PHYS 371: Electrons Laboratory
Laboratory Manual

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Contents

1 Laboratory general directions

1.1 General conduct in the laboratory

These directions are to be considered a guide to your laboratory work rather than a recipe to be followed blindly.

- *Come prepared*: Before the lab session, prepare by reading this manual and doing analysis in between sessions. Bring the necessary supplies and submit your report promptly. Also, if you ask the instructor for help between sessions bring your notes as it is much easier to talk about your data then to talk in general terms of the experiment.

- *Be safe*: Be careful with the equipment as you take data in order to protect yourself and the equipment. Especially, you will be using equipment which poses some hazards such as high-voltage power supplies. Listen to safety instructions and heed them. Also if a piece of equipment is not working even after you have followed all instructions, be careful what you fiddle with! Some fiddling is good, but if you are planning anything major (like taking a piece of equipment apart), it is EXTREMELY IMPORTANT to ask the instructor first. Also it is good lab practice not to eat or drink in the lab: you might damage the equipment and/or eat or drink something you don't intend to!

- *Understand the purpose of the experiment*: Think of the purpose of the experiment so that you know why you are doing each step. A scientist or engineer knows what is to be measured before the experiments begins (i.e., Are you measuring a constant, verifying a law?). In addition, professional scientists and engineers are paid for their work and consequently do not do things without a good reason. Do not depend upon your partner to do your thinking for you. Plan your laboratory activities for the greatest economy of time and effort. Often a few minutes of preliminary manipulation of the apparatus will save an hour of repeating the experiment due to taking worthless data hurriedly. Laboratory work is a joint venture. Carry your share of the responsibility. Don't leave the lab without making sure you understand your data and have recorded sufficient information to allow the data to be analyzed later. If you are unsure of a procedure, ask a laboratory assistant.

- *Record data completely and honestly*: Record what you observe, not what you hope it should be (or what someone else says it is). Keep a good laboratory book and record your data and the steps you take! Write down data in a neat and organized way. Don't think it is a trivial task. Feel free to ask your instructor for advices on how to accomplish this. The goal is to be able to read and understand what you did after you leave the laboratory. Do not be tempted to falsify data (in this course or later in your career). Sometimes there may be pressure to do this. Those who succumb to the pressure are usually found out and the ramifications are very serious. These incidents are damaging to society and the credibility of the scientific professions. The careers of the scientists responsible are typically severely damaged by these incidents.

- *Cross-check your data*: There are a variety of honest mistakes that an experimentalist can make in recording or in analyzing data. One possible mistake is to assume that an observed signal is due to the sample when it is actually due to the instrumentation. Another mistake is to reach the wrong conclusion in analyzing data. It may be difficult to avoid this pitfall. Perhaps the best advice is to urge caution, especially if you have some doubts. Publishing a mistake and then publishing a retraction is embarrassing and may damage one's career. For this lab, if the final result has intolerably large errors (more than 25% unless otherwise indicated), repeat the experiment after finding out what is wrong. If you find that the error cannon be reduced, you are expected to explain why this is the case. You are not being graded on the "correctness" of your data.

- *Investigate*: Look out for refinements in technique and equipment which indicate a carefully and well-performed experiment with the least error possible. For example, in an experiment with pulleys and a force table, one refinement is to make sure that all pulleys are the same size. Feel free to innovate and improve the apparatus and procedure if you can. Record these changes in your report. Also, when doing an experiment, it is important for the experimentalist not to accept the data as correct without adequately questioning it. Always be curious.

1.2 Guidelines to prepare yourself to perform an experiment: Run plan

The following questions are typically answered before hand by someone well prepared to perform an experiment. In this class, you are expected to answer these questions during the first session dedicated to an experiment. After discussing with your lab partner, write down your plan and turn in your answers to the instructor before leaving class.

- What are the goals of this experiment?

- What is the method used to achieve these goals? Write down a list of steps detailing the sequence of your work.

- What quantities will I measure? You may want to prepare tables that you will fill during the data taking.

1.3 Reporting experimental work

In addition to planning and thinking about the experiment to be done and optimizing the equipment and technique as indicated, there are several other important aspects of experimental work. These are: (1) organizing data and results into graphs and tables to aid in interpreting the experiment, (2) error analysis, and (3) writing a report to help in communicating the results to other scientists. The error analysis treatment will be explained in its own chapter later in this manual. A very good guide for how to write a report and organize your data was produced by the Pomona College of California and can be found at:

http://www.physics.pomona.edu/sixideas/labs/LRM/LR08.pdf

or on the web site of the class. The grading system for the lab reports is given in Table 1. Various aspects of report writing are discussed below.

1.3.1 Sections in the scientific report

The following sections should be included in the technical report. It is good to look at textbooks to see how equations, graphs, figures, tables and statements are handled.

- *Abstract:* The abstract should be a very short and concise description of what is in the report. Its purpose is to help others who may be looking up the literature on a particular subject. They should be able to read the abstract in a matter of seconds and know if this report will be of any help to them. Only when they decide that it will be of use will they read through the actual report. This cuts down on the time spent searching the literature. The abstract should contain a short statement of: (1) the purpose of the experiment, (2) the methods used, (3) the results, and (4) the conclusions reached as to verification of theory and the causes of any significant errors. For this class, the abstract should not be longer than five complete sentences.

- *Introduction:* The introduction section is meant to provide the reader with the answers to two very important questions: "What is the question that your experiment is supposed to answer?" and "Why is answering this question interesting (and/or important)?". This section often begins with a brief summary of current knowledge. It continues with a statement of a problem that this knowledge raises, a brief description of the experiment presented in your paper, and how it addresses the question asked. Detailed descriptions are not appropriate in this section. The point is to provide a concise picture of your purposes and a broad survey of your approach.

- *Theory:* This section should contain a complete exposition of the theory, including the physical phenomena and equations behind the experiment as found in various textbooks of physics. It should include derivations (or references derivations) of the formulas used in the experiment from from underlying principles. Figures may be included. Be sure to include proper citations to this lab manual or other sources. Remember that all variables and symbols must be defined and used consistently; do not use the same variable or symbol for more than one quantity! Equations should be numbered sequentially and reference where appropriate. Use your textbooks for examples of how to properly include equations grammatically into your text. For the purposes of this course, I would encourage you not to put an excessive amount of effort into derivations and figures for the theory section. I would prefer to see most of your efforts directed towards the exposition of your own work in the other sections.

- *Experimental apparatus and procedure:* This section should include a description of the equipment and how it was used. A schematic line drawing of the equipment and/or any critical components should be included. Finally, this section should include a complete description of the data that were taken as well as the rational for taking them. The data should be organized into tables if appropriate. The quantity and

units must be given, with sufficient and reasonable significant figures. This section should also include estimates of any expected systematic errors and random errors.

- *Analysis:* This section should include at least three sub-sections: (1) Method of analysis, (2) Presentation of the results, and (3) Discussion of the results. The various calculated values should be included here in clear tabular form, as appropriate. Graphs and tables should have titles and should help you to see trends and relations. Use your and other textbooks as models for your tables and graphs. Table columns and the axes of graphs must give the quantities and their units. Proper handling of significant figures is important. The slope and intercept of a graph often have important physical significance and should be explained. The discussion should include an interpretation of the graphs and tables. The level of agreement between the experiment and theory should also be discussed. It may also be appropriate to discuss problems with the experiment or possible improvements here.

- *Conclusions:* The conclusions should contain a summary of the outcome(s) of the experiment: the experimental results (with errors) and their agreement (or not) with theory. The conclusions reached must be explained and justified. Remarks about possible further research or other issues can also be included. The conclusion is important and should show considerable thought about the experiment. Note that in the working world, many people will only read the introduction and conclusion of your report. There is often some redundancy between between the discussion subsection of the analysis and the conclusions section. I suggest that you limit this redundancy by only including the *most important* conclusions and issues in this section.

- *Bibliography:* The bibliography must contain a list of all works cited in the report. It is important that sufficient information be provided so that somebody else can find the reference. Your format should be consistent; a good reference for formatting is:

  `http://authors.aps.org/STYLE/ms.html#footnotes`

  which gives the format used by the Physical Review family of journals (the most widely-used physics journals in the United States). An example of an item in a bibliography is:
  F.V. Hunt, *Electro-acoustics* (John Wiley, New York, 1954), p. 124.

An outline of your report could for example look like this:

ABSTRACT

1. INTRODUCTION

    A. Motivation (that is what are the goals of your experiment)

    B. Summary of the experiment you performed

2. THEORETICAL BACKGROUND

3. EXPERIMENTAL PROCEDURE

   A. Description of apparatus including the operating ranges of the devices

   B. Description of experimental procedure

   C. Description in plain English of the observed phenomena

   D. Description of sources of errors separating the systematic and random error

4. ANALYSIS[1]

   A. Method of analysis including precision estimation

   B. Presentation of results

   C. Discussion of results

   D. (optional) Suggestions for future improvements

5. CONCLUSIONS

   A. Summary of results

   B. Pertinence of results to the questions raised in the introduction

1.3.2 General comments on style

Your report is expected be of high quality. The report must be original, well organized, complete and precise. It must be correct with respect to grammar and spelling. After writing the report, read over it, watch for lack of precision and for ambiguity. Each sentence should present a clear message. Here are a few tips:

- With regard to precision, you should qualify what is written so that there is no question as to what was done or concluded. For example, instead of saying, "The temperature dependence of the surface tension of water was measured", you should be more precise and complete and say, "The temperature dependence of the surface tension of water was measured in the range 20 C to 60 C". Avoid using only adjectives as they are subject to interpretation and always try to make quantitative statements.

- The data and results should be well organized in tables and the graphs should be neat with legends on each axis explaining the quantity plotted and its units, a title and a caption. Figure 1 is an example of a graph with a proper caption and legend. Tables, figures and equations should be numbered for easy access (for example see Table 2, Figure 4, Equation 39). All tables and figures must be referenced and explained in the report. Tables and figures may be interspersed throughout the text, or included at the very end of the report.

[1] If your experiment had multiple goals or parts you might want to create multiple analysis sections.

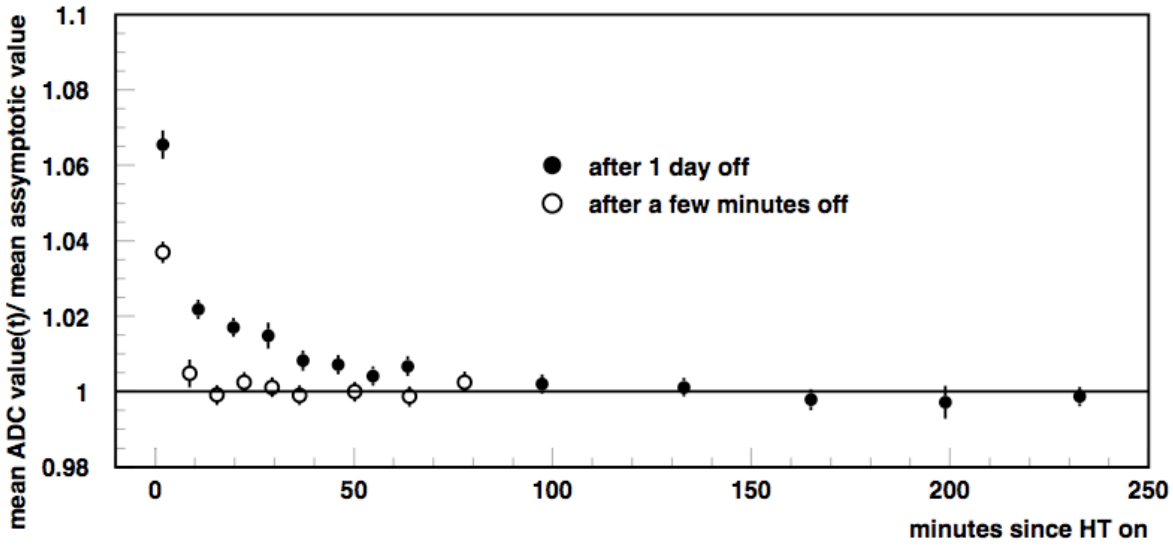

Figure 6: *Warming up of the FL PMT. Using a radioactive source, the position of the mean peak in ADC spectrum was measured as a function of time. Data were taken after the tube had been off for one day (solid circles) and for only a few minutes (open circles).*

Figure 1: *Example of the proper way to present a figure in a report. Observe the legend on each axis, the use of different symbols to disentangle the data and the numbering of this figure. Note how the caption points spells out what the figure is about.*

- Avoid using the first person, "I" or "we" in writing. Instead of writing "We weighed the frogs and put them in a glass jar", write "The frogs were weighed and put in a glass jar". Be consistent in the use of the tense throughout the paper - do not switch between past and present. Using past tense is standard. Be sure that pronouns refer to antecedents. For example, in the statement "Sometimes cecropia caterpillars are in the cherry trees but they are hard to find," does "they are hard to find" refer to the caterpillars or the trees?

- Use metric system of measurements. Abbreviation of units are used without a following period (for example 2 mm or 6 g). Number should be written as numerals when they are greater than ten or when they are associated with measurements. For example: 2 mm or 6 g but two explanations or six factors. Spell out numbers that begin sentences.

Additional References

D. Preston and E. Dietz, *The Art of Experimental Physics* (Wiley, New York, 1991).
Physical Review Style and Notation Guide (`http://authors.aps.org/STYLE/`). This is a good source of grammar, punctuation, abbreviation, etc... information.
The undergraduate lab manual for the Ohio University 250 series.

Student: **Lab:** (0: not acceptable, → 3: correct)

	your pts	weight	total
Structure of the report (A)		**1.**	
Summarize the entire paper in abstract		0 1 2 3	
State the problem and summarize procedure in introduction		0 1 2 3	
Introduce all relevant variables in the theory section		0 1 2 3	
Show considerable after-though about the experiment in the conclusion		0 1 2 3	
Presentation (B)		**1.**	
Quote results with proper precision, significant figures, and units		0 1 2 3	
Display relevant equations properly		0 1 2 3	
Give appropriate legends and headings to tables, graphs and sketches		0 1 2 3	
Use logical transition words and sentences to tie together the report		0 1 2 3	
Procedure (C)		**1.**	
Provide a sketch and a description of the experimental setup		0 1 2 3	
Report the precision and operating range of the apparatus		0 1 2 3	
Completely discuss all precautions taken to make a good measurement		0 1 2 3	
Describe all measurements as well as the rational to take them		0 1 2 3	
Describe in plain English the phenomena you are observing*		0 1 2 3	
Analysis (D)		**3.**	
Report the data on proper tabular form		0 1 2 3	
Show sample calculation(s)		0 1 2 3	
Recognize and quantify the main sources of uncertainty**		0 1 2 3	
Show details of your precision determination		0 1 2 3	
Present well-chosen graphs of the data that includes error bars		0 1 2 3	
Completeness (E)		**2.**	
Fulfill all of the experimental objectives of the lab		0 1 2 3	
Fulfill all of the theoretical objectives of the lab		0 1 2 3	
Thoroughness of data taking		0 1 2 3	
Final grade: A+B+C+D+E			**/100**

*This should be at least 5 sentences long. **List separately random and systematic errors.

Comments:

Table 1: *The grading system for the PHYS 371 the lab reports. Note that the sum of points is actually 102, so there are two "bonus points" (in some sense).*

2 Statistics and treatment of experimental data

2.1 Physical concepts

There is no such thing as a perfect experiment. Each measurement contains some degree of uncertainty due to the limits of instruments and the people using them. In physics, these of uncertainties are called errors (think of the error as the difference between the measured value and the true underlying value). We will consider two distinct error concepts: accuracy and precision. Precision refers to how close together a group of measurements actually are to each other. It is often called random error or statistical error. The accuracy of the measurement refers to how close the measured value is to the true or accepted value, after averaging over many measurements so that the error due to precision is negligible. The accuracy is often called the systematic error. Precision has nothing to do with the true or accepted value of a measurement, so it is quite possible to be very precise and totally inaccurate. In many cases, when precision is high and accuracy is low, the fault can lie with the instrument.[2]

2.2 Reading and homework assignments

The following book is the required reading for this class[3].
P.R. Bevington and D.K. Robinson, "Data reduction and error analysis for the physical sciences," ISBN 0-07-247227-8.

- For the second class: Read Chap 1, work out exercises 1.3 and 1.5.
- For the third class: Read Chap 2, work out exercises: 2.8, 2.13.
- For the fourth class: Read Chap 3, work out exercises: 3.5, 3.6.
- For the fifth class: Read Chap 4, work out exercises: 4.5 and 4.12 (skip t-distribution question).
- For the sixth class: Read Chap 6, work out exercise given in class.

2.2.1 Important formulas

Straight average: This method applies when one performs a set of N measurements of a given quantity x_i with equal precision $\Delta x_i = \Delta x$ for each measurement. For this sample, the mean value is $\bar{x}$, the standard deviation of the sample is σ_x and should be equal to Δx, and the precision on the mean is $\Delta\bar{x}$:

$$\bar{x} = \frac{1}{N}\sum x_i \qquad \sigma_x = \sqrt{\frac{1}{N-1}\sum (x_i - \bar{x})^2} \qquad \Delta\bar{x} = \frac{\sigma_x}{\sqrt{N}}. \tag{1}$$

[2]This section is based on a document of the Fordham Preparatory School.

[3]Another excellent reference is: .R. Taylor, "An introduction to error analysis, The study of uncertainties in physical measurements," ISBN 0-935702-75

Weighted average: This method applies when one performs a set of N measurements of a given quantity x_i with different precisions Δx_i for each measurement. For this sample, the mean value is $\bar{x}$, and the precision on the mean is $\Delta\bar{x}$:

$$\bar{x} = \frac{\sum(x_i/\Delta x_i^2)}{\sum(1/\Delta x_i^2)} \qquad \Delta\bar{x} = \frac{1}{\sqrt{\sum(1/\Delta x_i^2)}}. \tag{2}$$

χ^2 minimization: A numerical comparison between the observed distribution of measurements x_i with precisions Δx_i and a function y can be defined via χ^2:

$$\chi^2 = \sum \frac{(x_i - y)^2}{\Delta x_i^2} \tag{3}$$

The number of degrees of freedom ν is defined to be the number of data points minus the number of free parameters in the function y. The "best fit" is determined by minimizing χ^2. The minimum value of χ^2 should be approximately equal to ν. The reduced χ^2 is defined to be $\chi^2_\nu = \chi^2/\nu$.

Linear fit: Suppose a set of measurements of x_i, y_i and Δy_i, from which one to extract a linear function of the form $y(x) = ax + b$. One way to extract the parameters a and b is to minimize the χ^2 between the data and the function. The solution for the least-squares fit of a straight line is:

$$a = \frac{EB - CA}{DB - A^2} \qquad b = \frac{DC - EA}{DB - A^2} \tag{4}$$

$$(\Delta a)^2 = \frac{B}{DB - A^2} \qquad (\Delta b)^2 = \frac{D}{DB - A^2} \tag{5}$$

where

$$A = \sum \frac{x_i}{(\Delta y_i)^2} \qquad B = \sum \frac{1}{(\Delta y_i)^2} \tag{6}$$

$$C = \sum \frac{y_i}{(\Delta y_i)^2} \qquad D = \sum \frac{x_i^2}{(\Delta y_i)^2} \tag{7}$$

$$E = \sum \frac{x_i y_i}{(\Delta y_i)^2} \qquad F = \sum \frac{y_i^2}{(\Delta y_i)^2} \quad \text{(not used)}. \tag{8}$$

Note that in many real life experiments, the measured quantities are $x_i \pm \Delta x_i$ and $y_i \pm \Delta y_i$. How to deal with errors on both x and y is a non-trivial and sometimes controversial issue. For the purpose of this class and in this case, if on supposes $y_i = f(x_i)$, one should translate Δx_i into a Δy_i error by replacing Δy_i with

$$\sqrt{(\Delta y_i)^2 + (\frac{\partial f}{\partial x_i}\Delta x_i)^2}. \tag{9}$$

2.3 Tutorial

This exercise is extracted from "An introduction to error analysis: The study of uncertainties in physical measurements," 2nd edition, J.R. Taylor, University Science Book.

Systematic errors sometime arise when an experimenter unwittingly measures the wrong quantity. Here is an example.

Gravity can be measured using a pendulum made of a steel ball suspended by a light string. Gravity g can then be extracted using the following formula:

$$g = 4\pi^2 l/T^2 \tag{10}$$

where l is the length of the pendulum and T is its period. A student tries to measure g using this method. He records six different lengths of the pendulum l and the corresponding periods T. He estimates that his precision on the measurement of the length is 0.1 cm and on the periods is 0.0005 s. His data are presented in table 2.

Length l (cm)	51.2	59.7	68.2	75.0	79.7	88.3
Period T (s)	1.4485	1.5655	1.669	2.098	1.8045	1.896

Table 2: *Measurements of the length and the period of the pendulum*

1. For each pair (l_i, T_i), he calculates g_i. He then calculates the mean g_m of those values, their standard deviation σ_m and their standard deviation of mean Δg_m, assuming all his errors are random. What is his answer for $g_m \pm \Delta g_m$? He now compares his answer with the accepted value $g = 979.6$ cm/s^2 and he is horrified to realize that his discrepancy is nearly 10 times larger than his uncertainty. Confirm this sad conclusion.

2. Studying his results, the student notices that the values g_i he extracted from his results seem to increase as the period T increases which might be the sign of systematic bias. He decides to check if his observation is correct.

   First he computes the precision of each of his measurements g_i, i.e.,

   $$(\Delta g)^2 = \left(\frac{\partial g}{\partial l}\Delta l\right)^2 + \left(\frac{\partial g}{\partial T}\Delta T\right)^2 \tag{11}$$

   Using his set of g_i and Δg_i values, he recomputes $g_w \pm \Delta g_w$ using the weighted average method. How does g_m and g_w compare? Why are they different? Which method should be used to compute the average value of g? Explain your answer.

   Then he plots $g_i \pm \Delta g_i$ as a function of T^2 and draws the line corresponding to g_w on his plots. Reproduce his plot. Finally, he computes the χ^2 of g_w compared to his data and concludes that indeed there is bias in his data. What is the χ^2, the reduced χ^2 and the probability of his data being independent of T^2? Why would you say, like the student did, that there is a bias in his data?

3. Having checked his calculations and found them to be correct, the student concludes that he must have overlooked some systematic errors. He is sure there was no problem with the measurement of the period T and that the problem must be in his measurement of l. So he asks himself: How large would a systematic error in the length l have to be so that his result would be the accepted value 979.6 cm/s^2? Show that the answer is approximately 1.5%.

4. In order to make a maximum usage of his data, the student decides to extract the best value of his systematic error on the length l by performing a linear fit of his data. First he rewrites Equation 10 to take into account his supposed systematic error l_0 such that he measured l when he should have measured $l - l_0$:

$$T^2 g + 4\pi^2 l_0 = 4\pi^2 l \tag{12}$$

   Then he plots $4\pi^2 l$ versus T^2 such that the slope of straight line is g and the offset is $4\pi^2 l_0$. Reproduce this plot, don't forget the error bars. Using the linear fit method, extract $g_l \pm \Delta g_l$, as well as $l_0 \pm \Delta l_0$. How does the reduced χ^2 of this fit compared to the case where l_0 was ignored? What does it mean?

5. This result would mean that the student's length measurement suffered a systematic error of about a centimeter – a conclusion he first rejects as absurd. As he stares at the pendulum, he realizes that 1 cm is about the radius of the steel ball and that the lengths he recorded were the lengths *of the string.* Because the correct length of the pendulum is the distance from the pivot to the *center* of the ball, his measurements were indeed systematically off by the radius of the ball (see Figure 2). He, therefore, uses a caliper to find the ball's diameter, which turns out to be 2.00±0.01 cm. Make the necessary corrections to his data and compute his final result for g_f with its uncertainties. Don't forget to take into account the precision on the ball radius. Justify your choice to use the straight average method or the weighted average method. What about the χ^2 of the corrected values of g?

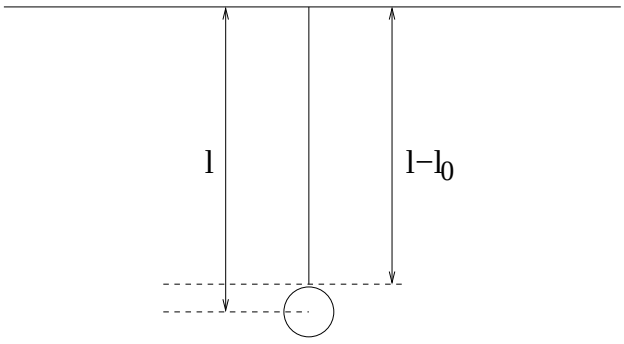

Figure 2: *A pendulum consists of a metal ball suspended by a string. The effective length of the pendulum (l) is the length of the string (l-l_0) plus the radius of the ball.*

3 The Hall Effect

The Hall Effect was discovered by E. Hall in 1879 while working on his doctoral thesis. Hall's experiments consisted of exposing thin gold leaf (and, later, using various other materials) on a glass plate and tapping off the gold leaf at points down its length. The effect is a potential difference (Hall voltage) on opposite sides of a thin sheet of conducting or semiconducting material (the Hall element) through which an electric current is flowing. This was created by a magnetic field applied perpendicular to the Hall element. The ratio of the voltage created to the amount of current is known as the Hall Resistance and is a characteristic of the material in the element. In 1880, Hall's experimentation was published as a doctoral thesis in the American Journal of Science and in the Philosophical Magazine. One very important feature of the Hall Effect is that it differentiates between positive charges moving in one direction and negative charges moving in the opposite. The Hall Effect offered the first real proof that electric currents in metals are carried by moving electrons, not by protons. The Hall Effect also showed that in some substances (especially semiconductors), it is more appropriate to think of the current as positive "holes" moving rather than negative electrons.

3.1 Physical concept

The Hall Effect comes about due to the nature of the current flow in the conductor. Current consists of many small charge-carrying "particles" (typically electrons) which experience a force (called the Lorentz Force) when in the presence of a magnetic field (see figure 3) . When a perpendicular magnetic field is absent, there is no Lorentz Force and the charge follows an approximate 'line of sight' path. When a perpendicular magnetic field is present, the path is curved perpendicular to the magnetic field due to the Lorentz Force. The result is an asymmetric distribution of charge density across the Hall element perpendicular to the 'line of sight' path the electrons would take in the absence of the magnetic field. As a result, an electric potential is generated between the two ends.

For a metal or for a n- or p-type semiconductor[4] at normal temperature, electric conduction is due to a single carrier type with charge q and mobility μ. If a magnetic field $\vec{B}$ is applied along the axis $\hat{z}$ as on Figure 4, then the carrier of charge q experiences a Lorentz Force :

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{13}$$

If the definition of carrier mobility is expanded to include magnetic forces, the drift velocity is given by :

$$\vec{v} = \pm\mu(\vec{E} + \vec{v} \times \vec{B}) \tag{14}$$

where the plus and minus sign is used for holes and the minus sign is used for electrons. For this configuration of fields, the drift velocity components are related by

$$v_x = \pm\mu(E_x + v_y B_z) \quad v_y = \pm\mu(E_y - v_z B_x) \quad v_z = \pm\mu E_z \tag{15}$$

[4]You may want to refresh your knowledge on semiconductors

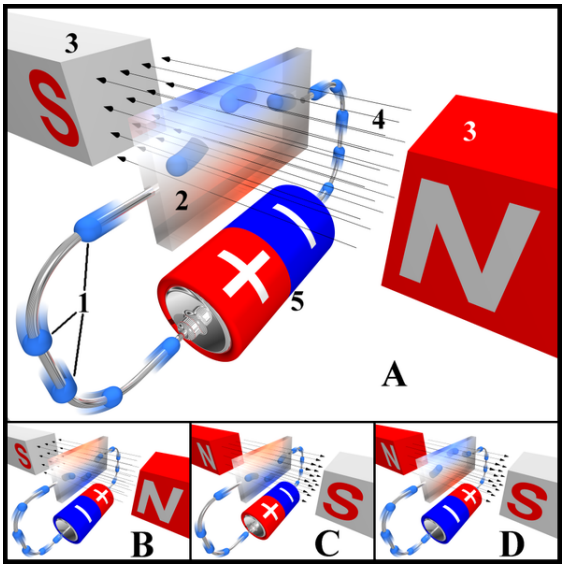

Figure 3: *Hall effect diagram, showing electron flow (rather than conventional current). Legend:1. Electrons (not conventional current!), 2. Hall element, or Hall sensor, 3. Magnets, 4. Magnetic field, 5. Power source. In drawing "A", the Hall element takes on a negative charge at the top edge (symbolised by the blue color) and positive at the lower edge (red color). In "B" and "C", either the electric current or the magnetic field is reversed, causing the polarization to reverse. Reversing both current and magnetic field (drawing "D") causes the Hall element to again assume a negative charge at the upper edge. This figure is taken from Wikipedia.*

The components of the current density $\vec{J} = nq\vec{v}$ are thus:

$$J_x = \pm\mu(nqE_x + J_yB_z) \quad J_y = \pm\mu(nqE_y - J_zB_x) \quad J_z = \pm\mu nqE_z \tag{16}$$

If the current is supplied by a DC source of current attached to Faces 1 and 2 of the sample, then in the steady state we must have $J_y = J_z = 0$. This implies that there is a component of $\vec{E}$ transverse to the current flow:

$$E_y = \frac{J_xB_z}{nq} \tag{17}$$

This is accounted for physically by the buildup of a static charge distribution on Faces 3 and 4 which produces an electric field along the y axis that compensates for the forces of $\vec{B}$ on the carriers. The Hall constant R_H is defined by

$$R_H = \frac{E_y}{J_xB_z} = \frac{1}{nq} \tag{18}$$

We can restate this definition in terms of experimentally measured quantities by referring to Figure 4 and noting that the Hall voltage $V_H = V_3 - V_4$ is expressed in term of E_y as

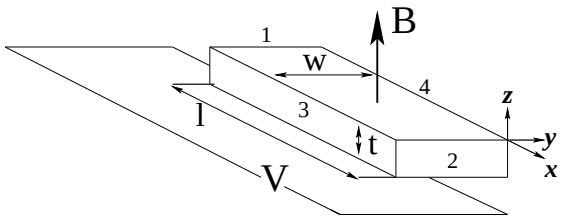

Figure 4: *A Hall sample is a conducting sample through which a current I is flowing. For the Hall voltage to appear, the sample needs to be in region where a magnetic field* **B** *exists. The current I flows from face 1 to face 2. The resulting Hall Voltage appears between face 3 and 4, that is perpendicularly to both the current I and the magnetic field* **B**. *This figure is taken from Preston.*

$V_H = wE_y$, and that $J_x = I/wt$. Thus, the experimental Hall Coefficient normalized by the thickness t of the sample is expressed in terms of measurable quantities as

$$\frac{R_H}{t} = \frac{V_H}{IB_z} \quad (m^2/C) \tag{19}$$

Note that the carrier density in this case can be calculated directly from a measurement of R_H, and that the sign of R_H can be used to determine the sign of the charge carriers. This, of course, means that careful analysis of the signs of I, B_z, as well as V_H needs to be performed.

3.1.1 Thermo-magnetic effects

Because of several thermo-magnetic effects, the measured voltage $V_{34} = V_3 - V_4$ may not be the "true" Hall voltage V_H (see figure 4). Reference[5] gives a concise description of the Nernst Effect, the Righi-Leduc Effect, and the Ettingshausen Effect, along with suggestions for reducing their respective contributions to measured voltages. The Ettingshausen Effect cannot be separated from the Hall Effect in the present experiment but will be small in samples with good bulk thermal conduction. The Nernst and Righi-Leduc effects, along with any residual IR drop, can be compensated by averaging V_{34} over both directions of current and field. Thus the Hall voltage corresponding to a current I and a field B should be taken as an average of four separate measurements:

$$V_H = \frac{1}{4}\left(V_{34}(I,B) - V_{34}(I,-B) - V_{34}(-I,B) + V_{34}(-I,-B)\right) \tag{20}$$

3.1.2 To go further

- What is the quantum Hall Effect?

[5]O. Lindberg, Proc IRE 40, 1414 (1952): A brief discussion of the Hall effect in semiconductors and the thermo-magnetic effects that complicates measurements of the Hall voltages

3.2 Experimental setup and procedure suggestion

The Hall apparatus used in this lab is a product of the Leybold Cie. A copy of the operating manual for this device is available on the class web site, it describes the apparatus as well as suggest a procedure for the experiment. Figure 5 is extracted from this manual and describes roughly how the apparatus should be setup. The Silver foil you will be working with is 50 μm thick. Setting up the experiment has revealed it self as a little tricky, here are some typical results to consider:

- When the electromagnet is setup correctly, one measures a magnetic field of ~ 0.7 T for a current of ~ 5 A,

- When the whole apparatus is correctly setup, the Hall voltage for a Silver foil sunk into a ~ 0.7 T magnetic field and traversed by a current of 15 A is 16 μV.

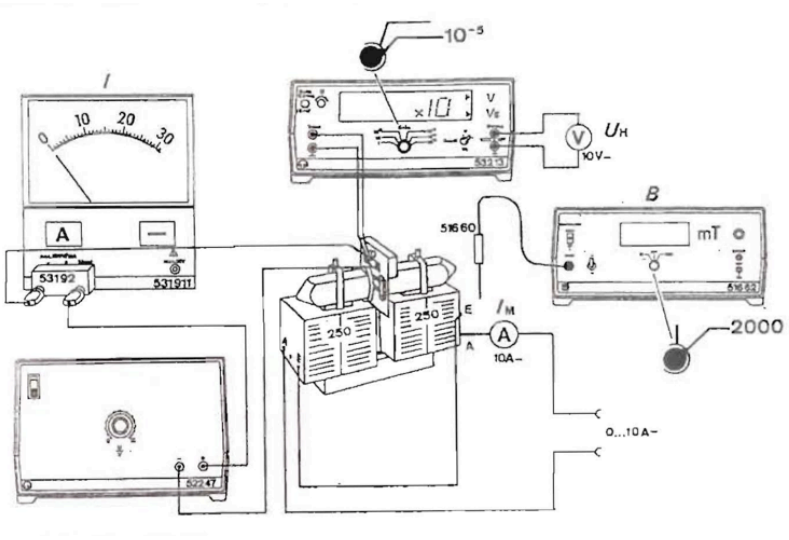

Figure 5: *Typical arrangement for the Hall apparatus.*

3.3 Lab objectives

- Verify the proportionality of the Hall voltage with the magnetic flux density and the current through the sample.

- Determine the Hal constant R_H and the charge carrier concentration of Silver and compare to literature. Do not forget to quote your results with their error bars. The literature value of R_H for Silver is 8.9 10^{-11} m^3/C, the concentration of charge carrier for Silver is 6.6 10^{28} m^{-3}.

- Determine the polarity of the charge carriers in Silver

3.4 Preliminary questions

The following questions should help you prepare for this lab:

- Describe at least 4 industrial applications of the Hall Effect.

- What are magnetic hysteresis and magnetic saturation for an electromagnet?

- Because of several thermo-magnetic effects, the voltage V_{34} measured on a semiconductor, may not be the "true" Hall voltage V_H (see section 3.1.1). For example $V_{34} = V_H + V_{bias}$. What are the name of the effects creating V_{bias}? For your data taking, how will you deal with this issue?

- Suppose you measure R_H/t for a fixed magnetic field B_0 and three different currents through the Hall probe. Suppose you know B_0 and I to 1% relative precision and each V_{34} to 1%. What is the final relative precision on R_H/t ? How much of this is random, and how much of this is systematic?

- The number density of free electrons in gold is 5.90 10^{28} electrons per cubic meter. If a metal strip of gold 2 cm wide carries a current of 10 A, how thin would it need to be to produce a Hall Voltage of at least one 1 mV. What would the drift velocity of the electrons be in this case? (Assume a perpendicular field of 5000 Gauss)

References

The discussion of the Hall Effect is taken up from "Wikipedia, The Free Encyclopedia" as well as from "The art of experimental physics", D. Preston, ISBN 0-471-84748-8.

4 The ratio Charge-to-Mass of the electron

J.J. Thomson first measured the charge-to-mass ratio of the fundamental particle of charge in a cathode ray tube in 1897. A cathode ray tube basically consists of two metallic plates in a tube which has been evacuated and filled with a very small amount of background gas. One plate is heated and "particles" boil off of the cathode and accelerate toward the other plate which hold a positive potential. The gas in between the two plates inelastically scatters the electrons, emitting light which shows the path of the particles. The charge-to-mass ratio of the particle can be measured by observing their motion in an applied magnetic field. Thomson repeated his measurement of e/m many time with different metals and also different gases. Having reached the same value of e/m every time, he concluded that a fundamental particle having a negative charge e and a mass 2000 times less than the lightest atoms existed in all atoms. Thomson named these particle "corpuscles," but we now know them electrons. In this lab, you will essentially repeat Thomson's experiment and measure e/m for electrons.

4.1 Physical concept

The mass-to-charge ratio is a physical quantity that is widely used in the electrodynamics of charged particles, e.g., in electron optics and ion optics. It appears in the scientific fields of lithography, electron microscopy, cathode ray tubes, accelerator physics, nuclear physics, auger spectroscopy, cosmology and mass spectrometry. The importance of the mass-to-charge ratio is that according to classical electrodynamics two particles with the same mass-to-charge ratio move in the same path in a vacuum when subjected to the same electric and magnetic fields.
The magnetic force F_m acting on a charged particle of charge q moving with velocity v in a magnetic field B is given by the equation

$$F_m = q\vec{v} \times \vec{B} \tag{21}$$

In the case where the initial velocity is perpendicular to the magnetic field, the equation can be written in scalar form as:

$$F_m = evB \tag{22}$$

Since the electrons are moving in a circle, they must be experiencing a centripetal force of magnitude

$$F_c = mv^2/r \tag{23}$$

where m is the mass of the electron, v is its velocity, and r is the radius of the circular motion. Since the only force acting on the electrons is that caused by the magnetic field, $F_m = F_c$. So Equations 22 and 23 can be combined to give

$$\frac{e}{m} = \frac{v}{Br} \tag{24}$$

Therefore, in order to determine e/m, it is only necessary to know the velocity of the electrons, the magnetic field and the radius of the circle described by the electrons. Suppose electrons are accelerated from a quasi-null speed through the accelerating potential

V, gaining kinetic energy equal to their charge times the accelerating potential. Therefore, $eV = 1/2mv^2$. The velocity of the electron is:

$$v = \sqrt{\frac{2eV}{m}} \tag{25}$$

Suppose also that the magnetic field in which the electron beam travels is produced near the axis of a pair of Helmholtz coils. The magnetic field B is, therefore:

$$B = \frac{N\mu_0 I}{(5/4)^{3/2}a} \tag{26}$$

where μ_0 is the magnetic permeability[6], I is the current through the Helmholtz coil, a is the radius of the Helmholtz coils, and N is the number of turns on each Helmholtz coil. In this case, the final formula for e/m is:

$$e/m = \frac{2V(5/4)^3 a^2}{(N\mu_0 I r)^2} \tag{27}$$

4.1.1 To go further

Mass spectroscopy is a widely used technique. How does it work ? How does it relate to the e/m experiment?

4.2 Experimental setup

A stream of electrons is accelerated by having them fall through a measured potential difference. This stream is projected into a uniform magnetic field, perpendicular to the velocity vector of the electrons that cause the electrons to bend into a circular path. An electron beam tube with Helmholtz coils will be provided. The circular path of the electrons will be used to measure the the ratio e/m. A copy of the operating manual for the CENCO apparatus you will be using is available on the class website.

4.3 Lab objectives

To measure the ratio of e/m for electrons and to compare it to the accepted world data.

4.4 Determining the radius of curvature

This lab requires that one determine the radius of the the electron trajectory from measurements of the trajectory in Cartesian coordinates. How does one do this? One approach is to choose three points from the trajectory, and use those points to determine the radius, since three points (in a 2-dimensional plane) uniquely determine a circle. I suggest utilizing two points from near the respective ends of the trajectory and one from near the middle (why?).

[6] $\mu_0 = 4\pi 10^{-7}\mathrm{Tm/A}$

The following formulas are from the Wolfram MathWorld website (given below). Assuming an arbitrary triangle with sides of length a, b, and c, one can define the the semiperimeter s:

$$s \equiv \frac{1}{2}(a+b+c). \tag{28}$$

The area A of the triangle is then given by Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)} \tag{29}$$

and the circumradius R is given by

$$R = \frac{abc}{4A}. \tag{30}$$

The circumradius is the radius of the circumscribed circle – in other words the radius of the circle which intersects the three vertices of the triangle. It is thus also the radius of our electron trajectory. Note that the lengths a, b, and c can be easily determined from the Cartesian coordinates using the Pythagorean Theorem.

4.5 Preliminary questions

The following questions should also help you to prepare for this lab:

- What are Helmholtz coils and what are they used for? Draw a diagram showing the magnetic field profile along the axis crossing the center of the coils for 3 different setups : 1) the coils are separated by a distance larger than their radius, 2) the coils are separated by a distance equal to their radius, 3) the coils are separated by a distance smaller than their radius.

- Suppose you perform 4 measurements of e/m by choosing one magnetic field and varying the accelerating voltage. Suppose you know the current through the coil, the accelerating voltage and the radius of the electron trajectory to 1%. What is the final relative precision of e/m? Which part of the error is systematic precision and which part is random?

- Assuming the Helmholtz coils have a radius of 10 cm, 130 turns and are powered with 2A. The accelerating voltage is typically 100 V. How does the magnetic field they create compare to Earth magnetic field?

- Classical mechanics was used to derive the equations used in this experiment. Evaluate the value of $\beta = v/c$ used in determining the charge-to-mass ratio of the electron for the setup described above. Based on these values of β, can relativistic corrections be neglected?

- Draw a diagram with the following features: a region of space containing a magnetic field pointing into the page, the path an electron would follow if injected into this space with a certain velocity initially pointing upward, and the path a proton would follow if injected in this same space with the same velocity.

References

- The SSS Theorem, from the Wolfram MathWorld website:
`http://mathworld.wolfram.com/SSSTheorem.html`

5 Electron diffraction

A primary tenet of quantum mechanics is the wave-like properties of matter. In 1924, graduate student Louis de Broglie suggested in his dissertation that since light has both particle-like **and** wave-like properties, perhaps all matter might also have wave-like properties. He postulated that the wavelength of object was given by $l = h/p$, where $p = mv$ is the momentum. This was quite a revolutionary idea, since there was no evidence at the time that matter behaved like waves. De Broglie received a Nobel prize in 1929 for his discovery of the wave nature of electron. In 1927, however, C. Davisson and L. Germer, and G. Thomson independently discovered experimental proof of the wave-like properties of matter -particularly electrons. Davisson and Thomson won the Nobel Prize in 1937 for their experimental discovery of the diffraction of electrons by crystal. Not only was this discovery important for the foundation of quantum mechanics, but electron diffraction is an extremely important tool used to study new material.

5.1 Physical concept

De Broglie's Law states that:

$$\lambda = \frac{h}{p} \tag{31}$$

where λ is the electron wave length, h is the Planck's Constant and p is the electron momentum. For an electron accelerated through a potential V, the momentum p is given by the following relation:

$$p = \sqrt{2meV} \tag{32}$$

where m is the mass of the electron and e its charge. The above assumes a non-relativistic approximation. Substituting in the de Broglie's relation:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{h^2/2me}{V}} \tag{33}$$

And when h, m and e are substituted:

$$\lambda\,(nm) = \sqrt{\frac{1.505}{V\,(Volts)}} \tag{34}$$

5.1.1 Bragg's law

The case of waves (electromagnetic waves such as x-rays or "matter" waves such as electrons) scattering off a crystal lattice is similar to light being scattered by a diffraction grating. However, the three-dimensional case of the crystal is geometrically more complex than the two- (or one-) dimensional diffraction grating case. Bragg's Law governs the position of the diffracted maxima in the case of the crystal. A wave diffracted by a crystal behaves as if it were reflected off the planes of the crystal. Moreover there is an outgoing diffracted wave only if the path length difference between rays "reflected" off adjacent planes are an

integral number of wavelengths. Figure 6 shows the extra path length off the electron beam scattering of two parallel planes as well as the diffraction patterns produce by electron beams of two different crystal structure.

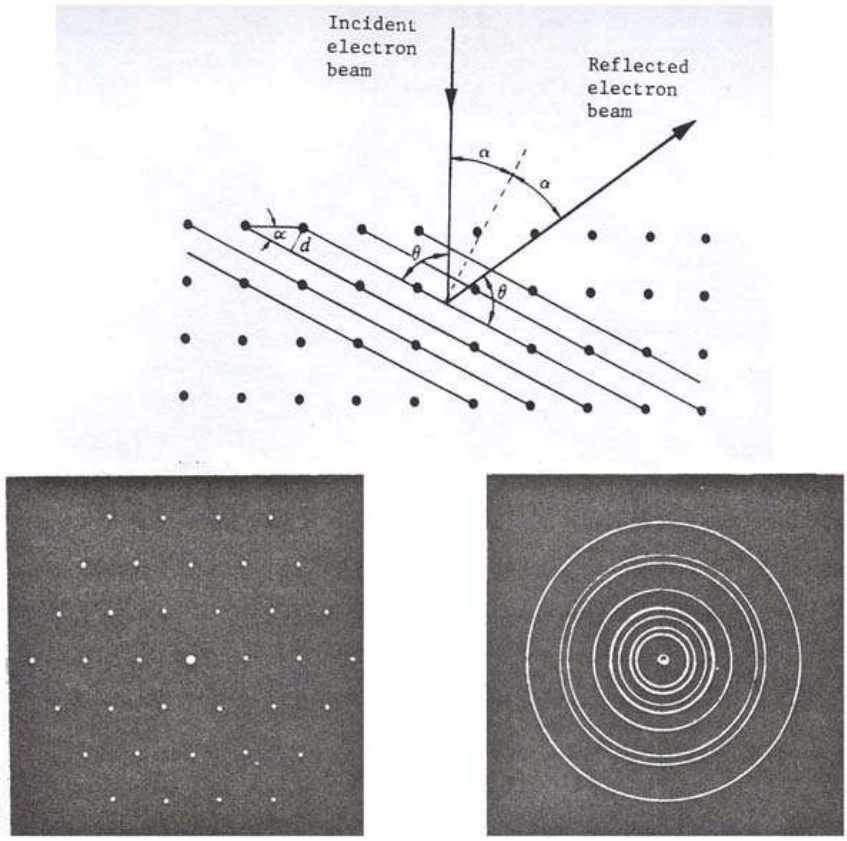

Figure 6: *Top: A beam incident on a crystal structure which planes are separated by a distance d. Bottom left: Diffraction pattern when scattering off a single crystal. Bottom right: Diffraction pattern off a polycrystal.*

The extra path length of the lower ray may be shown to be $2d\sin\theta$ so that maxima in the diffraction pattern will occur when:

$$2d\sin(\theta) = n \qquad\qquad n = 0, 1, 2... \tag{35}$$

This is Bragg's Law. Furthermore, the beam is deflected a total angle 2θ. Thus, suppose one observes the diffraction pattern on a screen. The maxima will registered as spots or rings on the screen. The distance of the spot from the incoming beam axis will be R, such that:

$$R = D\tan(deflection) = D\tan(2\theta) \sim 2D\theta \tag{36}$$

where D is the distance from target to screen. Combining Equations 35 and 36 and taking $\sin(\theta) \sim \theta$, then:

$$R = \frac{n\lambda D}{d} \tag{37}$$

Note, that obtaining a diffraction maximum requires that two conditions be met. Not only must the angle of deflection bear an appropriate relationship to d and λ, but also the crystal orientation must be correct to provide an apparent "reflection" off the crystal planes. The way the crystals are oriented relative to the incoming beam will thus determine the appearance of the diffraction pattern.

5.1.2 Electron diffraction patterns

In relation to diffraction patterns, it is interesting to consider three types of solid matter: single crystals, polycrystals and amorphous materials.

1. Single crystals:
   Single crystals consist of atoms arranged in an orderly lattice. Some types of crystal lattices are simple cubic, face center cubic (FCC), and body center cubic (BCC). In general, single crystals with different crystal structures will cleave into their own characteristic geometry. You may have seen single crystals of quartz, calcite, or carbon (diamond). Single crystals are the most ordered of the three structures. An electron beam passing through a single crystal will produce a pattern of spots. From the diffraction spots, one can determine the type of crystal structure (FCC, BCC) and the "lattice parameter" (i.e., the distance between adjacent (100) planes). Also, the orientation of the single crystal can be determined: If the single crystal is turned or flipped, the spot diffraction pattern will rotate around the center beam spot in a predictable way.

2. Polycrystalline materials:
   Polycrystalline materials are made up of many tiny single crystals. Most common metal materials (copper pipes, nickel coins, stainless steel forks) are polycrystalline. Also, a ground-up powder sample appears polycrystalline. Any small single crystal "grain" will not in general have the same orientation as its neighbors. The single crystal grains in a polycrystal will have a random distribution of all the possible orientations. A polycrystal, therefore, is not as ordered as a single crystal. An electron beam passing through a polycrystal will produce a diffraction pattern equivalent to that produced by a beam passing through a series of single crystals of various orientations. The diffraction pattern will, therefore, look like a superposition of single crystal spot patterns: a series of concentric rings resulting from many spots very close together at various rotations around the center beam spot. From the diffraction rings, one can also determine the type of crystal structure and the "lattice parameter." One cannot determine the orientation of a polycrystal, since there is no single orientation and flipping or turning the polycrystal will yield the same ring pattern.

3. Amorphous materials:
   Amorphous materials do not consist of atoms arranged in ordered lattices, but in hodgepodge random sites. Amorphous materials are completely disordered. The electron diffraction pattern will consist of fuzzy rings of light on the fluorescent screen.

The diameters of these rings of light are related to average nearest neighbors distances in the material.

5.1.3 Diffraction gratings

In optics, a diffraction grating is an optical component with a regular pattern, which splits and diffracts light into several beams travelling in different directions. The directions of these beams depend on the spacing of the grating and the wavelength of the light so that the grating acts as the dispersive element. Because of this, gratings are commonly used in monochromators and spectrometers. When scattering an electron beam off a crystal, the atoms that seat on regular spots on the lattice of this crystal acts like the many slits of an optical diffraction grating. Figure 7 summarizes the important features of a diffraction grating light curve when both interference and diffraction effect are taken into account. A good discussion of the combined effect of diffraction and interference can be found in the book by Melissinos[7].

5.1.4 To go further

The particle electron produces diffraction patterns, thus revealing its wave-like properties. Which experiment shows that light which produces diffraction patterns has particle-like properties? Hint: What is the Compton effect?

5.2 Experimental setup

The apparatus is an Electron diffraction tube (555 626) from "KEP-KLINGER, EDUCATIONAL PRODUCTS CORP.". In order to operate the tube, a 10 kV high-voltage power supply is used. The tube is mounted on a dedicated stand. The target is a polycrystalline graphite foil. Knowing the distance from the screen to the target crystal (13.5 cm), an investigation into the crystal structure can be carried out using Bragg's law. A copy of the operating manual for this device is available on the class web site, it describes how to operate the tube safely.

5.3 Lab objectives

The objectives of this experience are :

- For the two diffraction rings, measure R versus V, the accelerating voltage. Does your data agree with the V dependence predicted by de Broglie's Law? Hint: To check the validity of a relationship, evaluate the χ^2 of your data compared to your hypothesis.
- Determine the lattice plane spacings of the target. What is the precision of your extraction? How do your results compare to literature values? Note: with this device that shows only two rings, the lattice spacings accessible for the graphite target are the ones shown on figure 8

[7] "Experiments in Modern Physics", A. Melissinos and J, Napolitano, ISBN:0-12-489851-3

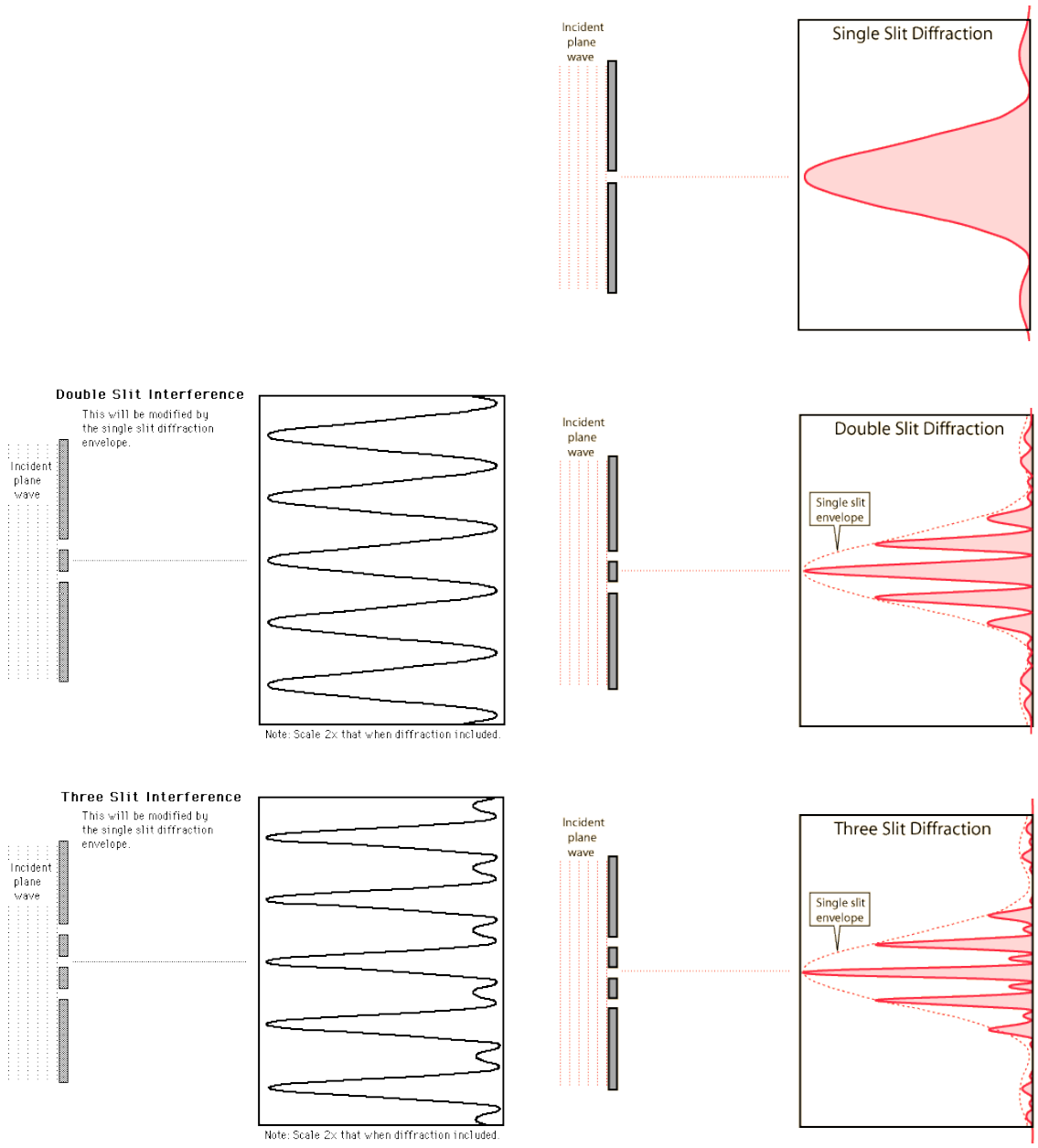

Figure 7: *Light curves produced by a plane wave passing through one (top row), two (middle row) or three (bottom row) slits. The left column shows the light curves obtained if one considers only interference effect while the right column shows the combined effect of interference and diffraction effects. This plots are extracted form the "hyperphysics" web site.*

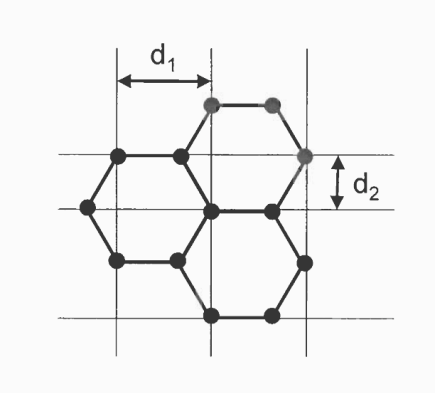

Figure 8: *Lattice plan spacing observable with the electron tube used in this experiment.*

5.4 Preliminary questions

The following questions should also help you to prepare for this lab:

- How do you steer an electron beam to a specific position? In the diffraction experiment, how can you be sure that you are looking at diffraction patterns created by electrons and not by light?

- What is the χ^2 test? What is it used for?

- How do electron microscopes relate to the wave-particle duality properties of electrons? What is the advantage of an electron microscope compared to a visible microscope?

- Suppose you observe the diffraction pattern produced by an electron beam interacting with a crystal of lattice parameter d_1. If the radius of the ring produced by the first maximum is R_1, what is the radius of the ring produced by the second maxima? Now suppose that the crystal you are looking at has two relevant lattice parameters d_1 and d_2 (see figure 8), suppose $d_2 = \alpha d_1$, how does the radius R_1 of the first maximum ring produced by d_1 compare to the radius of the first maximum ring produced by d_2?

- An electron and a photon each have kinetic energy equal to 50 keV. What are their de Broglie wavelengths?

- To verify de Broglie's law you will plot R versus $1/\sqrt{(V)}$. The data will line up on a straitht line. To compute the χ^2 , you will first only consider the error on the radius Then you will add the precision on V according to the last sentence of section 2.2.1. Suppose you are observing a diffraction pattern produced by a lattice spacing d=125 μm on a screen 135 cm away from the sample. Suppose you measure R and V to 1%. By how much does the precision on R increases when you include or not the precision on V ?

Reference

Lab manual of the Physics Department of the Toronto University, Canada.
Lab manual of the Physics Department of the Austin College.

6 The Millikan's oil drop experiment

In 1910, Robert Millikan published the details of an experiment that proved beyond doubt that charge was carried by discrete positive and negative entities, each of which had an equal magnitude. He was also able to measure the unitary charge (which we now recognize to be the electron) accurately. Millikan received the Nobel Prize in 1923 "for his work on the elementary charge of the electricity and on the photoelectric effect." In order to measure the charge of the electron, Millikan carefully balanced the gravitational and electric forces on tiny charged droplets of oil suspended between two metal electrodes. Knowing the electric field, the charge on the droplet could be determined. Repeating the experiment for many droplets, he found that the values measured were always multiple of the same number. He interpreted this as the charge on a single electron. This experiment has since been repeated by generations of physics students, although it is rather difficult to do properly.

The oil-drop experiment appears in a list of Science's 10 Most Beautiful Experiments originally published in the New York Times. See

http://physics.nad.ru/Physics/English/top10.htm.

6.1 Physical concept

In the experiment, a small charged drop of oil is observed in a closed chamber between two horizontal parallel plates. By measuring the velocity of the fall of the drop under gravity and its velocity of rise when the plates are at a high electrical potential difference, data is obtained from which the charge of the drop may be computed. In the experiment, the oil drops are subjected to three different forces: viscous, gravitational and electrical. By analyzing these various forces, an expression can be derived which will enable measurement of the charge on the drop and determination of the unit charge of the electron.

When there is no electric field present, the drop under observation falls slowly, subject to the downward pull of gravity and the upward force due to the viscous resistance of the air to the motion of the drop. The resistance of a viscous fluid to the steady motion of a sphere is obtainable from Stroke's Law from which the retarding force acting on the sphere is given by:

$$F_r = 6\pi a\eta v \tag{38}$$

where a is the radius of the sphere, η is the coefficient of viscosity of the fluid, and v is the velocity of the sphere. For an oil drop of mass m, which has reached constant or terminal velocity, v_g, the upward retarding force equals the downward gravitational force and

$$F_r = mg = 6\pi a\eta v_g \tag{39}$$

Now let an electrical field, E, be applied between the plates in such a direction as to make the drop move upward with a constant velocity, v_E. The viscous force again opposes its

motion but acts downward in this case. If the oil drop has an electrical charge, q, when it reaches constant velocity, the forces acting on the drop are again in equilibrium and

$$Eq - mg = 6\pi a\eta v_E \tag{40}$$

Solving Equation 40 for mg and equating to Equation 39, one obtains

$$q = \frac{6\pi a\eta}{E}\ (v_E + v_g) \tag{41}$$

The electrical field is obtained by applying a voltage, V, to the parallel plates of the condenser which are separated by a distance, d. Therefore,

$$q = \frac{6\pi a\eta d}{V}\ (v_E + v_g) \tag{42}$$

From Equation 42, it is seen that for the same drop and with a constant applied voltage, a change in q results only in a change of v_E and

$$\Delta q = C\Delta v_E \tag{43}$$

When many values of Δv_E are obtained, it is found that they are always integral multiples of a certain small value. Since this is true for Δv_E, the same must be true for Δq; that is, the charge gained or lost is the exact multiple of a unit charge. Thus the discreteness of the electrical charge may be demonstrated without actually obtaining a numerical value of the charge.

In Equation 42, all quantities are known or measurable expect a, the radius of the drop. To obtain the value of a, Stoke's Law is used. It states that when a small sphere falls freely through a viscous medium, it acquires a constant velocity,

$$v_g = \frac{2ga^2(\alpha - \alpha_1)}{9\eta} \tag{44}$$

where α is the density of the oil, α_1 the density of the air, and η, as stated previously, is the viscosity of air. Since the density of the air is very much smaller than the density of oil, α_1 is negligible and Equation 44 is reduced to

$$a = \sqrt{\frac{9\eta v_g}{2g\alpha}} \tag{45}$$

Substituting this value of a in Equation 42 gives an expression for q in which all the quantities are known or measurable:

$$q = \frac{6\pi d}{V}\sqrt{\frac{9\eta^3}{2\alpha g}}\,(v_E + v_g)\sqrt{v_g} \tag{46}$$

Millikan, in his experiments found that the electric charge resulting from the measurements seemed to depend somewhat on the size of the particular drop used and on the air pressure. He suspected that the difficulty was inherent in Stroke's Law which he found not to hold for

very small drops. It is necessary to make a correction, dividing the velocities by the factor $(1+\frac{b}{pa})$ where p is the barometric pressure (in cm of Hg), b is a constant of numerical value 6.16×10^{-6}, and a is the radius of the drop. The value of this correction is sufficiently small so that the rough value of a obtained in Equation 45 may be used in calculating it. The corrected charge on the drop is, therefore, given by:

$$q = \frac{6\pi d}{V}\sqrt{\frac{9\eta^3}{2\alpha g}}\left(1+\frac{b}{pa}\right)^{-\frac{3}{2}}(v_E+v_g)\sqrt{v_g} \tag{47}$$

Using the MKS system throughout, the charge q will be in Coulomb when: $b = 6.17\times10^{-6}$
p is the pressure in cm of mercury
a is the radius in meters
$\eta = 1.827\times10^{-5}$ N.s/m^{-2} is the air viscosity at 18^{o}C
g is the acceleration of gravity
v_E is the velocity in m/s
d is the distance in m
V is the potential difference between plates in volts
α is the density of oil.

For this experiment, the oil used is the doped with small latex shells. The shells have a 1.02 μm diameter. The density of the solution is 1.05 g/ml and there is 1.7% solids.

6.1.1 How do the drops get charged?

The oil used is the type that is usually used in vacuum apparatus. This is because this type of oil has an extremely low vapour pressure. Ordinary oil would evaporate away under the heat of the light source and so the mass of the oil drop would not remain constant over the course of the experiment. Some oil drops will pick up a charge through friction with the nozzle as they are sprayed, but more can be charged by exposing them to an ionizing radiation source (such as an x ray tube).

6.1.2 To go further

- Millikan and his contemporaries were only able to measure integer values of electron charge? Has anyone succeeded in measuring fractional values of electron charge? Hint: look up fractional quantum Hall effect, or the 1998 Nobel Prize for Physics. Also what is the electric charge of the quarks ? What are quarks?

- What are alpha particles? What are common sources of alpha particles? How much matter stops a beam of alpha particles?

- How does a CCD camera work?

6.2 Experimental setup

The oil drop apparatus you will use in this lab is the oil drop apparatus from CENCO #71263. A copy of the operating manual is available on the class web site. It consists

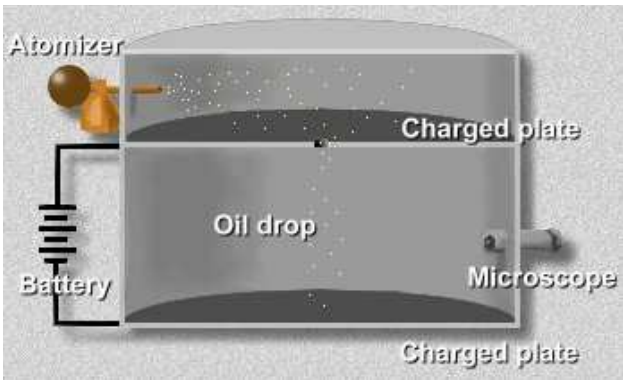

Figure 9: *Sketch of the oil drop apparatus.*

basically of two plates separated by a three millimeter illuminated gap (see Figure 9). A microscope equipped with a CCD camera allows us to watch the droplet motion within the gap. In operation, a cloud of oil is sprayed into the chamber above the top plate and charged droplets fall through a small hole in this plate and into the gap between the plates. The oil drop apparatus is equipped with two individually operated alpha ray sources; one in the chamber above the top plate, and one in the gap between the plates. The upper source is used to insure that the droplets are ionized before they enter the gap, and the bottom is used to change the number of electrons on the droplet inside the gap.
To operate the apparatus:

1. Level the apparatus on the work surface by adjusting the legs until the level, built into the cover of the oil chamber, indicates that the instrument is level.

2. The condenser should be disassembled and cleaned of oil regularly. Be sure to clean the holes and the caps too.

3. A small block of Aluminum is provided. This blocks is marked on of its side with scratched 0.4 mm apart. Use this block to calibrate the distance travelled by the drops on the TV screen.

4. Set the polarity switch to the neutral position and open the top alpha source by turning it until the indicating handle is vertical. Remove the top cover and spray a little oil into the chamber above the top plate. Replace the cover. A great many drops should be visible at first. If not, the holes in the top plate may be clogged, the atomizer might not be atomizing or the oil might be to thick.

5. It is advisable to choose a drop which moves slowly when the electric field is applied. This indicates that the charge on the drop is small, therefore probably being only a small multiple of that of the electron. For this experiment to succeed it is essential to measure a large number of drops with small electric charge, that is drops with large charge are pretty much useless.

6. If the drop is too small, it will not travel up and down in a straight line but will waiver back and forth. This is due to the Brownian Motion. If the drop tends to drift out

of focus, move the microscope gently in and out of focus but not while timing a rise or a fall. If drifting is excessive, re-level the entire instrument. Practice following the drop up and down to develop the technique of observing it and controlling its motion before starting to record data.

Pieces of wisdom:

- To calibrate the distance on the screen to the actual distance, you will need to open the top of the apparatus. First release the two clips holding it on as well as the connector that provides voltage to the upper plate. Focus on an object of known or measured dimensions.

- Measure the time of rise and fall for the same charge several times. Use the average. A sudden change in velocity indicates that the charge on the droplet or the mass of the droplet has suddenly changed, therefore, voiding the trial. Do this for at least fifteen different charges. You can alter the charge with the bottom radiation source. Set the polarity switch to neutral and open the bottom alpha source.

6.3 Lab objectives

Verify that the charge of the droplets are quantized. That is, verify that each droplet charge, Q, is an integer multiple of a common elementary charge e. The more droplets you measure, the better your result; plan on measuring at least 20 droplets with very similar total electric charge.
You should plot the electric charge of each droplet versus the droplet number, once the droplets have been ordered by their electric charge. Do not forget to include error bars. If this plot fail to reveal the electric charge quantization, you might want to plot for each droplet $Q/e - n$ (do not forget error bars). You will have to estimate n for each droplet as the integer the closest from Q/e.

6.4 Preliminary questions

The following questions should help you to prepare for this lab:

- What are the forces acting on the droplets in the Millikan's oil drop experiment? What is the effect of each of these forces on the droplets?

- In the context of this experiment, what is the role of the atomizer? Does it charge the droplet?

- The Millikan apparatus has two parallel plates separated by approximately 3 mm and a high-voltage power supply. Each time you start a new observation with zero field and a squirt from the atomizer you will see a myriad of droplets falling through the field of view. Your problem will be to pick droplet of similar size and total electric charge of few e. In this experiment, you will use oil that is enriched with little latex spheres. How much time does it take for such a little sphere to drop a distance of 1 mm?

- Once you succeed in trapping a little latex sphere, you will need to quickly decide if it carries a charge of a few electron (e.g. 1-3e). Calculate the time it takes for a little latex sphere to rise by 1mm if it is pulled up by a 200 V potential and carries an electric charge of 1 e. What is this time, if the sphere carries an electric charge of 3 e.

- Suppose you measure the rise time of a droplet to be 7.0±0.1 s, the time down to be 31.0 ± 0.1 s, the distance the droplet travels up and down to be 0.80 ± 0.01 mm. Suppose you work at atmospheric pressure and with a voltage of 200V. To which precision can you evaluate the charge of the droplet? Do not consider the precision on the correction to Stoke's law.

Reference

"Instructions for the Hoag-Millikan oil drop apparatus", The Welch scientific company, Cat # 0620.

7 The Franck-Hertz Experiment

In 1914, James Franck and Gustav Hertz performed an experiment which demonstrated the existence of excited states in mercury atoms, helping to confirm the quantum theory which predicted that electrons occupied only discrete, quantized energy states. In 1925, Franck and Hertz were awarded the Nobel Prize on Physics *for their discovery of the laws governing the impact of an electron upon an atom.* In the Franck-Hertz apparatus, electrons are accelerated and collected after passing through a gas of mercury. As the Franck-Hertz data shows (see figure 10), when the accelerating voltage reaches 4.9 volts, the current suddenly drops, indicating the sharp onset of a new phenomenon which takes enough energy away from the electrons that they cannot reach the collector. This drop is attributed to inelastic collisions between the accelerated electrons and atomic electrons in the mercury atoms. The sudden onset suggests that the mercury electrons cannot accept energy until it reaches the threshold for elevating them to an excited state. This 4.9 volt excited state corresponds to a strong line in the ultraviolet emission spectrum of mercury at 254 nm (a 4.9eV photon). Drops in the collected current occur at multiples of 4.9 volts since an accelerated electron which has 4.9 eV of energy removed in a collision can be re-accelerated to produce other such collisions at multiples of 4.9 volts. This experiment was strong confirmation of the idea of quantized atomic energy levels.

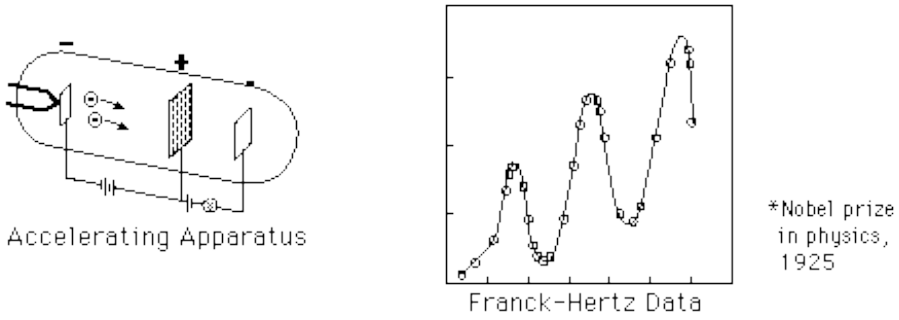

Figure 10: *Sketch of the Franck-Hertz apparatus (left) and graph of typical data (rigth). Thermionic electrons are emited by a cathode (rigth), accelerated in capsule containing mercury and collected on a anode (left). The data show the current collected as a function of the accelerating voltage, the peaks reveal the first excited state of Mercury.*

7.1 Physical concept

In the Franck-Hertz experiment, an electron beam is produced by thermionic emission from a filament. The electrons are accelerated, pass through the vapor, and are then retarded (decelerated) by a few volts before collection at the anode. This all takes place in a tube contained within an oven that controls the tubes temperature and thus the mercury vapor density. The setup is illustrated schematically in Figure 11.

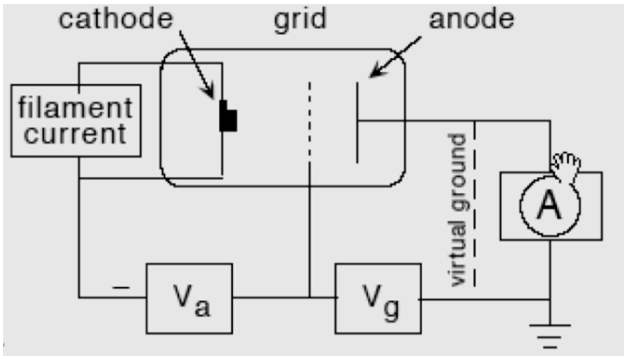

Figure 11: *The Franck-Hertz tube and electrical connections. The cathode is a filament heated by the current provided from a 6.3 V supply. The voltage V_a accelerates the electrons toward the grid. A voltage, V_g slows the electrons as they travel between the grid and the anode. The current of electrons reaching the anode is measured by a picoammeter/electrometer.*

Consider first what happens as the filament is heated, thus raising the average energy of conduction electrons in the metal. As the temperature increases, a greater fraction of the electrons have kinetic energy exceeding the work function W_{Pt} of the cathode material (platinum) . Some of these even reach the anode and a small current (e.g., nanoamps) flows. When the accelerating voltage V_a is increased from zero, filament electrons are accelerated toward the grid and a much greater current reaches the anode. If V_g =0, some of the emitted electrons reach the anode and the electron current is registered. The path of current in this circuit passes through the two power supplies, the filament, the mercury vapor in the tube, and through the ammeter. Not all the electrons make it to the anode: the beam really isnt a beam, and the trajectories spread out over a large range of angles. The planar geometry does improve the situation. As the accelerating voltage increases, the anode current increases because the electron trajectories are more focused, and the electrons are deflected less by scattering from the atoms of vapor in the tube. The anode current is observed to rise faster than linearly with V_a.

We now take into account electron collisions with the mercury atoms in the vapor. Elastic scattering is one channel that results in deflection of the electrons from their original trajectory. In an elastic scattering, the atom recoils in its ground state like a hard sphere, so the electron loses very little energy. Inelastic scattering is also possible. In this process, kinetic energy is lost to excitation or ionization of mercury atoms. The excitation energy is $E_{ex} \sim 5$ eV, so when $(eV_a - W_{Pt}) < E_{ex}$, no such excitation is possible. For electron kinetic energy greater than E_{ex}, it is, in principle, possible for the inelastic process to occur and decrease the electron kinetic energy by about E_{ex} for each mercury atom an electron excites.

The scattering process is best represented by a cross section σ - the effective area of a mercury atom presented to the electrons as they move through a vapor. Imagine looking through the vapor with total thickness t and a number density n atoms/cm^3 from the perspective of an electron with some initial trajectory. The vapor may look partly transparent with an areal density of nt atom/cm^2 in the electrons path. The total fraction of that area

taken up by mercury atoms from which the electron can scatter is nt , a measure of the probability the electron will scatter while propagating through the vapor. The cross section is an effective area and depends on the nature of the interaction, initial energy, final energy and direction, and other quantum mechanical features of the process. The elastic and inelastic cross sections are generally quite different: the inelastic excitation cross section can be a resonant process with a relatively large cross section.

As an electron passes through the vapor, it gains energy in the accelerating field produced by V_a. At the point where the electrons energy exceeds E_{ex}, inelastic scattering becomes possible, and the electron scatters with a short scattering length and loses almost all of its kinetic energy so that the process begins again. Thus multiple inelastic scatterings are possible as an electron moves from cathode to grid.

The final energy of electrons that reach the grid is the initial energy, eV_a, minus energy lost in scatterings. A small retarding voltage between the grid and anode deflects electrons with energy less than $eV_g + W_{Cu}$, where W_{Cu} is the work function of the anode. This energy is a few eV. Electrons with greater energy reach the anode and the electron current is registered by the ammeter. For electrons that lose total energy ΔE by scattering, the requirement for detection (reaching the anode) is $\Delta E \leq e(V_a - v_g - \Delta W)$, where ΔW, the contact potential, is the difference of work functions of the cathode and anode. The anode current is therefore a measure of the integral number of electrons that satisfy this inequality. These raw data must be analyzed to determine the electron energy loss spectrum and extract E_{ex}.

7.1.1 Energy levels of Mercury

A mercury atom has 80 electrons. For an atom in the ground state the K, L, M, and N shells of mercury are filled and the O and the P shells have the following electrons: O shell ($5s^2$, $5p^6$, $5d^{10}$) and P shell ($6s^2$). The energy levels of mercury, which are relevant to this experiment, are shown in figure 12. In this figure the energy levels are labeled with two notations:
- nl where n is the principal quantum number and l is the orbital angular momentum quantum number designated by $s(l = 0)$ and $p(l = 1)$.
- $^{2S+1}L_f$, where S, L, and J are the total spin quantum number, total orbital angular momentum quantum number, and total momentum quantum number.
The $^1P_1$ and $^3P_1$ are ordinary states, having lifetimes of about 10^{-8}s before decaying to $^1S_0$ ground state by photon emission. The $^3P_2$ and $^3P_0$ are meta-stable states, having lifetimes of about 10^{-3}s or 10^5 times as long as an ordinary state. Hence the probability per second of an electron making a transition from either the $^3P_2$ or $^3P_0$ state to the $^1S_0$ ground state by photon emission is 10^5 times smaller than the transition from either the $^1P_1$ and $^3P_1$ state to the $^1S_0$ ground state. Thus some transitions are forbidden while other are allowed. The allowed transitions for photon emission are indicated by the two arrows on the left of figure 12, and the four arrows on the right indicate energy spacing in units of electron volts.

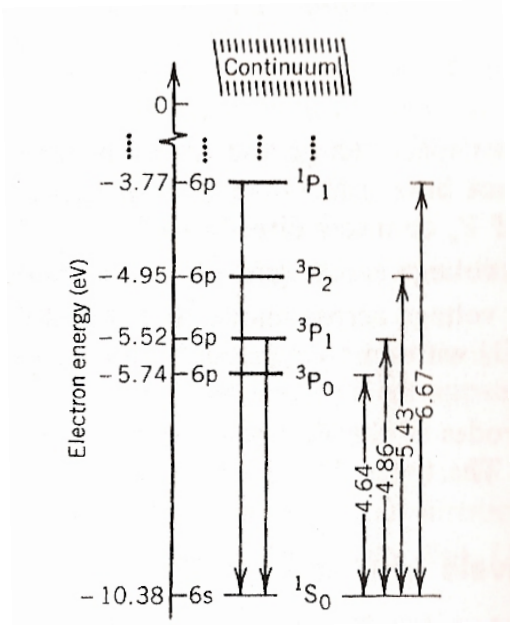

Figure 12: *Energy levels of mercury that are relevant to this experiment. Energy level separation in electron volts is indicated.*

7.1.2 Atomic excitation by inelastic electron scattering

An electron traveling from the cathode K toward the anode A has a mean free path l given by[8] :

$$l = \frac{1}{\sqrt{2}\pi d^2 n} \quad [m] \tag{48}$$

where $d \sim 302$ pm is the diameter of a mercury atom and n is the number of atoms per unit volume. At the end of one mean free path, the electron has gained a kinetic energy K from the electric field E:

$$K = eEl \quad [J] \tag{49}$$

where e is the electron charge and E is the electric field established by the the accelerating voltage V_a. If l is long, K is large. The number density n is very sensitive to the tube temperature; therefore l and, hence, K are very temperature sensitive.

7.1.3 Loss of kinetic energy in an elastic collision

The loss of kinetic energy by an electron when it collides elastically with a mercury atom is greatest when the collision is head-on. For an elastic head-on collision with the mercury

[8]See, D. Halliday and R. Resnick, "Physics", Part 1, 3d edition, Wiley, NewYork 1978. Mean free path is discussed on pages 522-524.

atom initially at rest, the change in electron energy is given by[9]:

$$\Delta K = \frac{4mM}{(m+M)^2} K_0 \quad [J] \tag{50}$$

where m and M are the masses of an electron and a mercury atom and K_0 is the initial electron energy.

7.1.4 Ionization potential of mercury

The Franck Hertz tube can be connected such that the ionization potential of mercury can be measured. In this case, the anode is connected so that the anode is maintained at a negative potential of a couple of volts with respect to the cathode. In this condition, it is energetically impossible for any electron to reach the anode. When the grid potential is made positive with respect to the cathode, electrons are accelerated to the grid. Some pass to through the grid into the space between the the grid and the anode, but all are eventually puled back and collected by the grid. However, if the electrons enter the space between the grid and anode have sufficient energy to ionize the mercury atoms, then the resulting positive mercury ions are drawn to the anode and a positive current is registered. The measure of the anode current as a function of the grid to cathode potential difference yields a measure of the ionization potential. The first difficulty is to determine the optimal oven temperature. If the vapor pressure of the mercury is too low, then the electrons entering the grid-to-anode region with sufficient energy to cause ionization will have a small probability of collision (long mean free path) and the ion current collected by the anode will be too low. If the vapor pressure of mercury is too high, then the electrons will suffer inelastic collisions in the space between the cathode and the grid as soon as they energy slightly exceed the energy required to raise mercury atoms to their first excited state and will therefore never attain sufficient energy to cause ionization.

7.2 Experimental setup and procedure suggestion

In this experiment, we will use a version of the Franck Hertz tube Mounted on Front-Plate with Mercury Filling and Heating Chamber from Klinger KEP (Model KA6040-KA6041), the Franck Hertz tube control unit KA6045, read out by an digital oscilloscope. A copy of the operating manual for this device is available on the class web site, it describes the Franck Hertz tube, the power supply used in this experiment as well as suggestions for the data taking.

The Franck-Hertz tube is contained within an oven, which is a metal box with a thermostatically controlled heater and terminals for connections to the tube. A thermometer can be inserted through a hole in the top of the oven. The tube has a parallel system of electrodes in order to produce a fairly uniform electric field. The distance between the filament/cathode and the perforated grid is much larger than the mean free path of electrons through the mercury vapor under normal operating conditions. A platinum ribbon with a

[9]See K. Rossberg, "A first course in Analytical Mechanics", Wiley, New York, 1983. Collision of two particles is discussed on page 157-160

small barium-oxide spot serves as a direct heated cathode. An electrode connected with the cathode limits the current and suppresses secondary and reflected electrons, making the electric field more uniform. In order to avoid current leakage along the hot glass wall of the tube, a protective ceramic ring is fused in the glass as a feed-through to the counter electrode. The tube is evacuated and coated inside with a getter which absorbs traces of air that leaked in during the manufacturing process and over the lifetime of the tube.

Before making any connections, be sure you understand the circuit. Figure 13 shows the setup used for this experiment, this figure is extracted from the manufacturer manual. The web site gives a list of important caution and tips that you should read before starting your experiment. When using the scope, couple the channel to DC to provide a good zero point for voltage. Also we will use the 60 Hz functionality of the power supply to minimize temperature correlated drifts of the measured voltages.

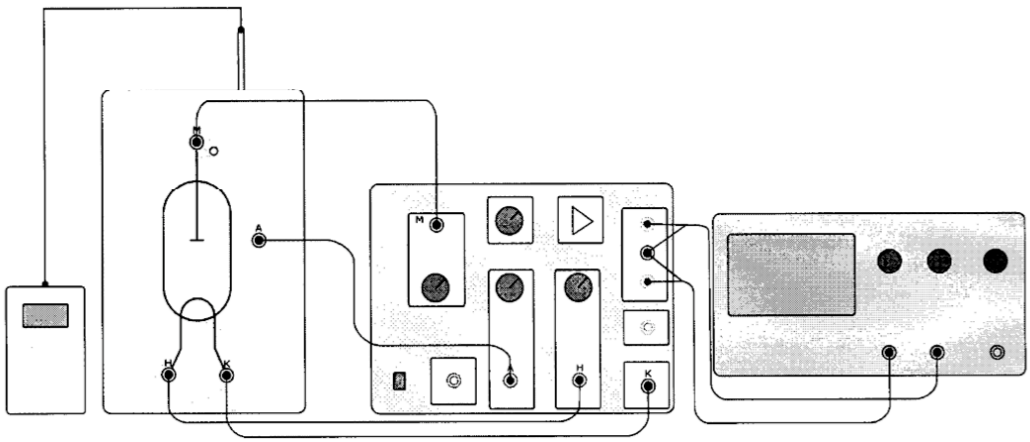

Figure 13: *The Franck-Hertz tube and electrical connection used in this experiment.*

7.3 Lab objectives

To measure the energy of the 1st excited state of Hg and the work function of the heating filament.
For this lab, you are asked to:

- in the theory section, explain in at least ten full sentences how the Bohr model of the atoms explains the quantization of excited states,
- use both peaks and valleys of your data to extract your results,
- use the result of your linear regression, to present a residual plot of your data.

7.4 Preliminary questions

The following questions should help you prepare for this lab:

- What is an ion? What is an excited state of an atom? How do those two concepts contrast? Which energy is larger ionization potential or first excited state, explain.

- Why does the Franck-Hertz apparatus have a cathode, anode and grid? Why is the retarding voltage needed to observe the Franck-Hertz effect?

- Considering that the energy of the first excited state of the mercury atom is about 5.0 eV above that of the ground state, what is the maximum amount of energy that an electron with 4.0 electron volts of kinetic energy can impart to a mercury atom with which it collides? Same question for a 6.0 eV electron.

- What is vapor pressure? How does it apply to the Franck-Hertz apparatus you are going to work with?

- What is the mean free path of electrons propagating in a gas (see equation 48) at 180 ^{o}C? At this temperature, the vapor pressure of Mercury is 1.1728 kPa.

- Suppose a voltage V is maintained between a cathode and an anode. The cathode and the anode are separated by a distance $d = 10$cm. Suppose and electron is released at rest from a filament attached to the cathode, the electron will be accelerated to the anode. At what distance d' from the cathode will this electro reach an energy of 5 eV if $V = 6$V, 12V or 24V?

References

The discussion of theFranck-Hertz Effect is copied from:

- the HyperPhysics web site: http://hyperphysics.phy-astr.gsu.edu/Hbase/hph.html

- The lab manual of the Michigan University: http://instructor.physics.lsa.umich.edu/adv-labs/Franck_Hertz/franck-hertz.pdf

- The Franck-Hertz experiment web page at MIT: http://web.mit.edu/8.13/www/07.shtml

- D. Preston and E. Dietz, The Art of Experimental Physics, Wiley, New York, 1991.