Phys 735: Homework VI

1. Scattering by a Separable Potential

Consider the separable potential

$$V(\mathbf{k}', \mathbf{k}) = \lambda g(k') g(k). \tag{1}$$

The function $g(k)$ is often called *form factor*. This type of potential was introduced by Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954) as special type of nucleon-nucleon potential, for which the scattering problem can be solved exactly. The potential of Eq. (1) is usually referred to as Yamaguchi potential. It's relevance today is that arbitrary functions can be expressed in a separable basis.

(a) *(4 p)* Verify that the off-energy-shell t-matrix for this potential has the form

$$\langle \mathbf{k}' | t(E) | \mathbf{k} \rangle = \frac{\lambda g(k') g(k)}{1 - \lambda J(E)}, \tag{2}$$

where $E = k_0^2 / 2m$ is the energy. Give the explicit form of $J(E)$.

(b) *(3 p)* Relate the value of the zero momentum form factor $g(k = 0)$ to the scattering length.

(c) *(3 p)* Express the bound-state condition for this potential in terms of the integral $J(E)$.

(d) *(3 p)* Deduce the number of bound states that this potential supports.

(e) *(3 p)* Express the condition for this potential to produce a Breit-Wigner resonance at energy $E_r = k_r^2 / 2m$.

(f) *(3 p)* Show that the width of this resonance is

$$\Gamma = B g^2(k_r), \tag{3}$$

where B is a constant that you should determine.