 5.1 is applied. Label "B" and "C" refers to two variations of that OBEP with an increased tensor force. The full lines denote Dirac-Brueckner results, the dashed curves stand for calculations with free spinors ("conventional relativistic Brueckner theory").

The constant part of the scalar potential, A_0, and of the vector potential, B_0, are displayed versus the Fermi-momentum, k_F, in Fig. 8.4 and 8.5 respectively. Generally we find a smoother density-dependence of these quantities compared to other authors.

More details about these calculations and more results are given in Ref. 194.

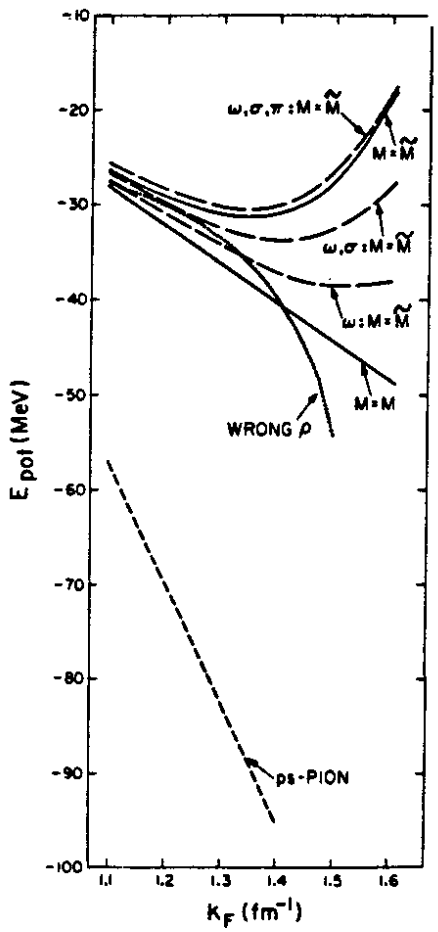


FIG. 8.3. "Potential energy," E_{pot}, (i.e. second term on r.h.s. of Eq. (7.6)) versus Fermi momentum, k_F, for the present OBEP. Starting with the conventional result labeled "M=M" the single mesons are switched successively on to the Dirac-Brueckner approach indicated by "M=M̃." The dotted curve shows the result when the M occurring in the Lagrangian for the ρNN-interaction, Eq. (8.8), is incorrectly replaced by M̃. The short dashed curve is obtained by using the ps-coupling for the pion.

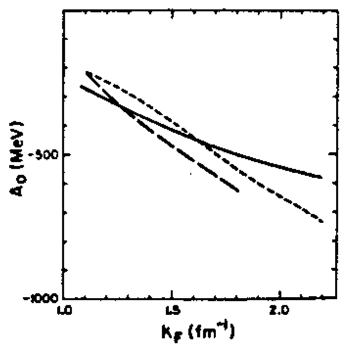


FIG. 8.4. Constant part of the scalar potential, A_0, versus Fermi-momentum, k_F. The full line displays the results using the present OBEP. The long and short dashed curve are from Ref. 189 and Ref. 190 respectively.

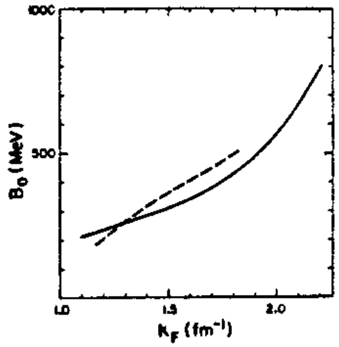


FIG. 8.5. Constant part of the vector potential, B_0, versus Fermi-momentum, k_F. The full line is obtained from the present OBEP; the dashed from the work of Ref. 189.

9. SUMMARY, CONCLUSIONS AND OUTLOOK

Briefly summarizing we want to state:

(i) Meson theory performed consistently and without fictitious terms is a quantitative theory for the NN-interaction. The necessity for the introduction of additional degrees of freedom is not indicated up to $E_{LAB} \approx 300$ MeV. The work presented is the first clean check of these questions.

(ii) The comprehensive field theoretic model for the NN interaction developed in this work presents a sound basis for studying several important issues in nuclear physics, like meson-current corrections, medium modifications of the nuclear force and others.

(iii) The empirical nuclear matter saturation can be explained quantitatively within a relativistically extended Brueckner theory applying a realistic and comprehensive meson-exchange model for the nuclear force. This is a parameter free calculation.

(iv) The contributions to the NN-interaction stemming from 2π- and $\pi\rho$-exchanges can well be approximated by a suitably chosen σ-boson. Even the relativistic medium effects of those explicit diagrams are simulated well by the σ. This justifies the use of OBEP in future relativistic nuclear structure calculations.

(v) Thus, the relationship between the free NN-interaction and nuclear structure in a relativistic framework is successfully established.

An important open question is the contributions from 3- and 4-body correlations in nuclear matter within the Dirac-Brueckner approach.

On the basis of the success of the Dirac-Brueckner approach to nuclear matter, future work should be devoted to the following topics: a) nucleon-nucleus scattering and b) structure of finite nuclei in a relativistic approach based on forces tested successfully in nuclear matter. Moreover, it is worthwhile to check the relativistic approach in other subfields of nuclear structure physics.

ACKNOWLEDGEMENT

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APPENDIX A: FORMULAE FOR THE RELATIVISTIC MOMENTUM SPACE OBEP AND THE RELATED TWO-BODY EQUATIONS[194]

Defining helicity states $|\lambda\rangle$ by

$$\frac{\vec{\sigma}\cdot\vec{q}}{2|\vec{q}|}\,|\lambda\rangle = \lambda|\lambda\rangle \tag{A1}$$

and abbreviating $\lambda = \pm 1/2$ by $\pm$, we consider six independent NN amplitudes:

$$\begin{aligned}
V_1^J &\equiv \langle ++|V^J(q',q)|++\rangle \\
V_2^J &\equiv \langle ++|V^J(q',q)|--\rangle \\
V_3^J &\equiv \langle +-|V^J(q',q)|+-\rangle \\
V_4^J &\equiv \langle +-|V^J(q',q)|-+\rangle \\
V_5^J &\equiv \langle ++|V^J(q',q)|+-\rangle \\
V_6^J &\equiv \langle +-|V^J(q',q)|++\rangle
\end{aligned} \tag{A2}$$

where the arguments (q',q) have been suppressed on the left hand side of all six equations. For convenience, we will later use the following combinations of helicity amplitudes:

$$\begin{aligned}
{}^0V^J &\equiv V_1^J - V_2^J \\
{}^1V^J &\equiv V_3^J - V_4^J \\
{}^{12}V^J &\equiv V_1^J + V_2^J \\
{}^{34}V^J &\equiv V_3^J + V_4^J \\
{}^{55}V^J &\equiv 2V_5^J \\
{}^{66}V^J &\equiv 2V_6^J
\end{aligned} \tag{A3}$$

We will represent the OBE contributions in these partial wave helicity amplitudes, the principal derivation of which is described in the review article by Erkelenz.* Therefore, we will not repeat it here. Note, however, that in the present work there are a few small, but decisive changes with regard to the quoted reference, namely in the present work

*Ref. 104, p.197-208.

(i) the meson propagator is (e.g. for a scalar meson):

$$\frac{1}{-(\vec{q}\,'-\vec{q})^2-m_\alpha^2} \tag{A4}$$

(ii) for the pseudoscalar mesons (π,η) the pseudovector (pv) coupling is used.

(iii) Due to the zeroth component of the meson momentum being zero the ρ-exchange is different from that of Ref. 104.

Because of these alterations the formula of Ref. 104 do not apply in all details. Therefore, to avoid any misunderstandings, we give below the final formulae explicitly for the different couplings involved in the OBE model presented in Sec. 5.1. We use the following definitions and abbreviations:

$$E \equiv \sqrt{\vec{q}^{\,2}+M^2}\ ,\ E' \equiv \sqrt{\vec{q}\,'^2+M^2} \tag{A5}$$

$$z_\alpha \equiv \frac{\vec{q}\,'^2+\vec{q}^{\,2}+m_\alpha^2}{2q'q} \tag{A6}$$

$$\begin{aligned}
Q_J^{(1)}(z_\alpha) &\equiv z_\alpha Q_J(z_\alpha) - \delta_{J0} \\
Q_J^{(2)}(z_\alpha) &\equiv \frac{1}{J+1}\left(Jz_\alpha Q_J(z_\alpha) + Q_{J-1}(z_\alpha)\right) \\
Q_J^{(3)}(z_\alpha) &\equiv \sqrt{\frac{J}{J+1}}\left(z_\alpha Q_J(z_\alpha) - Q_{J-1}(z_\alpha)\right) \\
Q_J^{(4)}(z_\alpha) &\equiv z_\alpha Q_J^{(1)}(z_\alpha) - \frac{1}{3}\delta_{J1} \\
Q_J^{(5)}(z_\alpha) &\equiv z_\alpha Q_J^{(2)}(z_\alpha) - \frac{2}{3}\delta_{J1} \\
Q_J^{(6)}(z_\alpha) &\equiv z_\alpha Q_J^{(3)}(z_\alpha) + \frac{\sqrt{2}}{3}\delta_{J1}
\end{aligned} \tag{A7}$$

with $Q_J(z_\alpha)$ the Legendre functions of the second kind, which occur in the analytic solutions of the integral for the partial wave decomposition

$$I_\alpha^{(0)} \equiv \int_{-1}^{+1} dt\, \frac{P_J(t)}{(\vec{q}\,'-\vec{q})^2+m_\alpha^2} = \frac{1}{q'q}\,Q_J(z_\alpha) \tag{A8}$$

with $P_J(t)$ the Legendre polynomials. The integral representations of Eq. (A7) are:

$$I_\alpha^{(1)} \equiv \frac{Q_J^{(1)}(Z_\alpha)}{q'q} = \int_{-1}^{+1} dt \frac{tP_J(t)}{(\vec{q}'-\vec{q})^2+m_\alpha^2}$$

$$I_\alpha^{(2)} \equiv \frac{Q_J^{(2)}(Z_\alpha)}{q'q} = \int_{-1}^{+1} dt \frac{\frac{J}{J+1} tP_J(t)+ \frac{1}{J+1} P_{J-1}(t)}{(\vec{q}'-\vec{q})^2+m_\alpha^2}$$

$$I_\alpha^{(3)} \equiv \frac{Q_J^{(3)}(Z_\alpha)}{q'q} = \sqrt{\frac{J}{J+1}} \int_{-1}^{+1} dt \frac{tP_J(t)-P_{J-1}(t)}{(\vec{q}'-\vec{q})^2+m_\alpha^2}$$

$$I_\alpha^{(4)} \equiv \frac{Q_J^{(4)}(Z_\alpha)}{q'q} = \int_{-1}^{+1} dt \frac{t^2P_J(t)}{(\vec{q}'-\vec{q})^2+m_\alpha^2}$$

$$I_\alpha^{(5)} \equiv \frac{Q_J^{(5)}(Z_\alpha)}{q'q} = \int_{-1}^{+1} dt \frac{\frac{J}{J+1} t^2P_J(t)+ \frac{1}{J+1} tP_{J-1}(t)}{(\vec{q}'-\vec{q})^2+m_\alpha^2}$$

$$I_\alpha^{(6)} \equiv \frac{Q_\alpha^{(6)}(Z_\alpha)}{q'q} = \sqrt{\frac{J}{J+1}} \int_{-1}^{+1} dt \frac{t^2P_J(t)-tP_{J-1}(t)}{(\vec{q}'-\vec{q})^2+m_\alpha^2} \qquad \text{(A9)}$$

These formulae can be verified with the help of the recurrence relations

$$tP_J(t) = \frac{J+1}{2J+1} P_{J+1}(t) + \frac{J}{2J+1} P_{J-1}(t) \qquad \text{(A10)}$$

and

$$Z_\alpha Q_J(Z_\alpha) = \frac{J+1}{2J+1} Q_{J+1}(Z_\alpha) + \frac{J}{2J+1} Q_{J-1}(Z_\alpha) + \delta_{Jo}$$

Instead of using the Legendre functions of the second kind and their combinations Eq. (A7), one can as well evaluate the integrals Eq. (A8) and (A9) numerically, since that can be done fast and reliably. This latter method has the advantage that the products of propagator and cutoffs need not to be decomposed (see below); the cutoffs are just included in the integrands of Eq. (A8) and (A9).

<u>Pseudovector coupling (π,η):</u>

$$\mathcal{L}_{pv} = \frac{f_{ps}}{m_{ps}} \bar{\psi}\gamma_5\gamma^\mu\psi\partial_\mu\phi_{ps} \qquad \text{(A12)}$$

$$^{o}V_{pv}^J = C_{pv}\left(F_{pv}^{(0)}I_{ps}^{(0)} + F_{pv}^{(1)}I_{ps}^{(1)}\right)$$

$$^{1}V_{pv}^J = C_{pv}\left(-F_{pv}^{(0)}I_{ps}^{(0)} - F_{pv}^{(1)} I_{ps}^{(2)}\right)$$

$$^{12}V_{pv}^J = C_{pv}\left(F_{pv}^{(1)}I_{ps}^{(0)} + F_{pv}^{(0)}I_{ps}^{(1)}\right)$$

$$^{34}V_{pv}^J = C_{pv}\left(-F_{pv}^{(1)}I_{ps}^{(0)} - F_{pv}^{(0)}I_{ps}^{(2)}\right)$$

$$^{55}V_{pv}^J = C_{pv}F_{pv}^{(2)}I_{ps}^{(3)}$$

$$^{66}V_{pv}^J = -C_{pv}F_{pv}^{(2)}I_{ps}^{(3)} \qquad \text{(A13)}$$

with

$$C_{pv} = \frac{f_{ps}^2}{2\pi^2 m_{ps}^2} \frac{M^2}{E'E}$$

and

$$F_{pv}^{(0)} = E'E - M^2 + \frac{1}{4M^2}(E'-E)^2(E'E+3M^2)$$

$$F_{pv}^{(1)} = -\left[M^2 - \frac{1}{4}[E'-E]^2\right] \frac{q'q}{M^2}$$

$$F_{pv}^{(2)} = -\frac{1}{M}(E'-E)\left(\frac{1}{4}(E'-E)^2+E'E\right)$$

<u>Scalar coupling (σ,δ):</u>

$$\mathcal{L}_s = g_s\bar{\psi}\psi\phi_s \qquad \text{(A14)}$$

$$^{o}V_s^J = C_s\left(F_s^{(0)}I_s^{(0)} + F_s^{(1)}I_s^{(1)}\right)$$

$$^{1}V_s^J = C_s\left(F_s^{(0)}I_s^{(0)} + F_s^{(1)}I_s^{(2)}\right)$$

$$^{12}V_s^J = C_s\left(F_s^{(1)}I_s^{(0)} + F_s^{(0)}I_s^{(1)}\right)$$

$$^{34}V_s^J = C_s\left(F_s^{(1)}I_s^{(0)} + F_s^{(0)}I_s^{(2)}\right)$$

$$^{55}V_s^J = C_sF_s^{(2)}I_s^{(3)}$$

$$^{66}V_s^J = C_sF_s^{(2)}I_s^{(3)} \qquad (A15)$$

with

$$C_s = \frac{g_s^2}{8\pi^2E'E}$$

and

$$F_s^{(0)} = -(E'E + M^2)$$

$$F_s^{(1)} = q'q$$

$$F_s^{(2)} = E'+E$$

Vector and tensor coupling (ρ,ω):

$$\mathcal{L}_v = g_v\bar{\psi}\gamma_\mu\psi\phi^\mu + \frac{f_v}{2M}\,\psi\sigma_{\mu\nu}\psi\partial^\mu\phi_v^\nu \qquad (A16)$$

One obtains three terms which are characterized by their coupling constants.

Vector-vector:

$$^{0}V_v^J = C_v(2E'E - M^2)I_v^{(0)}$$

$$^{1}V_v^J = C_v\left(E'E\cdot I_v^{(0)} + q'qI_v^{(2)}\right)$$

$$^{12}V_v^J = C_v\left(2q'q\cdot I_v^{(0)} + I_v^{(1)}\right)$$

$$^{34}V_v^J = C_v\left(q'q\cdot I_v^{(0)} + E'EI_v^{(2)}\right)$$

$$^{55}V_v^J = -C_vEI_v^{(3)} \qquad (A17)\ \text{Cont'd}$$

$$^{66}V_v^J = -C_vE'I_v^{(3)} \qquad (A17)$$

with

$$C_v = \frac{g_v^2}{4\pi^2E'E}$$

Vector-tensor:

$$^{0}V_{vt}^J = C_{vt}M\left((q'^2+q^2)I_v^{(0)} - 2q'qI_v^{(1)}\right)$$

$$^{1}V_{vt}^J = C_{vt}M\left(-(q'^2+q^2)I_v^{(0)} + 2q'qI_v^{(2)}\right)$$

$$^{12}V_{vt}^J = C_{vt}M\left(6q'qI_v^{(0)} - 3(q'^2+q^2)I_v^{(1)}\right)$$

$$^{34}V_{vt}^J = C_{vt}M\left(2q'qI_v^{(0)} - (q'^2+q^2)I_v^{(2)}\right)$$

$$^{55}V_{vt}^J = C_{vt}(E'q^2 + 3Eq'^2)I_v^{(3)}$$

$$^{66}V_{vt}^J = C_{vt}(Eq'^2 + 3E'q^2)I_v^{(3)} \qquad (A18)$$

with

$$C_{vt} = \frac{g_v\left(\frac{f_v}{M}\right)}{8\pi^2E'E}$$

(Note, that in relativistic nuclear matter calculations when $M \to \tilde{M}$ the factor (f_v/M) has to be kept unchanged as it represents the tensor coupling constant.)

Tensor-tensor:

$$^{0}V_t^J = C_t\left\{(q'^2+q^2)(3E'E+M^2)I_v^{(0)} + [q'^2+q^2-2(3E'E+M^2)]q'qI_v^{(1)} - 2q'^2q^2I_v^{(4)}\right\}$$

$$^{1}V_t^J = C_t\left\{[4q'^2q^2+(q'^2+q^2)(E'E-M^2)]I_v^{(0)} + 2(E'E+M^2)q'q\cdot I_v^{(1)} - (q'^2+q^2+4E'E)q'q\cdot I_v^{(2)} - 2q'^2q^2\cdot I_v^{(5)}\right\} \qquad (A19)\ \text{Cont'd}$$

$$ {}^{12}V_t^J = C_t \Big\{ \left[4M^2-3(q'^2+q^2)\right]q'qI_V^{(0)} + \left[6q'^2q^2-(q'^2+q^2)(E'E+3M^2)\right]I_V^{(1)} + 2(E'E+M^2)q'q\cdot I_V^{(4)} \Big\} $$

$$ {}^{34}V_t^J = C_t \Big\{ -\left[q'^2+q^2+4E'E\right]I_V^{(0)} - 2q'^2q^2I_V^{(1)} + \left[4q'^2q^2+(q'^2+q^2)(E'E-M^2)\right]I_V^{(2)} + 2(E'E+M^2)q'q\cdot I_V^{(5)} \Big\} $$

$$ {}^{55}V_t^J = C_tM \Big\{ \left[E'(q'^2+q^2)+E(3q'^2-q^2)\right]I_V^{(3)} - 2(E'+E)q'q\cdot I_V^{(6)} \Big\} $$

$$ {}^{66}V_t^J = C_tM \Big\{ \left[E(q'^2+q^2)+E'(3q^2-q'^2)\right]I_V^{(3)} - 2(E'+E)q'q\cdot I_V^{(6)} \Big\} \quad \text{(A19)} $$

with

$$ C_t = \frac{(f_V/M)^2}{32\pi^2E'E} $$

(The note below Eq. (A18) applies again.)

Note, that for the isovector bosons π, δ and ρ the corresponding potentials have to be multiplied by a factor $\vec{\tau}_1\cdot\vec{\tau}_2$. The use of cutoffs means that the propagator

$$ \frac{1}{(\vec{q}\,'-\vec{q})^2+m_\alpha^2} $$

has to be replaced by

$$ \frac{1}{(\vec{q}\,'-\vec{q})^2+m_\alpha^2}\left(\frac{\Lambda_\alpha^2-m_\alpha^2}{(\vec{q}\,'-\vec{q})^2+\Lambda_\alpha^2}\right)^2 \quad \text{(A20)} $$

This expression can be applied directly in the case of the numerical integration of Eq. (A8) and (A9). When using the Legendre functions of the second kind Eq. (A20) has to be replaced by

$$ \frac{1}{(\vec{q}\,'-\vec{q})^2+m_\alpha^2} - \frac{\Lambda_{\alpha,2}^2-m_\alpha^2}{\Lambda_{\alpha,2}^2-\Lambda_{\alpha,1}^2}\,\frac{1}{(\vec{q}\,'-\vec{q})^2+\Lambda_{\alpha,1}^2} + \frac{\Lambda_{\alpha,1}^2-m_\alpha^2}{\Lambda_{\alpha,2}^2-\Lambda_{\alpha,1}^2}\,\frac{1}{(\vec{q}\,'-\vec{q})^2+\Lambda_{\alpha,2}^2} \quad \text{(A21)} $$

with

$$ \Lambda_{\alpha,1} = \Lambda_\alpha + \varepsilon $$

$$ \Lambda_{\alpha,2} = \Lambda_\alpha - \varepsilon; \qquad \varepsilon \ll \Lambda_\alpha $$

(ε=10 MeV is a suitable choice.)

R-matrix equation (Thompson) for spin singlet states:

$$ {}^0R^J(q',q) = {}^0V^J(q',q) - \mathbb{P}\int_0^\infty k^2dk\,\frac{{}^0V^J(q',k)\,{}^0R^J(k,q)}{2E_k - 2E_q} \quad \text{(A22)} $$

where

$$ {}^0V^J = \sum_{\alpha=\pi,\eta,\sigma,\delta,\omega,\rho} {}^0V_\alpha^J \quad \text{(A23)} $$

and

$$ E_k \equiv \sqrt{M^2+\vec{k}^2}\ ,\ E_q \equiv \sqrt{M^2+\vec{q}^2}\ . $$

$\mathbb{P}$ denotes the principal value.

For the coupling constants, masses and cutoff masses of the various mesons summed up in Eq. (A23) see Table 5.1.

Phase-shift relation:

$$ \mathrm{tg}\,{}^0\delta^J(q) = -\frac{\pi}{2}\,qE_q\,{}^0R^J(q,q) \quad \text{(A24)} $$

with

$$ E_{LAB} = \frac{2q^2}{M} $$

The formulae for the spin triplet states are given in Ref. 104 (pp.225-228); note, however, that those formulae have to be modified concerning the two-nucleon propagator (compare Eq. (A22)) and concerning the factor in the phase-shift relation (compare Eq. (A24)).

The transformation of the partial-wave helicity amplitudes, in which the potential is explicitly presented here, into a LSJ-state basis, which is commonly used in nuclear structure calculations, can be done with the help of the following formulae:

Notation:

$$ V_{L',L}^{JST} $$

Spin singlet:

$$V^{J0T}_{J,J} = {}^{0}V^{J}$$

Spin triplet uncoupled:

$$V^{J1T}_{J,J} = {}^{1}V^{J}$$

Spin triplet coupled:

$$V^{J1T}_{J-1,J-1} = \frac{1}{2J+1}\left\{J\,{}^{12}V + (J+1)\,{}^{34}V^{J} + 2\sqrt{J(J+1)}\;{}^{56}V^{J}\right\}$$

$$V^{J1T}_{J+1,J+1} = \frac{1}{2J+1}\left\{(J+1)\,{}^{12}V^{J} + J\,{}^{34}V^{J} - 2\sqrt{J(J+1)}\;{}^{56}V^{J}\right\}$$

$$V^{J1T}_{J-1,J+1} = \frac{\sqrt{J(J+1)}}{2J+1}\left\{{}^{12}V^{J} - {}^{34}V^{J} - \frac{J}{\sqrt{J(J+1)}}\,{}^{55}V^{J} + \frac{(J+1)}{\sqrt{J(J+1)}}\,{}^{66}V^{J}\right\}$$

$$V^{J1T}_{J+1,J-1} = \frac{\sqrt{J(J+1)}}{2J+1}\left\{{}^{12}V^{J} - {}^{34}V^{J} + \frac{(J+1)}{\sqrt{J(J+1)}}\,{}^{55}V^{J} - \frac{J}{\sqrt{J(J+1)}}\,{}^{66}V^{J}\right\}$$

with

$${}^{56}V^{J} = V^{J}_{5} + V^{J}_{6}$$

APPENDIX B: EXPRESSIONS FOR A NON-RELATIVISTIC OBEP IN R-SPACE

Following the lines of Sec. 3, however, keeping terms up to momentum squared, the one-meson-exchange contributions are first simplified in q-space, such that they can be transformed analytically into r-space. Such q-space expressions are:[195,133]

pseudoscalar mesons (π,η)

$$V_{ps}(\vec{k},\vec{p}) = -\frac{g^2_{ps}}{4M^2}\,\frac{\vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}}{\vec{k}^2+m^2_{ps}}\;;$$

$\vec{k} \equiv \vec{q}\,'-\vec{q}$ with $\vec{q}\,'$ and $\vec{q}$ as in Fig. 3.5.

scalar mesons (σ,δ)

$$V_{s}(\vec{k},\vec{p}) = -\frac{g^2_{s}}{\vec{k}^2+m^2_{s}}\left[1 - \frac{\vec{p}^{\,2}}{2M^2} + \frac{\vec{k}^2}{8M^2} - \frac{i}{2M^2}\,\vec{S}\cdot(\vec{k}\times\vec{p})\right]\;;$$

where $\vec{p} \equiv 1/2(\vec{q}\,'+\vec{q})$ and $\vec{S} = 1/2(\vec{\sigma}_1+\vec{\sigma}_2)$.

vector mesons (ω,ρ)

$$V_{v}(\vec{k},\vec{p}) = \frac{1}{\vec{k}^2+m^2_{v}}\left\{g^2_{v}\left[1 + \frac{3}{2}\frac{\vec{p}^{\,2}}{M^2} - \frac{\vec{k}^2}{8M^2} + \frac{3i}{2M^2}\,\vec{S}\cdot(\vec{k}\times\vec{p}) - \vec{\sigma}_1\cdot\vec{\sigma}_2\,\frac{\vec{k}^2}{4M^2} + \frac{1}{4M^2}\,\vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}\right]\right.$$
$$+ \frac{g_v f_v}{2M}\left[-\frac{\vec{k}^2}{M} + \frac{4i}{M}\,\vec{S}\cdot(\vec{k}\times\vec{p}) - \vec{\sigma}_1\cdot\vec{\sigma}_2\,\frac{\vec{k}^2}{M} + \frac{1}{M}\,\vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}\right]$$
$$\left. + \frac{f^2_v}{4M^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{k}^2 + \vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}\right]\right\}$$

After a Fourier transformation one obtains the following r-space potentials:

pseudoscalar mesons (π,η)

$$V_{ps}(r) = \frac{1}{12}\,\frac{g^2_{ps}}{4\pi}\,m_{ps}\left\{\left(\frac{m_{ps}}{M}\right)^2 Y(m_{ps}r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + Z(m_{ps}r)\,S_{12}\right\}$$

scalar mesons (σ,δ)

$$V_s(r) = \frac{-g_s^2}{4\pi} m_s \left\{ \left[1 - \frac{1}{4}\left(\frac{m_s}{M}\right)^2\right] Y(m_s r) + \frac{1}{4M^2}\left[\nabla^2 Y(m_s r) + Y(m_s r)\nabla^2\right] + \frac{1}{2} Z_1(m_s r)\vec{L}\cdot\vec{S} \right\}$$

vector mesons (ω,ρ)

$$\begin{aligned} V_v(r) = \frac{g_v^2}{4\pi} m_v &\left\{ \left[1 + \frac{1}{2}\left(\frac{m_v}{M}\right)^2\right] Y(m_v r) - \frac{3}{4M^2}\left[\nabla^2 Y(m_v r) + Y(m_v r)\nabla^2\right] \right. \\ &\left. + \frac{1}{6}\left(\frac{m_v}{m}\right)^2 Y(m_v r)\vec{\sigma}_1\cdot\vec{\sigma}_2 - \frac{3}{2} Z_1(m_v r)\vec{L}\cdot\vec{S} - \frac{1}{12} Z(m_v r)S_{12} \right\} \\ &+ \frac{1}{2}\frac{g_v f_v}{4\pi} m_v \left\{ \left(\frac{m_v}{M}\right)^2 Y(m_v r) + \frac{2}{3}\left(\frac{m_v}{M}\right)^2 Y(m_v r)\vec{\sigma}_1\cdot\vec{\sigma}_2 \right. \\ &\left. - 4Z_1(m_v r)\vec{L}\cdot\vec{S} - \frac{1}{3} Z(m_v r)S_{12} \right\} \\ &+ \frac{f_v^2}{4\pi} m_v \left\{ \frac{1}{6}\left(\frac{m_v}{M}\right)^2 Y(m_v r)\vec{\sigma}_1\cdot\vec{\sigma}_2 - \frac{1}{12} Z(m_v r)S_{12} \right\} \end{aligned}$$

with

$$Y(x) = e^{-x}/x$$

$$Z(x) = \left(\frac{m_\alpha}{M}\right)^2 \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) Y(x)$$

$$Z_1(x) = \left(\frac{m_\alpha}{M}\right)^2 \left(\frac{1}{x} + \frac{1}{x^2}\right) Y(x)$$

$$M = 938.926 \text{ MeV}/c^2$$

Apply a factor $\vec{\tau}_1\cdot\vec{\tau}_2$ for π, ρ, and δ. For the application of the cutoffs see Eq. (A21) from which the consequences for the r-space potential can be drawn easily. The parameters for the potential given in this appendix are in Table 5.2. The full potential is defined by the sum of the six one-meson-exchange contributions.

APPENDIX C: NUCLEAR MATTER FORMULAE FOR THE RELATIVISTIC DIRAC BRUECKNER APPROACH

The calculations are done in partial wave decomposition. The Brueckner equation, Eq. (7.3), for an uncoupled partial wave state is:

$$G_L(q',q,A,k_F) = \tilde{V}_L(q',q,A) - \int_0^\infty k^2 dk\, \tilde{V}_L(q',k,A) \frac{Q(k,k_F)}{e(k,q,A)} G_L(k,q,A,k_F) \tag{C1}$$

The extension to coupled cases is straightforward. The CM momentum, P, is set to zero for consistency with the potential and the way it was derived. $\vec{q}$,q' and k are relative momenta; e.g. $q = 1/2|\vec{k}_1-\vec{k}_2|$ with $\vec{k}_1$ and $\vec{k}_2$ as in Eq. (7.3). The wiggle on top of the potential, $\tilde{V}_L$, indicates that in the expressions for the OBEP of Appendix A one has to do the following replacements:

$$M \rightarrow \tilde{M} = M + A$$

and

$$E \rightarrow \tilde{E} \equiv \sqrt{\tilde{M}^2+\vec{q}^2}\ ,$$

$$E' \rightarrow \tilde{E}' \equiv \sqrt{\tilde{M}^2+\vec{q}'^2} \tag{C2}$$

due to the fact that the Dirac spinors Eq. (8.3) are used in nuclear matter. Q denotes the Pauli projector which for P=0 assumes the simple form:

$$Q(k,k_F) = \begin{cases} 0 & \text{for } k < k_F \\ 1 & \text{for } k > k_F \end{cases} \tag{C3}$$

The energy-denominator in the Brueckner equation is:

$$e(k,q,A) = 2\tilde{E}_k - 2\tilde{E}_q \tag{C4}$$

with

$$\tilde{E}_k \equiv \sqrt{\tilde{M}^2+\vec{k}^2} \quad \text{and} \quad \tilde{E}_q \equiv \sqrt{\tilde{M}^2+\vec{q}^2}$$

The scalar potential A remains to be determined. This can be done with the help of the following formualae. From Eq. (8.7) and (8.4) we know that

$$\varepsilon(k) = \frac{M\tilde{M}+k^2}{\tilde{E}_k} + U(k) \qquad \text{(C5)}$$

with

$$k \equiv |\vec{k}|$$

and

$$U(k) = \sum_{J,S,T,L} (2J+1)(2T+1) \times \left\{ \left[4 \int_0^{1/2(k_F-k)} q_0^2 dq_0 + \int_{1/2(k_F-k)}^{1/2(k_F+k)} q_0^2 dq_0 \, \frac{k_F^2-(2q_0-k)^2}{2q_0 k} \right] G_{LL}^{JST}(q_0,q_0,A,k_F) \right\} \qquad \text{(C6)}$$

(J ≡ total angular momentum, T ≡ total isospin.)

On the other hand, because of the ansatz Eq. (8.2), we also have:

$$\varepsilon(k) = \tilde{E}_k + B; \qquad 0 \leq k < \infty \qquad \text{(C7)}$$

("continuous choice")

Computing ε for two different k ($k_1 = (1/2k_F)$ and $k_2 = k_F$ is an appropriate choice) according to Eq. (C5) and Eq. (C6) and then applying Eq. (C7) determines A and B.

The procedure to be done separately for each k_F is as follows: One starts with a guess for A (e.g. -600 MeV), solves the Brueckner equation Eq. (C1) to obtain G and evaluates U(k) [Eq. (C6)] with those G for two different k. From that one obtains with the help of Eq. (C5) and (C7) a new value for A (and B). With the new A one starts all over again till the old and the new A agree sufficiently well, i.e. selfconsistency is achieved. From the G-matrix elements, computed with the selfconsistent A, one evaluates the energy per nucleon according to the following formula [compare Eq. (7.6) and Eq. (8.5)-(8.7)]:

$$\frac{E}{N}(k_F) = \frac{3}{k_F^3} \int_0^{k_F} q_0^2 dq_0 \, \frac{q_0^2+M\tilde{M}}{\tilde{E}_{q_0}} + \sum_{JSTL} (2J+1)(2T+1) \times \int_0^{k_F} q_0^2 dq_0 \left(2-3\frac{q_0}{k_F}+\frac{q_0^3}{k_F^3}\right) G_{LL}^{JST}(q_0,q_0,A,k_F) - M \qquad \text{(C8)}$$

It is most practical to use in all calculations units of MeV for momenta, masses and energies (conversion factor: $\hbar c$ = 197.3286 MeV·fm). The potential V, $\tilde{V}$ and the G-matrix are then in units of MeV^{-2}.

Concerning the derivation of some of the formulae given in this Appendix and some computational hints see Ref. 196 and 197.

REFERENCES

1. For an introduction into QCD see e.g.: W. Marciano and H. Pagels, Phys. Reports 36, 137 (1978).
2. For a recent review of lattice gauge theories see: M. Creutz, L. Jacobs and C. Rebbi, Phys. Reports 95, 201 (1983); and Janos Polonyi, Lectures at this workshop.
3. G.E. Brown and M. Rho, Phys. Lett. 82B, 177 (1979); G.E. Brown, M. Rho and V. Vento, Phys. Lett. 84B, 383 (1979).
4. A. Mittal and A.N. Mitra, Phys. Rev. D29, 1408 (1984).
5. Yu You-Wen and Zhang Zong-Ye, Nucl. Phys. A426, 557 (1984).
6. T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12, 2060 (1975).
7. A.W. Thomas, S. Théberge and G.A. Miller, Phys. Rev. D24, 216 (1981); A.W. Thomas, Advances in Nucl. Phys. 13, 1 (1983).
8. K. Maltman and N. Isgur, Phys. Rev. D29, 952 (1984); N. Isgur, Lectures at this workshop.
9. K. Holinde, Phys. Lett. 157B, 123 (1985).
10. C.S. Warke and R. Shanker, Phys. Rev. C21, 2643 (1980).
11. J. Burger and M. Hofmann, Phys. Lett. 148B, 25 (1984); and preprint, Erlangen, 1985.
12. M. Harvey, J. Letourneux and B. Corazo, Nucl. Phys. A424, 428 (1984); N. Mankoc-Borstnik et al., Nucl. Phys. A395, 349 (1983), and contribution to this workshop.
13. A. Faessler, F. Fernanadez, G. Lübeck and K. Shimizu, Nucl. Phys. A402, 555 (1983).
14. O. Morimatsu, K. Yazaki and M. Oka, Nucl. Phys. A424, 412 (1984).
15. R.L. Jaffe, Lectures at this workshop.
16. G. t'Hooft, Nucl. Phys. B72, 461 (1974); B75, 461 (1974).
17. E. Witten, Nucl. Phys. B160, 57 (1979).
18. T.H.R. Skyrme, Proc. Roy. Soc. A260, 127 (1961); Nucl. Phys. 31, 556 (1962).
19. V. Vento, M. Rho, E. Nyman, J.H. Jun and G.E. Brown, Nucl. Phys. A345, 413 (1980).
20. G.E. Brown , A.D. Jackson, M. Rho and V. Vento, Phys. Lett. 140B, 285 (1984).
21. Lewis Carroll, Alice's Adventures in Wonderland (Macmillan, London, 1865).
22. S. Nadkarin, H.B. Nielsen and I. Zaked, "Bosonization Relations as Bag Boundary Conditions," preprint Niels Bohr Institut, NBI-HE-84-41, 1984.
23. G. Adkins and C.R. Nappi, Phys. Lett. 137B, 251 (1984).
24. M. Rho, Proc. Int. School of Physics "Enrico Fermi," 18-23 June 1984 (Varenna, Italy); K. Iketani, Kyushu University Preprint, 1984.
25. I. Zahed, Lectures at this workshop.
26. J. Chadwick, Proc. Roy. Soc. (London), A136, 692 (1932).
27. W. Heisenberg, Z. Physik 77, 1 (1932).

28. E. Majorana, Z. Physik 82, 137 (1933).
29. M. Tuve, N. Heydenburg and L. Hafstad, Phys. Rev. 50, 806 (1936).
30. G. Breit, E. Condon and R. Present, Phys. Rev. 50, 825 (1936).
31. H. Yukawa, Proc. Phys. Math. Soc. Japan 17, 48 (1935).
32. H. Yukawa, Proc. Phys. Math. Soc. Japan 19, 712 (1937); H. Yukawa and S. Sakata, Proc. Phys. Math. Soc. Japan 19, 1084 (1937).
33. H. Yukawa, S. Sakata and M. Taketani, Proc. Phys. Math. Soc. Japan 20, 319 (1938); H. Yukawa, S. Sakata, M. Kobayasi and M. Taketani 20, 720 (1938); the papers by Yukawa and other Japanese physicists from the first days in meson theory up to about 1950 are reprinted in Progr. of Theor. Phys. (Kyoto), Suppl. 1 and 2 (1955).
34. S.H. Neddermeyer and C.D. Anderson, Phys. Rev. 51, 884 (1937); J.C. Street and E.C. Stevenson, Phys. Rev. 51, 1005A (1937).
35. N. Kemmer, Proc. Roy. Soc. (London) A166, 127 (1938).
36. A. Proca, J. Phys. Radium 7, 347 (1936).
37. N. Kemmer, Proc. Cambridge Phil. Soc. 34, 354 (1938).
38. H.J. Bhabha, Proc. Roy. Soc. (London) A166, 501 (1938).
39. N. Kemmer, Proc. Roy. Soc. (London) A173, 91 (1939).
40. B. Clark, Contribution to this workshop.
41. G. Wick, Nature 142, 993 (1938).
42. J. Kellog, I. Rabi, N.F. Ramsey and J. Zacharias, Phys. Rev. 56, 728 (1939); 57, 677 (1940).
43. C. Møller and L. Rosenfeld, Kgl. Danske Vid. Selskab, Math-fys. Medd. 17, No. 8 (1940).
44. J. Schwinger, Phys. Rev. 61, 387 (1942).
45. J.M. Jauch and N. Hu, Phys. Rev. 65, 289 (1944).
46. H.A. Bethe, Phys. Rev. 57, 260, 390 (1940); for an application of Bethe's idea see also: W. Rarita and J. Schwinger, Phys. Rev. 59, 436, 556 (1941).
47. W. Pauli, Meson Theory of Nuclear Forces, (Interscience, New York, 1946).
48. G. Breit, Phys. Rev. 51, 248 (1937); G. Breit and J.R. Stehn, Phys. Rev. 53, 459 (1938).
49. L. Rosenfeld, Nature 156, 141 (1945).
50. L. Rosenfeld, Nuclear Forces (North-Holland Publ. Comp., Amsterdam, 1948).
51. A.E.S. Green, Phys. Rev. 73, 26 (1948); 75, 1926 (1949); 76, 460, 870 (1949).
52. A.E.S. Green, unpublished; private communication.
53. G. Wentzel, Quantum Theory of Fields (Interscience, New York, 1949).
54. M. Conversi, E. Pancini and O. Piccioni, Phys. Rev. 71, 209, 557 (1947).

55. G.P.S. Occhialini, C.F. Powell, C.M.G. Lattes and Muirhead, Nature 159, 186, 694 (1947); C.M.G. Lattes, G.P.S. Occhialini and C.F. Powell, Nature 160, 453, 486 (1947).
56. E. Gardner and C.M. Lattes, Science 107, 270 (1948).
57. M. Taketani, S. Nakamura and M. Sasaki, Prog. Theor. Phys. (Kyoto) 6, 581 (1951).
58. Progr. Theoret. Phys. (Kyoto), Suppl. 3 (1956).
59. P. Cziffra, M.H. MacGregor, M.J. Moravcsik and H.P. Stapp, Phys. Rev. 114, 880 (1959).
60. M.J. Moravcsik, Nucleon-Nucleon Scattering and the One-Pion-Exchange, UCRL 5886-T (1958).
61. G. Breit, M.H. Hull, K.E. Lassila and K.D. Pyatt, Phys. Rev. 120, 2227 (1960); G. Breit, Rev. Mod. Phys. 34, 776 (1962).
62. D.Y. Wong, Phys. Rev. Lett. 2, 406 (1959).
63. N.K. Glendenning and G. Kramer, Phys. Rev. 126, 2159 (1962).
64. J. Iwadare, S. Otsuki, R. Tamagaki and W. Watari, Progr. Theor. Phys. (Kyoto) 15, 86 (1956); 16, 455 (1956); for a summary see the contribution by these authors in Ref. 58, p.32.
65. T.E.O. Ericson and M. Rosa-Clot, Nucl. Phys. A405, 497 (1983).
66. M. Taketani, S. Machida and S. Onuma, Prog. Theor. Phys. (Kyoto) 7, 45 (1952).
67. K.A. Brueckner and K.M. Watson, Phys. Rev. 90, 699 (1953); 92, 1023 (1953).
68. K.A. Brueckner, M. Gell-Mann and M. Goldberger, Phys. Rev. 90, 476 (1953).
69. A. Klein, Prog. Theor. Phys. (Kyoto) 20, 357 (1958).
70. N. Hoshizaki and S. Machida, Prog. Theor. Phys. (Kyoto) 27, 288 (1962).
71. M.L. Goldberger, Proc. Midwestern Conf. on Theor. Phys., Purdue University, Lafayette, Ind. (USA), April 1960, pp.50-63.
72. H.A. Bethe, Scientific American, 189, 58 (1953).
73. M.J. Moravcsik, The Two-Nucleon Interaction (Clarendon Press, Oxford, 1963); for a more condensed review on the 1950's see also: R.J.N. Phillips, Repts. Progr. in Phys. 22, 562 (1959).
74. H.A. Bethe and F. de Hoffmann, Mesons and Fields, Vol. II, Mesons (Row, Peterson and Comp., Evanston, Ill., 1955).
75. Prog. Theor. Phys. (Kyoto), Suppl. 39 (1967).
76. R. Wilson, The Nucleon-Nucleon Interaction (Interscience, New York, 1963).
77. H.P. Stapp, T.J. Ypsilantis and N. Metropolis, Phys. Rev. 105, 302 (1957).
78. L. Eisenbud and W. Wigner, Proc. Nat. Acad. Wash. 27, 281 (1941); see also: S. Okubo and R.E. Marshak, Ann. of Phys. (N.Y.) 4, 166 (1958).

79. J.L. Gammel, R.S. Christian and R.M. Thaler, Phys. Rev. 105, 311 (1957).
80. J.L. Gammel and R.M. Thaler, Phys. Rev. 107, 291, 1337 (1957).
81. R. Jastrow, Phys. Rev. 81, 165 (1951).
82. M.G. Mayer, Phys. Rev. 75, 1969 (1949); 78, 16 (1950); O. Haxel, J.H.D. Jensen and H.E. Suess, Phys. Rev. 75, 1766 (1949).
83. P. Signell, R. Zinn and R. Marshak, Phys. Rev. Lett. 1 (1958) 416.
84. T. Hamada and I.D. Johnston, Nucl. Phys. 34, 382 (1962); for an improved version see: T. Hamada, Y. Nakamura and R. Tamagaki, Prog. Theor. Phys. 33, 769 (1965).
85. K.E. Lassila, M.H. Hull, H.M. Ruppel, F.A. McDonald and G.E. Breit, Phys. Rev. 126, 881 (1962).
86. R.V. Reid, Ann. of Phys. (N.Y.) 50, 411 (1968).
87. Y. Nambu, Phys. Rev. 106, 1366 (1957).
88. G. Breit, Proc. Natl. Acad. Sci. (U.S.), 46, 746 (1960); Phys. Rev. 120, 287 (1960).
89. J.J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960); Phys. Rev. 119, 1784 (1960).
90. W.R. Frazer and J.R. Fulco, Phys. Rev. 117, 1609 (1960).
91. B. Maglich, L. Alvarez, A. Rosenfeld and C. Stevenson, Phys. Rev. Lett. 7, 178 (1961).
92. See e.g. C. Alff et al., Phys. Rev. Lett. 9, 322 (1962); Refs. therein.
93. Particle Data Group, Phys. Lett. 50B, 74 (1974); find a complete list of earlier references from the 1960's therein.
94. Particle Data Group, Rev. Mod. Phys. 48, 5114 (1976).
95. R.A. Bryan, C.R. Dismukes and W. Ramsey, Nucl. Phys. 45, 353 (1963); R.A. Bryan and B.L. Scott, Phys. Rev. 135, B434 (1964).
96. A. Scotti and D.Y. Wong, Phys. Rev. Lett. 10, 142 (1963); Phys. Rev. 138, 145 (1965).
97. N. Hoshizaki, I. Lin and S. Machida, Prog. Theor. Phys. 27, 288 (1961).
98. S. Sawada, T. Ueda, W. Watari and M. Yonezawa, Prog. Theor. Phys. 28, 991 (1962).
99. R.S. McKean, Jr., Phys. Rev. 125, 1399 (1962).
100. A.E.S. Green and R.D. Sharma, Phys. Rev. Lett. 14, 380 (1965); A.E.S. Green and T. Sawada, Nucl. Phys. B2, 267 (1967).
101. M.M. Nagels, T.A. Rijken and J.J. de Swart, Phys. Rev. D17, 768 (1978).
102. R. de Tourreil, B. Rouben and D.W.L. Sprung, Nucl. Phys. A242, 445 (1975).
103. E.E. Salpeter and H.A. Bethe, Phys. Rev. 84, 1232 (1951).
104. K. Erkelenz, Phys. Reports 13C, 191 (1974).
105. K. Holinde and R. Machleidt, Nucl. Phys. A247, 495 (1975).

106. K. Holinde and R. Machleidt, Nucl. Phys. A256, 479 (1976).
107. J. Fleischer and J.A. Tjon, Nucl. Phys. B84, 375 (1975); Phys. Rev. D15, 2537 (1977); D21, 87 (1980); M.J. Zuilhof and J.A. Tjon, Phys. Rev. C22, 2369 (1980).
108. M.J. Moravcsik, Rep. Prog. Phys. 35, 587 (1972).
109. Rev. Mod. Phys. 39, 495 (1967); for the status of meson theory in that year see especially p.594.
110. S. Mandelstam, Rep. on Prog. in Phys. 25 (1962).
111. G.F. Chew, S-Matrix Theory of Strong Interactions (Benjamin, New York, 1961).
112. D. Amati, E. Leader and B. Vitale, Nuovo Cim. 17, 68; 18, 409, 458 (1960).
113. J. Binstock, Phys. Rev. D3, 1139 (1971); J. Binstock and R.A. Bryan, Phys. Rev. D4, 1341 (1971).
114. M. Chemtob, J.W. Durso and D.O. Riska, Nucl. Phys. B38, 141 (1972).
115. R. Vinh Mau et al., Phys. Lett. 44B, 1 (1973).
116. A.D. Jackson, D.O. Riska and B. Verwest, Nucl. Phys. A249, 397 (1975).
117. R. Woloshyn and A.D. Jackson, Nucl. Phys. A185, 131 (1972).
118. M. Lacombe et al., Phys. Rev. D12, 1495 (1975).
119. M. Lacombe et al., Phys. Rev. C21, 861 (1980).
120. R. Vinh Mau, in Mesons in Nuclei, Vol. I., eds. M. Rho and D. Wilkinson (North-Holland, Amsterdam, 1979), p.151.
121. G.E. Brown and A.D. Jackson, The Nucleon-Nucleon Interaction (North-Holland, Amsterdam, 1976).
122. M.H. Partovi and E.L. Lomon, Phys. Rev. D2, 1999 (1970).
123. R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).
124. F. Partovi and E.L. Lomon, Phys. Rev. D5, 1192 (1972); E.L. Lomon, Phys. Rev. D14, 2402 (1976); D22, 229 (1980).
125. H. Sugawara and F. von Hippel, Phys. Rev. 172, 1764 (1968).
126. K. Holinde and R. Machleidt, Nucl. Phys. A280, 429 (1977).
127. K. Holinde, R. Machleidt, M.R. Anastasio, A. Faessler and H. Müther, Phys. Rev. C18, 870 (1978).
128. K. Holinde, R. Machleidt, M.R. Anastasio, A. Faessler and H. Müther, Phys. Rev. C19, 948 (1979).
129. K. Holinde, R. Machleidt, A. Faesler, H. Müther, and M.R. Anastasio, Phys. Rev. C24, 1159 (1981); X. Bagnoud, K. Holinde and R. Machleidt, Phys. Rev. C24, 1143 (1981).
130. K. Holinde and R. Machleidt, Nucl. Phys. A372, 349 (1981).
131. X. Bagnoud, K. Holinde and R. Machleidt, Phys. Rev. C29, 1792 (1984).
132. R. Machleidt, in: Quarks and Nuclear Structure, ed. K. Bleuler, Lecture Notes in Physics (Springer Verlag, Heidelberg, 1984), Vol. 197, p.352.
133. R. Machleidt, K. Holinde and C. Elster, to be published.
134. P. Signell, Adv. Nucl. Phys. 2, 223 (1969).

135. E. Segrè, Nuclei and Particles (W.A. Benjamin, London, 1977).
136. E. Wigner, Phys. Rev. 43, 252 (1933).
137. R.A. Arndt et al., Phys. Rev. D28, 97 (1983).
138. A. Messiah, Quantum Mechanics (North-Holland, Amsterdam, 1966), Vol. I and II.
139. J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics, (McGraw-Hill, New York, 1964).
140. J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields (McGraw-Hill, New York, 1965).
141. F. Iachello, A.D. Jackson and A. Lande, Phys. Lett. 43B, 191 (1973); and earlier references therein.
142. E.M. Henley and G.A. Miller, in Mesons in Nuclei, eds. M. Rho and D.H. Wilkinson (North Holland, Amsterdam, 1979), p.406.
143. O. Dumbrajs et al., Nucl. Phys. B216, 277 (1983).
144. D. Lurié, Particles and Fields (Interscience, New York, 1968).
145. S.S. Schweber, An Introduction to Relativistic Quantum Field Theory (Row-Peterson, New York, 1961), pp.415-435.
146. D. Schütte, Nucl. Phys. A221, 450 (1974).
147. M.J. Zuilhof and J.A. Tjon, Phys. Rev. C24, 736 (1981).
148. S. Weinberg, Phys. Rev. Lett. 18, 188 (1967).
149. S.J. Brodsky, in Quarks and Nuclear Forces, eds. D. Fries and B. Zeitnitz, Springer Tracts in Modern Physics, Vol. 100, p.34.
150. Particle Data Group, Rev. Mod. Phys. 56, S1 (1984).
151. F.E. CLose, An Introduction to Quarks and Partons (Academic Press, London, 1979).
152. S. Godfrey and N. Isgur, Phys. Rev. D32, 189 (1985).
153. J.W. Durso, A.D. Jackson and B.J. Verwest, Nucl. Phys. A345, 471 (1980).
154. Q. Ho-Kim and D. Turcot, Phys. Rev. C22, 1352 (1980).
155. J.W. Durso, M. Saarela, G.E. Brown and A.D. Jackson, Nucl. Phys. A278, 445 (1977).
156. R. Vinh Mau et al., Phys. Lett. 44B, 1 (1973).
157. R. Dubois et al., Nucl. Phys. A377, 554 (1982).
158. R. Koch and E. Pietarinen, Nucl. Phys. A336, 331 (1980).
159. P. Kroll, Physics Data, 22, 1 (1981).
160. E. Pietarinen, Helsinki University, HU-/TFT-17-17.
161. W. Grein, Nucl. Phys. B131, 255 (1977).
162. W. Grein and P. Kroll, Nucl. Phys. A338, 332 (1980).
163. G.E. Brown and W. Weise, Phys. Reports 22C, 281 (1975).
164. R.M. Woloshyn and A.D. Jackson, Nucl. Phys. B64, 269 (1973).
165. R.H. Thompson, Phys. Rev. D1, 1738 (1970).
166. D. Schütte, Nucl. Phys. A221, 450 (1974).
167. K. Kotthoff, R. Machleidt and D. Schütte, Nucl. Phys. A264, 484 (1976).

168. R. Machleidt and K. Holinde, Phys. Lett. 152B, 295 (1985).
169. J.W. Negele, Phys. Rev. C1, 1260 (1970).
170. X. Campi and D.W.L. Sprung, Nucl. Phys. A194, 401 (1972).
171. C.J. Horowitz and B.D. Serot, Nucl. Phys. A368, 503 (1981).
172. D. Youngblood et al., Phys. Rev. Lett. 39, 1188 (1977).
173. B.D. Day, Rev. Mod. Phys. 39, 719 (1967).
174. H.A. Bethe, Ann. Rev. Nucl. Sci. 21, 93 (1971).
175. B.D. Day, Rev. Mod. Phys. 50, 495 (1978).
176. J.W. Negele, Rev. Mod. Phys. 54, 913 (1982).
177. J. Jeukenne, A. Lejeune and C. Mahaux, Nucl. Phys. A245, 411 (1975).
178. R. Machleidt and K. Holinde, Nucl. Phys. A350, 396 (1980).
179. B.D. Day, Phys. Rev. C24, 1203 (1981).
180. B.D. Day, Phys. Rev. Lett. 47, 226 (1981).
181. F. Coester, S. Cohen, B.D. Day and C.M. Vincent, Phys. Rev. C1, 769 (1970).
182. H.A. Bethe and M.B. Johnson, Nucl. Phys. A230 1 (1974).
183. K. Holinde, Phys. Reports 68, 121 (1981), Table 1 column 1 and 2.
184. R.A. Bryan and A. Gerstan, Phys. Rev. D6, 341 (1972).
185. R. de Tourreil and D.W.L. Sprung, Nucl. Phys. A201, 193 (1973).
186. Ref. 178: Δ1 is the result obtained in this work with the continuous choice for the single particle potential; for Δ2 the mesonic effects have been ignored.
187. K. Holinde and R. Machleidt, Nucl. Phys. A280, 429 (1977); for the result quoted in Fig. 7.2 a continuous choice is used.
188. M.R. Anastasio et al., Phys. Rev. C18, 2416 (1978); the result quoted in Fig. 7.2 refers to model MDFPΔ1 applied with a continuous choice and without mesonic effects.
189. M.R. Anastasio et al., Phys. Reports 100, 327 (1983).
190. B.D. Serot and J.D. Walecka, Advances in Nuclear Physics, eds. J.W. Negele and E. Vogt (Plenum Press, New York, 1985) Vol. 16, to be published.
191. C.J. Horowitz and B.D. Serot, Phys. Lett. 137B, 287 (1984).
192. R. Brockmann and R. Machleidt, Phys. Lett. 149B, 283 (1984).
193. R. Machleidt and R. Brockman, Phys. Lett. to be published.
194. R. Brockmann and R. Machleidt, to be published.
195. C. Elster, private communication.
196. M. Haftel and F. Tabakin, Nucl. Phys. A158, 1 (1970).
197. M.I. Haftel, Ph.D. Thesis, University of Pittsburgh (1969).