Phys 735: Homework VIII

due February 27, 2009

1. Lorentz Transformations
Lorentz transformations are a set of maps £: $M^4 \to M^4$: so that $\forall L \in$ £, there is $g = LgL^T$, where M^4 is Minkovski space with following metric g

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

These maps £ form a group.

(a) *(4 p)* Prove that in M^4, the boost along the x direction L_x can be written in terms of the exponentiation of generator L_1:

$$L_x = exp(-R_x L_1) \tag{1}$$

where $R_x = \tanh(v_x/c)$ and L_1 is the extension of σ_1 written as

$$L_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

where σ_1 is the Pauli matrix

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Hint: Remember how to set up a matrix algebra for calculating e^A, with A being a matrix

(b) *(3 p)* By the example of 3a, find the generators L_2 and L_3 which are also natural extension of σ_1, which can generate the boost along y and z direction respectively via exponentiation.

(c) *(3 p)* Verify that the commutator $[L_1, L_2]$ can generate a rotation along a certain direction and in a certain plane via exponentiation. Find the direction of rotation and the plane.

2. *(5 p)* Show the Lorentz invariance of the Klein-Gordon equation.