The Mirror Equation – Convex Mirrors

While ray diagrams can be used to determine the image location, size, orientation and type of the image formed by objects when placed at a given location in front of a concave mirror, they will not provide numerical information about the image distance and size. To obtain this type of information it is necessary to use the mirror equation and the magnification equation.

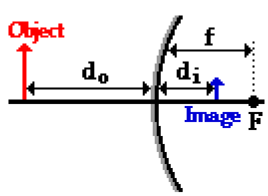
The *mirror equation* expresses the relationship between the object distance d_o, the image distance d_i and the focal length f

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The magnification equation relates the ratio of the image distance and object distance to the image height h_i and object height h_o

$$\mathrm{M} = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

These two equations can be combined to yield information about the image distance and image height if the object distance, object height, and focal length are known.



Note that for a convex mirror the focal point is behind the mirror and thus negative.

Example Problem 1:

A 4.0 cm tall light bulb is placed at a distance of 35.5 cm from a concave mirror having a focal length of -12.2 cm. Determine the image distance and image size of the light bulb.

1. Identify the known quantities in the problem
   $h_o = 4.0$ cm $\quad d_o = 45.7$cm $\quad$ f = -12.2 cm

2. The unknown quantities you wish to solve for are
   $d_i = ?\quad h_i = ?$

3. To determine the image distance, the mirror equation must be used. The following gives the solution and shows substitutions, algebraic steps and units in all lines

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{-12.2\ cm} = \frac{1}{35.5\ cm} + \frac{1}{d_i}$$

$$-0.0820\ cm^{-1} = 0.0282\ cm^{-1} + \frac{1}{d_i}$$

$$-0.110\ cm^{-1} = \frac{1}{d_i}$$

$$d_i = -9.07\ \text{cm}$$

Note that in the intermediate steps the fractions were not rounded, only in the final answer. The negative value for the distance indicates that the image is located behind the mirror.

4. To determine the image height, the magnification equation needs to be used. Since three of the four quantities in the equation are known (disregard M), the fourth quantity can be calculated:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4.0cm} = -\frac{-9.08cm}{35.5cm}$$

$$h_i = -4.0\text{cm} \cdot \frac{-9.08cm}{35.5cm}$$

$$h_i = 1.02\text{cm}$$

5. The image will be upright and smaller than the object.

+/- Sign conventions:

- Focal length f is – if the mirror is a convex mirror
- Image distance d_i is + if the image is real and located on the object's side of the mirror
- Image distance d_i is – if the image is virtual and located behind the mirror
- Image height h_i is + if the image is upright
- Image height h_i is – if the image is inverted.

Test your understanding:

1. A convex mirror has a focal length of -10.8 cm. An object is placed 32.7 cm from the mirror's surface. Determine the image distance.

2. Determine the focal length of a convex mirror that produces an image that is 16.0 cm behind the mirror, when the object is 28.5 cm from the mirror.