

Phys 726: Homework IV

due October 14, 2009

1. Scattering by a Yukawa Potential

A particle of mass m is scattered by a potential

$$V(\vec{r}) = V_0 \exp(-r/a)$$

- (a) (5 p) Find the differential cross section in the first Born approximation. Sketch the angular dependence for small and large k , where k is the wave number of the particle being scattered. At what k values does the scattering begin to be significantly non-isotropic? Compare this value with the one given by elementary arguments based on angular momentum.
- (b) (6 p) The criterion for the validity of the Born approximation is

$$|\Delta\Psi^{(1)}(0)/\Psi^{(0)}(0)| \ll 1,$$

where $\Delta\Psi^{(1)}$ is the first order correction to the incident plane wave $\Psi^{(0)}$. Evaluate this criterion explicitly for the present potential. What is the low- k limit of your result? Relate this to the strength of the attractive potential required for the existence of bound states. Is the high- k limit of the criterion less or more restrictive on the strength of the potential?

- (c) (5 p) If you put the Coulomb potential $V^C = -Ze^2/r$ in the expression for the Born approximation for the scattering amplitude, you obtain a non-existing expression (which one?). One can apply the mathematical trick of solving for a screened Coulomb potential of the form

$$V^R(r) = -\frac{Ze^2}{r} e^{-r/R} \quad (1)$$

and then consider the limit $R \rightarrow \infty$.

Derive the scattering amplitude for the screened Coulomb potential and consider the limit $R \rightarrow \infty$ to obtain the Coulomb Born amplitude and from it the classical Rutherford cross section

$$\frac{d\sigma^{born}}{d\Omega} = \frac{Z^2 e^4 m^2}{4p^2 \sin^4 \frac{\theta}{2}} \quad (2)$$

2. Resonance for a Square Well (6 p)

Use the exact solution for the spherical square well of depth V_0 to find the condition on the potential for the s-wave ($l = 0$) to produce a resonance at an energy $E_0 = (\hbar^2 k_0^2)/2m$. Can you relate your answer to bound state energies in the well?