The Mirror Equation – Concave Mirrors

While ray diagrams can be used to determine the image location, size, orientation and type of the image formed by objects when placed at a given location in front of a concave mirror, they will not provide numerical information about the image distance and size. To obtain this type of information it is necessary to use the mirror equation and the magnification equation.

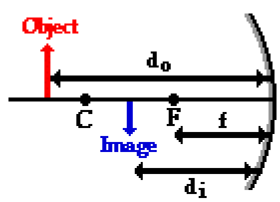
The *mirror equation* expresses the relationship between the object distance d_o, the image distance d_i and the focal length f

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The magnification equation relates the ratio of the image distance and object distance to the image height h_i and object height h_o

$$\mathrm{M} = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

These two equations can be combined to yield information about the image distance and image height if the object distance, object height, and focal length are known.



Example Problem 1:

A 4.0 cm tall light bulb is placed at a distance of 45.7 cm from a concave mirror having a focal length of 15.2 cm. Determine the image distance and image size of the light bulb.

1. Identify the known quantities in the problem
   $h_o = 4.0\text{cm}$ $d_o = 45.7\text{cm}$ $\text{f} = 15.2\text{ cm}$

2. The unknown quantities you wish to solve for are
   $d_i = ?$ $h_i = ?$

3. The center of the mirror is given by $C = 2f$. Thus, the object is located beyond the center of the mirror. According to ray diagrams you expect that the image is located between C and f, is smaller and inverted.

4. To determine the image distance, the mirror equation must be used. The following gives the solution and shows substitutions, algebraic steps and units in all lines are shown

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{15.2\ cm} = \frac{1}{45.7\ cm} + \frac{1}{d_i}$$

$$0.0658\ cm^{-1} = 0.0219\ cm^{-1} + \frac{1}{d_i}$$

$$0.0439\ cm^{-1} = \frac{1}{d_i}$$

$$d_i = 22.79\ \mathrm{cm}$$

Note that in the intermediate steps the fractions were not rounded, only in the final answer.

5. To determine the image height, the magnification equation needs to be used. Since three of the four quantities in the equation are known (disregard M), the fourth quantity can be calculated:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4.0\ cm} = -\frac{22.79\ cm}{45.7\ cm}$$

$$h_i = -4.0\ \mathrm{cm} \cdot \frac{22.79\ cm}{45.7\ cm}$$

$$h_i = -\ 1.99\ \mathrm{cm}$$

6. The negative value for the image height indicates that the image is inverted. In addition, the image is smaller than the object.

Example Problem 2

A 4.0 cm tall light bulb is placed at a distance of 8.3 cm from a concave mirror having a focal length of 15.2 cm. Determine the image distance and image size of the light bulb. Note that this is the same mirror as in Problem 1, however, the object is placed closer to the mirror.

1. Identify the known quantities in the problem
   $h_o = 4.0\ cm$ $d_o = 8.3\ cm$ $f\ =\ 15.2\ cm$

2. The unknown quantities you wish to solve for are
   $d_i = ?$ $h_i = ?$

3. The object is located between f and the surface of the mirror. According to ray diagrams you expect that the image is upright, virtual and enlarged.

4. To determine the image distance, the mirror equation must be used. The following gives the solution and shows substitutions, algebraic steps and units in all lines

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{15.2cm} = \frac{1}{8.3\ cm} + \frac{1}{d_i}$$

$$0.0658\ cm^{-1} = 0.120\ cm^{-1} + \frac{1}{d_i}$$

$$-\ 0.0547\ cm^{-1} = \frac{1}{d_i}$$

$$d_i = -\ 18.28\ \mathrm{cm}$$

   The image distance is negative, indicating that the images is located *behind* the mirror.

5. To determine the image height, the magnification equation needs to be used. Since three of the four quantities in the equation are known (disregard M), the fourth quantity can be calculated:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4.0\ cm} = -\frac{-18.28\ cm}{8.3\ cm}$$

$$h_i = -4.0\ \text{cm} \cdot \frac{-18.28\ cm}{8.3\ cm}$$

$$h_i = 8.81\ cm$$

6. The image height is positive, indicating that the image is upright. In addition, the image is larger than the object.

+/- Sign conventions:

- Focal length f is + if the mirror is a concave mirror
- Image distance d_i is + if the image is real and located on the object's side of the mirror
- Image distance d_i is – if the image is virtual and located behind the mirror
- Image height h_i is + if the image is upright
- Image height h_i is – if the image is inverted.

Test your understanding:

1. Determine the image distance and image height for a 5.0 cm tall object placed 30.0 cm from a concave mirror having a focal length of 15.0 cm.

2. Determine the image distance and image height for a 5.0 cm tall object placed 10 cm from a concave mirror having a focal length of 15 cm.