Phys 735: Homework II

due January 16, 2009

1. *[4 pts]*
The time evolution of a free state $|\varphi(t)\rangle$ is given by the Schrödinger equation

$$\frac{d}{dt}|\varphi(t)\rangle = -\frac{i}{\hbar}H_0|\varphi(t)\rangle$$

with the general solution

$$|\varphi(t)\rangle = e^{-\frac{i}{\hbar}H_0\ t}|\varphi(0)\rangle.$$

Here $H_0 = \mathbf{P}^2/2m$ is the free Hamiltonian. Consider a state

$$\varphi(\vec{x},t) \equiv \langle\vec{x}|\varphi(t)\rangle$$

and show that

$$\varphi(\vec{x},t) = \left(\frac{m}{2\pi i\hbar t}\right)^{\frac{3}{2}} \int d^3x' e^{\frac{i}{\hbar}\frac{m}{2t}(\vec{x}-\vec{x}')^2}\varphi(\vec{x},0)$$

2. *[8 pts]*
Inelastic scattering and reactions can be included into a single channel scattering formalism by viewing this process as 'absorbing' particles from the incident beam, where the absorption is described by a complex potential

$$V(r) = U(r) + iW(r). \tag{1}$$

1. Derive the continuity equation for a time-dependent Schroödinger equation with the potential of Eq. (1).
2. Show that this leads to the relation
$$\frac{\partial}{\partial t}\int d^3r|\psi|^2 = 2\int d^3rW(r)|\psi|^2 - \int r^2 d\Omega \cdot \frac{\psi^\star\nabla\psi - \psi\nabla\psi^\star}{2\mu i}. \tag{2}$$
3. Show that W(r) must be negative to be a 'sink' rather than a 'source' of flux.
4. Show that the optical theorem is valid when σ_t includes the non-elastic events.