**Phys. 726: Homework II**

due September 30, 2009

**1.** *[6 pts]*
Show that the general solution of the free, time-independent Schrödinger equation

$$H_0|\Psi\rangle = E|\Psi\rangle \tag{1}$$

with $E = \hbar^2 k^2/(2m) > 0$ can be written as

$$\Psi_k(\mathbf{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm} j_l(kr) Y_{lm}(\theta, \varphi) \tag{2}$$

Determine the coefficients $C_{lm}$.

**2.** *[8 pts]*
Inelastic scattering and reactions can be included into a single channel scattering formalism by viewing this process as 'absorbing' particles from the incident beam, where the absorption is described by a complex potential

$$V(r) = U(r) + iW(r). \tag{3}$$

1. Derive the continuity equation for a time-dependent Schroödinger equation with the potential of Eq. (3).
2. Show that this leads to the relation

$$\frac{\partial}{\partial t} \int d^3r |\psi|^2 = 2 \int d^3r W(r) |\psi|^2 - \int r^2 d\Omega \cdot \frac{\psi^\star \nabla \psi - \psi \nabla \psi^\star}{2\mu i}. \tag{4}$$

3. Show that W(r) must be negative to be a 'sink' rather than a 'source' of flux.
4. Show that the optical theorem is valid when $\sigma_t$ includes the non-elastic events.