Phys 735: Homework V

due February 6, 2009

1. Scattering by a Yukawa Potential
A particle of mass m is scattered by a potential

$$V(\vec{r}) = V_0 \exp(-r/a)$$

(a) *(5 p)* Find the differential cross section in the first Born approximation. Sketch the angular dependence for small and large k, where k is the wave number of the particle being scattered. At what k values does the scattering begin to be significantly non-isotropic? Compare this value with the one given by elementary arguments based on angular momentum.

(b) *(6 p)* The criterion for the validity of the Born approximation is

$$|\Delta\Psi^{(1)}(0)/\Psi^{(0)}(0)| \ll 1,$$

where $\Delta\Psi^{(1)}$ is the first order correction to the incident plane wave $\Psi^{(0)}$. Evaluate this criterion explicitly for the present potential. What is the low-k limit of your result? Relate this to the strength of the attractive potential required for the existence of bound states. Is the high-k limit of the criterion less or more restrictive on the strength of the potential?

(c) *(5 p)* If you put the Coulomb potential $V^C = -Ze^2/r$ in the expression for the Born approximation for the scattering amplitude, you obtain a non-existing expression (which one?). One can apply the mathematical trick of solving for a screened Coulomb potential of the form

$$V^R(r) = -\frac{Ze^2}{r} e^{-r/R} \tag{1}$$

and then consider the limit $R \longrightarrow \infty$.
Derive the scattering amplitude for the screened Coulomb potential and consider the limit $R \longrightarrow \infty$ to obtain the Coulomb Born amplitude and from it the classical Rutherford cross section

$$\frac{d\sigma^{born}}{d\Omega} = \frac{Z^2 e^4 m^2}{4p^2 \sin^4 \frac{\theta}{2}} \tag{2}$$