The Lens Equation

While ray diagrams can be used to determine the image location, size, orientation and type of the image formed by objects when placed at a given location in front of a lens, they will not provide numerical information about the image distance and size. To obtain this type of information it is necessary to use the lens equation and the magnification equation.

The lens equation expresses the quantitative relationship between the object distance (d_o), the image distance (d_i), and the focal length (f).

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The magnification equation relates the ratio of the image distance and object distance to the image height h_i and object height h_o

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

These two equations can be combined to yield information about the image distance and image height if the object distance, object height, and focal length are known.

The sign conventions used for the lens equation are:

+/- Sign conventions:

- Focal length f is + if the lens is a convex lens (converging lens)
- Focal length f is – if the lens is a concave lens (diverging lens)
- Image distance d_i is + if the image is real and located on the opposite side of the lens
- Image distance d_i is – if the image is virtual and located on the object's side of the lens
- Image height h_i is + if the image is upright (and thus also virtual)
- Image height h_i is – if the image is inverted (and thus also real)

Example Problem 1:

A 4.0 cm tall light bulb is placed at a distance of 45.7 cm from a convex lens having a focal length of 15.2 cm. Determine the image distance and image size of the light bulb.

1. Identify the known quantities in the problem

   $h_o = 4.0\ cm \qquad d_o = 45.7\ cm \qquad f = 15.2\ cm$

2. The unknown quantities you wish to solve for are

   $d_i = ? \qquad h_i = ?$

3. The object is located farther away from the lens than 2f = 30.4 cm. Thus, from *ray tracing* you expect that the image is located on the other side of the lens between f and 2f and will be inverted and smaller.

4. To determine the image distance, the lens equation must be used. The following lines represent the solution to the image distance; substitutions, algebraic steps and units are shown.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{15.2\ cm} = \frac{1}{45.7\ cm} + \frac{1}{d_i}$$

$$0.0658\ cm^{-1} = 0.0219\ cm^{-1} + \frac{1}{d_i}$$

$$0.0439\ cm^{-1} = \frac{1}{d_i}$$

$$d_i = 22.79\ cm$$

   Note that in the intermediate steps the fractions were not rounded, only in the final answer.

5. To determine the image height, the magnification equation needs to be used. Since three of the four quantities in the equation are known (disregard M), the fourth quantity can be calculated:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4.0\ cm} = -\frac{22.79\ cm}{45.7\ cm}$$

$$h_i = -4.0\ cm \cdot \frac{22.79\ cm}{45.7\ cm}$$

$$h_i = -\ 1.99\ cm$$

6. The negative value for the image height indicates that the image is inverted. In addition, the image is smaller than the object.

   From the calculations in this example it can be concluded that if a 4.00-cm tall object is placed 45.7 cm from a double convex lens having a focal length of 15.2 cm, then the image will be inverted, 1.99-cm tall and located 22.8 cm from the lens. The results of this calculation agree with the principles of ray tracing.

Example Problem 2:

A 4.00 cm tall light bulb is placed a distance of 8.30 cm from a double convex lens having a focal length of 15.2 cm. Note that this is the same object and the same lens as in Example 1, however, the object is placed closer to the lens. Determine the image distance and the image size.

1. Identify the known quantities in the problem
   $h_o = 4.0\ cm \qquad d_o = 8.3\ cm \quad f = 15.2\ cm$

2. The unknown quantities you wish to solve for are

   $d_i = ? \qquad h_i = ?$

3. The object is located between f and the surface of the lens. Thus, from ray tracing it is expected that the image will be located on the same side as the object and will be upright and enlarged.

4. To determine the image distance, the lens equation will have to be used. The following gives the solution and shows substitutions, algebraic steps and units in all lines.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{15.2\ cm} = \frac{1}{8.3\ cm} + \frac{1}{d_i}$$

$$0.0658\ cm^{-1} = 0.120\ cm^{-1} + \frac{1}{d_i}$$

$$-\,0.0547\ cm^{-1} = \frac{1}{d_i}$$

$$d_i = -\,18.28\ cm$$

The negative value for image distance indicates that the image is a virtual image located on the object's side of the lens.

5. To determine the image height, the magnification equation needs to be used. Since three of the four quantities in the equation are known (disregard M), the fourth quantity can be calculated:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4.0\ cm} = -\frac{-18.28\ cm}{8.3\ cm}$$

$$h_i = -4.0\ cm \cdot \frac{-18.28\ cm}{8.3\ cm}$$

$$h_i = 8.81\ cm$$

6. The calculation indicates that image will be enlarged, upright, 8.81 cm tall and located 18.3 cm from the lens on the object's side.

Example Problem 3:

This problem will use a diverging lens.

A 4.00 cm tall light bulb is placed a distance of 35.5 cm from a diverging lens having a focal length of -12.2 cm. Determine the image distance and the image size.

1. Identify the known quantities in the problem

   $h_o = 4.0\ cm \qquad d_o = 35.5\ cm \quad f\ =\ -12.2\ cm$

2. The unknown quantities you wish to solve for are

   $d_i = ? \qquad h_i = ?$

3. From ray tracing one can conclude that diverging lenses always produce images located on the object's side of the lens, and that the image is upright and virtual.

4. To determine the image distance, the lens equation must be used. The following gives the solution and shows substitutions, algebraic steps and units in all lines.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{-12.2\ cm} = \frac{1}{35.5\ cm} + \frac{1}{d_i}$$

$$-0.0820\ cm^{-1} = 0.0282\ cm^{-1} + \frac{1}{d_i}$$

$$-0.110\ cm^{-1} = \frac{1}{d_i}$$

$$d_i = -\ 9.09\ cm$$

Note that in the intermediate steps the fractions were not rounded, only in the final answer. The negative values for image distance indicate that the image is located on the object's side of the lens.

5. To determine the image height, the magnification equation needs to be used. Since three of the four quantities in the equation are known (disregard M), the fourth quantity can be calculated:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4.0\ cm} = -\frac{-9.09\ cm}{35.5\ cm}$$

$$h_i = -4.0\ cm \cdot \frac{-9.09\ cm}{35.5\ cm}$$

$$h_i = 1.02\text{cm}$$

The image will be upright and smaller than the object.

6. From the calculations it can be concluded that if a 4.00 cm tall object is placed 35.5 cm from a diverging lens having a focal length of -12.2 cm, then the image will be upright, 1.02 cm tall and located 9.08 cm from the lens on the object's side.

Test your understanding:

1. Determine the image distance and image height for a 5 cm tall object placed 30 cm from a convex lens having a focal length of 15 cm.

2. A magnified, inverted image is located at a distance of 32 cm from a convex lens with a focal length of 12.0 cm. Determine the object distance and tell whether the image is real or virtual.

3. A concave lens has a focal length of -10.8 cm. An object is placed 32.7 cm from the lens' surface. Determine the image distance.