Phys. 726: Homework I

due September 16, 2009

1. *(3 pts)*
Prove that

$$\sigma \cdot \mathbf{a} \, \sigma \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{b} + i\sigma \cdot (\mathbf{a} \times \mathbf{b})$$

2. *(5 pts)*
Suppose the neutron-proton force in the deuteron (ground-state) can be described by the potential

$$V = V_1(r) + V_2(r) \, \mathbf{s}_1 \cdot \mathbf{s}_2$$

where $\mathbf{s}_1$ and $\mathbf{s}_2$ are the spin operators of the proton and neutron, and the potentials V_1 and V_2 depend only on the neutron-proton separation r. What are the good quantum numbers of the deuteron?

If the additional term (called tensor force)

$$V' = V_3(r) \left(\frac{3(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})}{r^2} - \mathbf{s}_1 \cdot \mathbf{s}_2 \right) =: V_3(r) S_{12}$$

is added to the above potential V, what are now the good quantum numbers?

3. *(6 pts)*
Show that any positive integral power of each of these operators

$$(\mathbf{s}_1 \cdot \mathbf{s}_2) \quad \text{and} \quad S_{12} = \frac{3(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})}{r^2} - \mathbf{s}_1 \cdot \mathbf{s}_2$$

and also any product of these powers can be represented in the form of a linear combination of these operators and a unit matrix.