Phys 735: Homework VII

due February 20, 2009

1. Galilei Invariance of the free Schrödinger Equation *(8 pts)*

Derive the transformation law for the wave function $\Psi(\vec{x}, t)$ of a free particle under a Galilei transformation

$$\vec{x}' = \vec{x} + \vec{v}t + \vec{a}, \quad t' = t + b, \tag{1}$$

from the constraint that the Schrödinger equation has the same form in both systems of inertia

$$\Psi = \Psi(\vec{x}, t) \quad : \quad i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi \tag{2}$$

$$\Psi' = \Psi'(\vec{x}', t') \quad : \quad i\hbar \frac{\partial \Psi'}{\partial t'} = -\frac{\hbar^2}{2m} \Delta' \Psi' \tag{3}$$

How does the probability current

$$\vec{j} = \frac{\hbar}{2mi} (\Psi^* \vec{\Delta} \Psi - (\vec{\Delta} \Psi^*) \Psi) \tag{4}$$

transform?

Hints: There should be the following relation $\Psi'(\vec{x}', t) = e^{if(\vec{x}, t)} \Psi(\vec{x}, t)$ between the wave functions Ψ and Ψ' where f is a real function (why?). Assume that f is independent from Ψ and that eh derivatives exist and derive an equation for f from the above relation between the two wave functions and by using the Schrödinger equation.