Phys 735: Homework III

due January 23, 2009

1. Scattering from a Hard Sphere

A potential describing a hard sphere is given by

$$V(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases} \tag{1}$$

(a) *(2 pts)* Derive an expression for the phase shifts δ_l as function of the momentum.

(b) *(4 p)* Find the total cross-section for an incoming energy

$$E = \frac{\hbar^2 k^2}{2m}$$

in the two limits

- $k \to 0$
- $k \to \infty$

Compare your results for both cases with the classical cross section for scattering from a hard sphere, i.e. the result you derived in classical mechanics.

Hint: For $k \to \infty$ use the asymptotic forms of j_l and n_l to obtain a simple form for $\sin^2 \delta_l$. Furthermore, replace the sum over l by an integral, so that

$$\sigma = \sum_{l=0}^{l=ka} \sigma_l \approx \frac{4\pi}{k^2} \int_0^{ka} (2l+1) \sin^2 \delta_l \, dl$$

2. Cross Sections and Phase Shift Analysis

An experiment measures the differential cross section for the elastic scattering of two particles with wave vector k in the center of momentum to have the form

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{1}{k^2} e^{-2(1-\cos\theta)}. \tag{2}$$

1. *[1 pt]* Plot the differential cross section as function of the scattering angle θ for all allowed values of θ.

2. *[2 pts]* Without any detailed calculation, deduce the number of partial waves which contribute to the scattering and indicate if this is compatible with scattering from a finite range potential.

3. *[2 pts]* What must be the modulus of the angle-dependent scattering amplitude, $|f_E(\theta)|$?
   Remark: A complex number $z = x + iy = Re^{i\alpha}$ has modulus $R = \sqrt{x^2 + y^2}$ and phase α.

   Next, the experimentalist measures the total cross section for the same particles and finds it to have the form

$$\sigma_{tot} = \frac{4\pi}{k^2}. \tag{3}$$

4. *[2 pts]* What is the value of the scattering amplitude in forward direction, $f_E(0^o)$?

5. *[2 pts]* Assuming that the scattering amplitude has a constant phase, what is $f_E(\theta)$?

6. *[2 pts]* What is the total elastic (integrated elastic) cross section for this reaction? Comment on why this is the same or different from the total cross section.

7. *[2 pts]* Why must the phase shift $\delta_l(k)$ be complex for this reaction?

8. *[2 pts]* Find the $l = 0$ phase shift for this interaction.