Phys. 726: Homework III

due October 7, 2009

1. Cross Sections and Phase Shift Analysis

An experiment measures the differential cross section for the elastic scattering of two particles with wave vector k in the center of momentum to have the form

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{1}{k^2} e^{-2(1-\cos\theta)}. \tag{1}$$

1. *[1 pt]* Plot the differential cross section as function of the scattering angle θ for all allowed values of θ.

2. *[2 pts]* Without any detailed calculation, deduce the number of partial waves which contribute to the scattering and indicate if this is compatible with scattering from a finite range potential.

3. *[2 pts]* What must be the modulus of the angle-dependent scattering amplitude, $|f_E(\theta)|$?
   Remark: A complex number $z = x + iy = Re^{i\alpha}$ has modulus $R = \sqrt{x^2 + y^2}$ and phase α.

   Next, the experimentalist measures the total cross section for the same particles and finds it to have the form

   $$\sigma_{tot} = \frac{4\pi}{k^2}. \tag{2}$$

4. *[2 pts]* What is the value of the scattering amplitude in forward direction, $f_E(0^o)$?

5. *[2 pts]* Assuming that the scattering amplitude has a constant phase, what is $f_E(\theta)$?

6. *[2 pts]* What is the total elastic (integrated elastic) cross section for this reaction? Comment on why this is the same or different from the total cross section.

7. *[2 pts]* Why must the phase shift $\delta_l(k)$ be complex for this reaction?

8. *[2 pts]* Find the $l = 0$ phase shift for this interaction.

2. Scattering from a square well

Two particles of mass m scatter. The potential between them is approximated by an attractive square well:

$$V(r) = \begin{cases} -V_0 & r < b \\ 0 & r > b \end{cases} \tag{3}$$

1. *[3 pts]* Solve for $k \cot \delta_0$, where δ_0 is the S wave phase shift.
2. *[3 pts]* Show explicitly that the condition for the scattering amplitude for this partial wave,
$$f_0 = \frac{e^{i\delta_0(k)} \sin \delta_0(k)}{k}, \tag{4}$$
to have a pole on the positive imaginary k axis, is also the condition for this potential to produce a bound state.
3. *[2 pts]* Based on the above expression for f_0, verify that the scattering amplitude is an analytic function of the energy $E = k^2/2\mu$ with a branch cut from 0 to ∞, and a bound state pole on the negative axis.

Hint: The problems are taken from R.Landau's chapter 3