Physics 373 – Introductory Laboratory – Nucleons

Rutherford Scattering

Purpose: The purpose of this experiment is to acquaint the student with the experimental procedures necessary to measure an experimental cross section that can be directly compared to a theoretical prediction. It is based on the measurement of the Rutherford scattering cross section in α+Au elastic scattering using the Edwards Accelerator Laboratory at Ohio University.

List of Goals:

A). Understand the experimental layout and related equipment.

B). Perform measurements of the α yield as a function of scattering angle with a silicon detector.

C). Compute a differential cross section for Rutherford scattering.

D). Compare a measured differential cross section to a theoretical calculation.

E). Keep a detailed logbook of all relevant parts of this measurement.

F). Prepare a detailed formal lab report on all aspects of this investigation.

I. Pre-Experiment Approach and Details

◇ Be sure that you have spent adequate time researching the general topic so that you can appreciate the basic ideas outlined above under the *"List of Goals"*.

◇ One of our graduate students might be assisting us in setting up the experiment and collecting the data. Your main work will be in analyzing the data once it is in hand as well as assisting in positioning the silicon detector for each run.

◇ Come up with a run plan for how many different angles are required to compare against the Rutherford formula. Talk to your classmates to optimize this plan. What should be the required level of statistics at each point? Note that in this lab we will all take data together and will not split up into teams.

◇ Do not be surprised if the experiment does not go smoothly or takes more than 2 hours to complete. Experimental nuclear physicists sometimes spend a good deal of time waiting around for accelerator personnel to either deliver the beam or tune the beam to desired levels.

II. Pre-Experiment Checkout

a). After performing the essential pre-lab preparations, begin to understand the experimental setup. This includes understanding the purpose of each part of the system. The basic layout of the Rutherford scattering experiment is shown in Fig. 1.

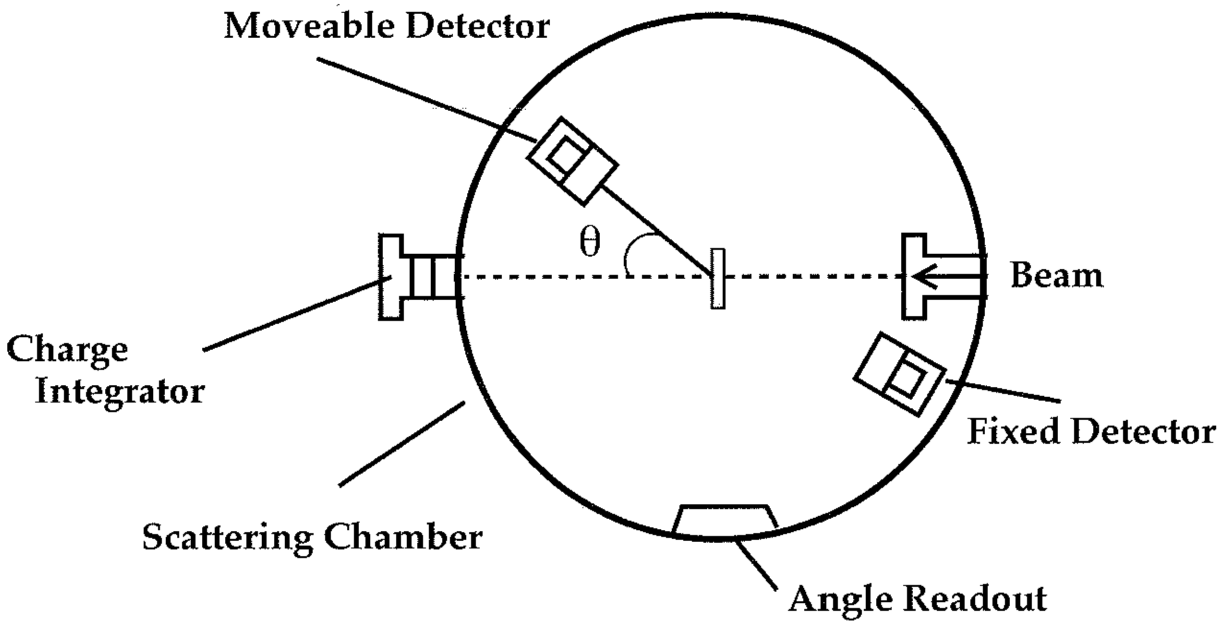


Figure 1: Major components of the Rutherford scattering experiment for this laboratory.

b). Go through the set-up step by step to understand how the electronics are laid out in both the experimental area and the control room. This basic layout and the purpose of each electronics module should be reasonably clear given the experience you have gained in the first part of this course. The manuals for each of the electronics used are available for review. A complete diagram is contained in Fig. 2. Be sure to provide clear and complete documentation in your logbook. Also begin to understand how to operate the data acquisition system and what the defined histograms mean. Make sure you understand where to record the computer dead time for each measurement. This is crucial in order to measure the Rutherford cross section.

c). Make sure you know how to set the angle of the movable silicon detector to the precise angle that you want and that the angle readback makes sense to you. Note that the movable detector is limited to angles in the range from 25° to 150° on one side of the beam and from 250° to 335° on the other.

d). Take measurements of the radial positions of the two silicon detectors. This is required to compute the detector solid angle.

e). Make sure you record the size of the apertures in from of the two silicon detectors. These are required for the solid angle computation.

f). Make sure that you understand how to read out the charge integrator for the alpha beam. This is required in order to determine the number of alphas incident on our gold foil.

g). Make sure you record the thickness of the gold foil (relevant units are $\mu g/cm^2$). This is necessary to compute the cross section.

h). Make sure you know how to set the angle of target (the gold foil). In this experiment we will want to minimize the material traversed by the scattered alpha particle in the gold foil. To accomplish this we will set the angle of the target to be $\theta_{scatt}/2$ for angles $\theta_{\alpha} \leq 90°$. Here θ_{scatt} is the defined angle of the silicon detector. For angles larger than 90°, we will set the target angle to be at 0° (i.e. normal to the incident beam flux).

III. Scattering Cross Section

The scattering of particles is described quantitatively by the differential cross section, $\frac{d\sigma}{d\Omega}$ [1]. We will consider the case when N_0 projectiles are incident on a target of uniform thickness described by n_1 target nuclei per unit area. The incident projectiles can in principle scatter to any angle with respect to the incident direction. The number of particles scattered into a solid angle $\Delta\Omega$ is given by

$$N_s = \frac{d\sigma}{d\Omega} N_0 n_1 \Delta\Omega. \tag{1}$$

In general, $\frac{d\sigma}{d\Omega}$ depends on the type of projectile, the type of target, the energy of the projectile, and the scattering angle. Note also that the above formula is technically only exactly true in the limit that $\Delta\Omega \to 0$, due to the fact that $\frac{d\sigma}{d\Omega}$ depends on the scattering angle. The number of scattered particles N_s is directly proportional to N_0, n_1, and $\Delta\Omega$ as one intuitively expects – in this picture, the differential cross section is the proportionality constant. It is often said that the differential cross section "contains the physics"...

The differential cross section has units of area divided by solid angle. While solid angle is technically dimensionless, we traditionally utilize the units of steradians (sr) to describe it, where $\Delta\Omega = 4\pi$ sr for an entire sphere. In nuclear and particle physics, it is also traditional to use the units of fermis (fm) to describe distances (1 fm=10^{-13} cm) and barns (b) to describe cross sections (1 b=10^{-24} cm^2).

IV. Rutherford Scattering Physics

The Rutherford scattering formula may be derived classically (or quantum-mechanically!) assuming the Coulomb potential between two particles (labeled 0 and 1):

$$U(r) = \frac{Z_0 Z_1 e^2}{4\pi\epsilon_0 r}, \tag{2}$$

where r is the distance between the particles, $Z_0 e$ is the charge of particle 0, $Z_1 e$ is the charge of particle 1, and ϵ_0 is usual electrostatics constant ($\epsilon_0 = 8.854187... \times 10^{-12}$ F/m).

The resulting differential cross section is

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}} = \frac{Z_0^2 Z_1^2 e^4}{16(4\pi\epsilon_0)^2 E_{\text{c.m.}}^2} \frac{1}{\sin^4(\theta_{\text{c.m.}}/2)},$$

where $E_{\text{c.m.}}$ and $\theta_{\text{c.m.}}$ are the center-of-mass kinetic energy and scattering angle, respectively. Note that the Rutherford differential cross section diverges as $\theta_{\text{c.m.}} \to 0$ and the total cross section is in fact infinite.

Since we will not be performing the experiment in the center-of-mass coordinate system, there is more work to do. The algebra of changing coordinate systems for the Rutherford formula is non-trivial and is seldom discussed in textbooks. The formulas below are taken from a paper by Sargood [2]. Let's now assume that label 0 refers to the projectile and label 1 refers to the target which is at rest. First we will define

$$K = \left(\frac{M_0 \cos\theta + \sqrt{M_1^2 - M_0^2 \sin^2\theta}}{M_0 + M_1} \right)^2, \tag{3}$$

where M_i are the masses and θ is the *laboratory* scattering angle, where $0°$ is by definition the direction of the incident projectile. We will also define the parameter

$$a = \sqrt{1 - \left(\frac{M_0 \sin\theta}{M_1} \right)^2}. \tag{4}$$

The differential cross section in the laboratory is given by:

$$\frac{d\sigma}{d\Omega} = \frac{Z_0^2 Z_1^2 e^4}{4a(4\pi\epsilon_0)^2 E_0^2} \frac{(a+\cos\theta)^2}{\sin^4\theta}, \quad \Rightarrow \frac{d\sigma}{d\Omega} \left(\frac{Z e Z e}{4\pi \epsilon_0 E_0} \right)^2 \frac{1}{\sin^4(\frac{\theta}{2})} \tag{5}$$

e theory

where E_0 is the kinetic energy of the incident projectile. The energy of the scattered projectile is given by $E_0' = K E_0$. Note that in the limit $M_0/M_1 \to 0$ ("light" projectile and "heavy" target) we have $K \to 1$ and $a \to 1$ and the lab system is also the center-of-mass system – one can also verify that the differential cross section formulas are equivalent in this limit using the trig identity

$$\frac{1+\cos\theta}{\sin^2\theta} = \frac{1}{2\sin^2(\theta/2)}. \tag{6}$$

It is convenient to expand Eqs. (4) and (5) in powers of M_0/M_1; the result is

$$\frac{d\sigma}{d\Omega} = \frac{Z_0^2 Z_1^2 e^4}{16(4\pi\epsilon_0)^2 E_0^2} \left[\frac{1}{\sin^4(\theta/2)} - 2\left(\frac{M_0}{M_1} \right)^2 \right] + O\left[\left(\frac{M_0}{M_1} \right)^4 \right]. \tag{7}$$

The fundamental constants can be inserted as follows

$$\frac{e^4}{(4\pi\epsilon_0)^2} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c} \right)^2 (\hbar c)^2 \tag{8}$$

$$= \left(\frac{1}{137.036} \right)^2 \times (197.327 \text{ MeV-fm})^2 \times \left(\frac{10 \text{ mb}}{\text{fm}^2} \right) \tag{9}$$

$$= \sout{20.375} \text{ mb-MeV}^2 \tag{10}$$

20.735

to yield "nuclear physics" units. The final formula is

$$\frac{d\sigma}{d\Omega} = 1.296 \left(\frac{\text{mb-MeV}^2}{\text{sr}}\right) \left(\frac{Z_0 Z_1}{E_0}\right)^2 \left[\frac{1}{\sin^4(\theta/2)} - 2\left(\frac{M_0}{M_1}\right)^2\right] \tag{11}$$

which can be used with E_0 in MeV to calculate the differential cross section in mb/sr.

V. Rutherford Scattering Experiment

The experiment will amount to acquiring data at a number of different scattering angles on either side of the incident beam direction. You will record the number of counts in each of our two silicon detectors. The naming convention is to call the fixed detector RBS and the movable detector PRD. In addition to summing up the counts in the ADC spectra, you will have to record the system live time (LT) for each measurement, as well as the real time duration (RT) of each run and the charge integrator value. The counts in the ADC spectra must be multiplied by the ratio RT/LT in order to determine the true number of particles which struck the detector during the real time duration of each run.

In defining E_0 for this experiment, use of the incident α beam energy is only an approximation. In general a better approximation is to use the average alpha energy in the target given by:

$$E_{\text{avg}} = \frac{E_i + E_f}{2}, \tag{12}$$

where E_i is the incident alpha energy and E_f is the alpha energy after passing through the gold foil. It would be relevant to compare the sensitivity of the final result with and without this correction.

Based on the previous discussion, the measured differential scattering cross section in units of cm^2/sr is given by:

$$\frac{d\sigma}{d\Omega} = \frac{N_s}{N_0 \cdot \Delta\Omega \cdot n_1} \tag{13}$$

where:

- N_s = the number of detected particles, corrected by the ratio RT/LT.
- N_0 = the number of beam particles which impinged on the scattering foil. This quantity is determined from the charge integrator. Remember that we are using a He^{2+} beam when converting the measured number of μC to the number of particles.
- $\Delta\Omega$ = the solid angle of the detector (sr). The solid angle is given by A_c/R^2, where A_c is the area of the detector (or the defining aperture) and R is the distance of separation between the aperture and the target.
- n_1 = the number of target nuclei per cm^2. This number will be given to you by your lab instructor.

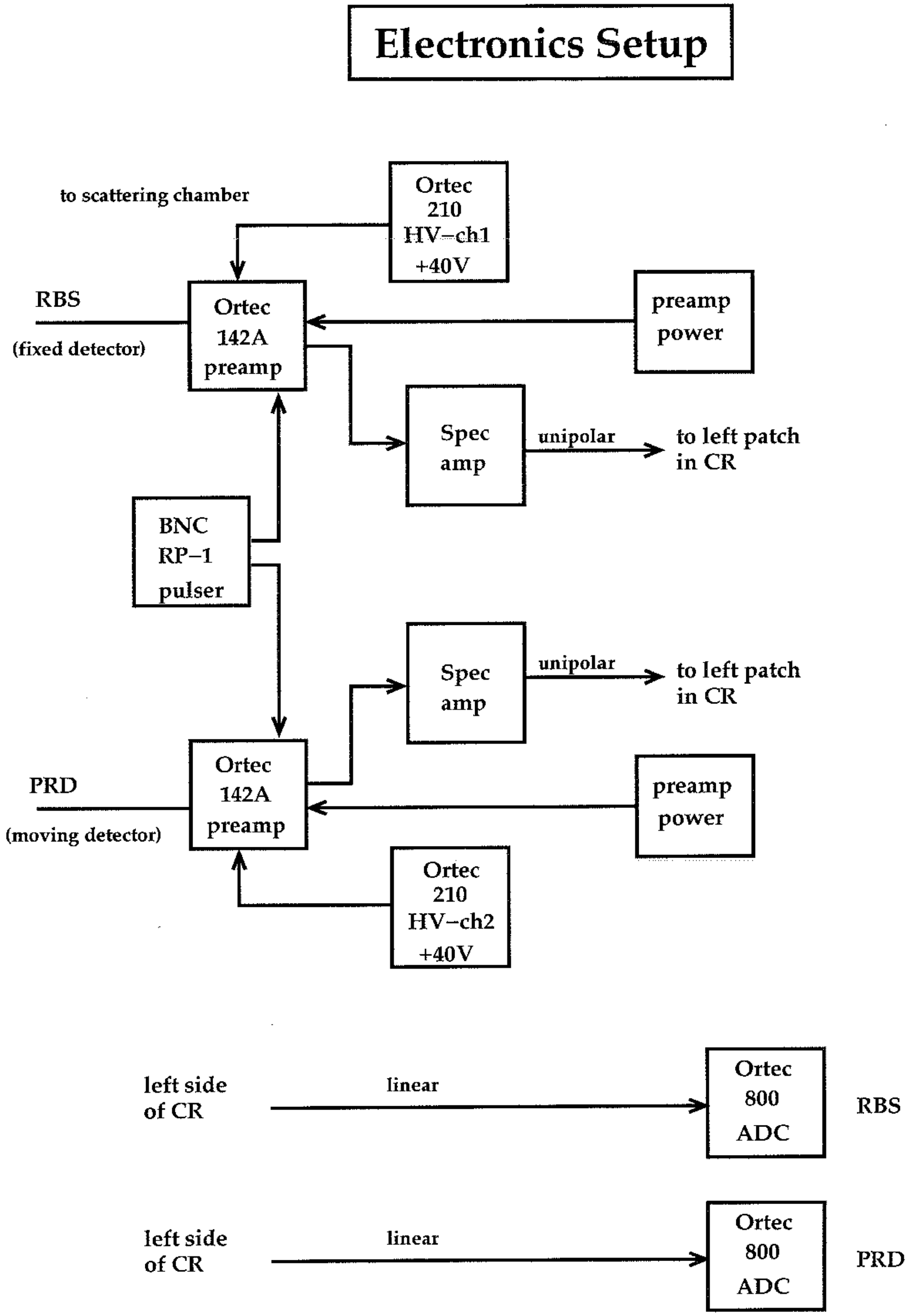


Figure 2: Electronics layout for the Rutherford scattering experiment.

Silicon Detectors – Background Information

Semiconductor charged particle detectors have been used extensively in experimental nuclear physics research for over 30 years, and have revolutionized nuclear particle detection. Silicon detectors can be used to measure a wide range of charged particles. This range includes protons and electrons as low as 20 keV up to fission fragments of energy over 100 MeV. The inherent resolution of ion-implanted and surface barrier detectors is surpassed only by magnetic spectrometers. The detector output pulses rise rapidly, hence they are well suited for fast ($\sim$1 ns) timing with coincidence circuitry or time-to-amplitude converters.

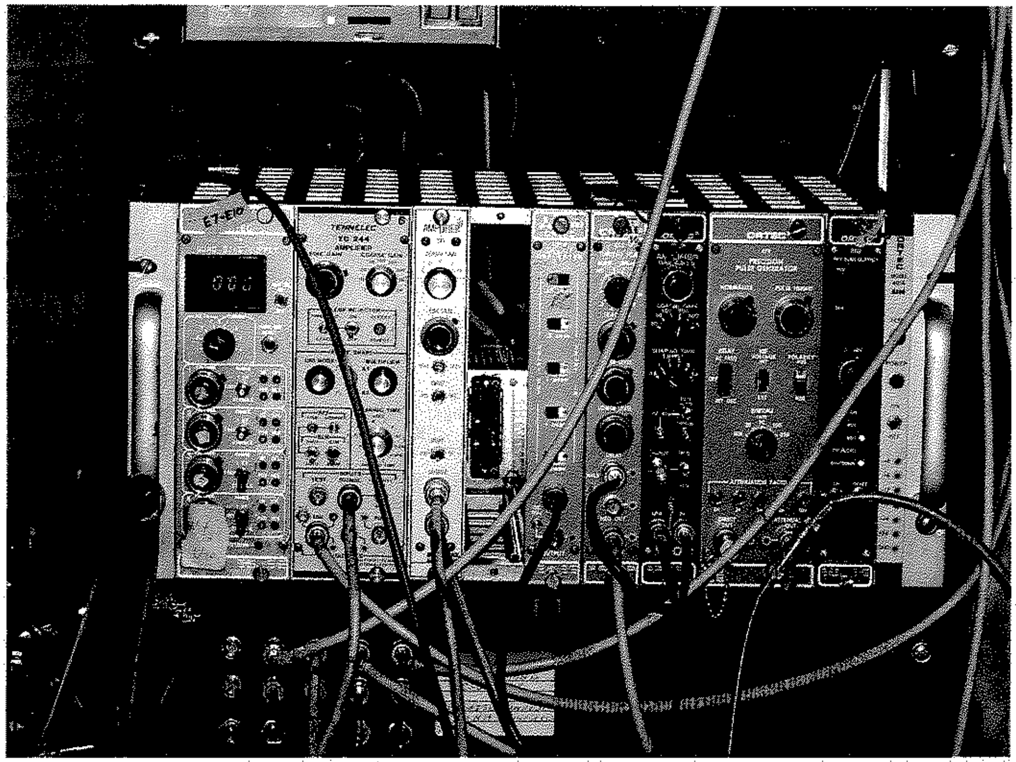
The efficiency of silicon charged particle detectors for their active volume is essentially 100%, and their energy vs. pulse height curves are linear over a rather impressive range. They also have good long-term pulse height stability. This is particularly noticed when they are contrasted with scintillation counters, gas proportional counters, or ionization chambers. The high efficiency of these detectors allows for straightforward analysis of cross sections via the extraction of detected particle yields from the accumulated spectra (provided data acquisition/computer dead times are accounted for accurately).

Solid state detectors can be thought of as a solid state ionization detector. When a charged particle enters the depletion region of the detector, it loses energy primarily by making electron-hole pairs in the silicon. For each electron-hole pair that is made, the initial charged particle must lose 3.6 eV. In this experiment we will be measuring alpha particles that have an energy less than 3 MeV. If a 3 MeV alpha particle enters the detector, roughly 8×10^5 electron-hole pairs will be produced. The detector is reverse-biased and these electron-hole pairs are collected to produce the output pulse of the detector. Since a large number of charge carriers is produced, the statistical variation in the number collected is small and hence very good energy resolution is possible. The "partially depleted" silicon detector used in this experiment has a resolution of roughly 20 keV.

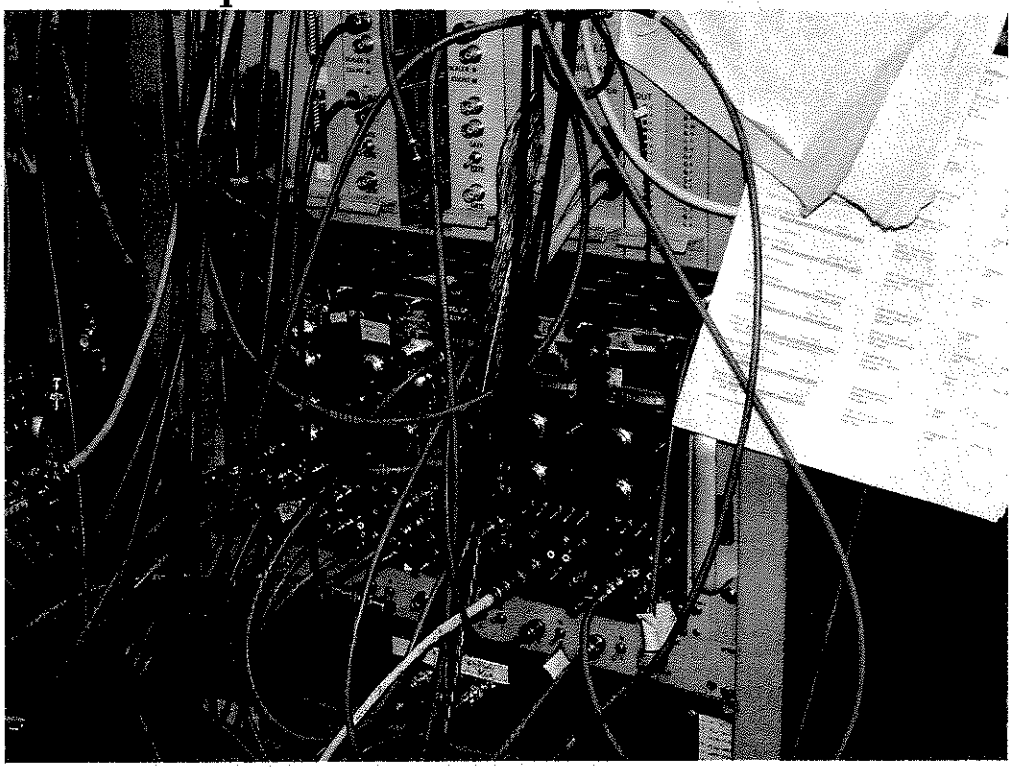
The three main parameters that define a silicon surface-barrier detector are its resolution, active area, and depletion depth. The shape of a typical detector is a circular disk. Thus the active area is simply the area of the face of this disk (provided it is placed normal to the incident flux of radiation). The depletion depth of a detector is synonymous with the sensitive depth of the detector. For any given experiment, this depth must be sufficient to completely stop all the incident charged particles that are to be measured. The detector's ability to do this is dependent upon both the energy and the particle type. Fig. 3 is a range-energy curve for five of the more common charged particles. From it, the maximum depth can be determined for the maximum energy of a particle type. From Fig. 3, note that a 3 MeV alpha particle is completely stopped with about 20 μm of silicon.

Acknowledgments

This document was written by Carl Brune, Daniel Carman, and Andreas Schiller (Ohio University). Many of the details in this write-up were extracted from the Rutherford scattering writeup contained in Ortec Application Note AN-34 *"Experiments in Nuclear Science"* (1976), as well as the Rutherford scattering writeup contained within *"Laboratory Investi-*



Experimental Area Electronics



Counting Room (CR) Electronics

Outline instructions for venting and pumping of L15 RBS chamber

Venting the Chamber

1. Close the two gate valves on the beam line.
2. Ensure that there is no bias voltage on the detectors.
3. Turn off the ion gauge.
4. Close the valve from the chamber to the diffusion pump.
5. Open the N_2 tank valve. Do not change the regulator.
6. Open the chamber vent valve.
7. Turn the chamber crank handle until a slight resistive force is felt.
8. When the chamber lid breaks seal turn off the N_2 tank and close the vent valve.
9. Open the chamber using the crank handle. After the chamber lid clears the positioning bolts place the small block under the lever to tilt the chamber lid to completely open.

Placing the chamber under vacuum

1. Close the chamber lid using the crank handle. When the chamber lid is slightly above the positioning bolts, tilt the chamber lid to a horizontal position, and remove the small block from the lever.
2. Ensure that the guide holes on the chamber lid are over the positioning bolts and finish lowering the lid using the crank handle.
3. Ensure that the vent valve and N_2 tank valve are closed.
4. Close the valve between the diffusion pump and roughing pump.
5. Open the valve between the roughing pump and chamber.
6. Monitor the pressure in the chamber. At $\approx$ 50 mtorr close the roughing pump valve to the chamber.
7. Open the roughing valve to the diffusion pump.
8. Open the valve between the diffusion pump and the chamber.
9. Monitor the pressure in the chamber. When the pressure reading drops off scale turn on the ion gauge.
10. Monitor the pressure. The pressure should reach the 10^{-7} torr range before opening gate valves on the line and running beam.

AN34 Experiment 4

Alpha Spectroscopy with Surface Barrier Detectors

Equipment Needed from ORTEC

Charged Particle Detector U-014-050-100
142A Preamplifier
4001A/4002D NIM Bin and Power Supply
575A Spectroscopy Amplifier
807 Vacuum Chamber
428 Detector Bias Supply
480 Pulser
C-36-12 Cable
C-24-12 Cables (4)
C-24-1 Cables (2)
C-29 BNC Tee Connector
TRUMP-2K-32 Plug-In MCA with MAESTRO-32 Software (other ORTEC MCA may be used)
ALPHA-PPS-115 Portable Vacuum Pump Station

Other Equipment Needed

Alpha Source Kit
Personal Computer with Windows 98/2000/XP or NT
Oscilloscope

Purpose

The purpose of this experiment is to familiarize the student with the use of silicon charged-particle detectors and to study some of the properties of alpha-emitting isotopes.

Applicability

Semiconductor charged-particle detectors have been used extensively in experimental nuclear research for over 30 years, and have revolutionized nuclear particle detection. Publications in nuclear journals indicate that semiconductor detectors are now used almost exclusively for the detection of charged particles. Semiconductor gamma and x-ray detectors have contributed even more significantly in their own field of photon spectroscopy.

Semiconductor charged-particle detectors can be used through an extensive range of energies including 20 keV electrons on one end of the spectrum and 200 MeV heavy ions on the other. The inherent resolution of ion-implanted and surface barrier detectors is surpassed only by magnetic spectrometers. The detector output pulses rise rapidly; hence they are well suited for fast (~1ns) timing with coincidence circuitry or time-to-amplitude converters (TACs).

The efficiency of silicon charged-particle detectors for their active volume is essentially 100%, and their energy vs. pulse-height curves are linear over a rather impressive range. They also have good long-term pulse-height stability. This is particularly noticed when they are contrasted with scintillation counters, gas proportional counters, or ionization chambers. Finally, their compact size make them easily adaptable to almost any counting geometry. The remaining fact of particular interest in the educational market is that they are relatively inexpensive.

It should take about 6 hours to complete all parts of Experiment 4. The parts are written so that they can be completed in two 3-hour laboratory periods, or certain parts can be easily omitted if equipment time is not available.

Alpha Sources

CAUTION: Alpha sources offer a potential contamination problem. Never touch the face of a source with your fingers. Most alpha sources are electrodeposited onto platinum disks. The actual radioactive source is usually a spot ~1 mm diameter

deposited in the geometrical center of the disk. If you look carefully, you may be able to see the deposited spot. The ^{210}Po source in the alpha source kit has been evaporated onto a silver disk, and the disk covered with a piece of plastic with a hole through the center for transmission of the alpha particles. ALWAYS handle an alpha source by the edge of the mounting disk.

Ion-Implanted and Surface Barrier Detectors

There are three main parameters that define silicon charged-particle detectors: resolution, active area, and depletion depth. ORTEC model numbers reflect each of these three parameters in that order. The U-014-050-100 is an ULTRA (ion-implanted) detector with a resolution of 14 keV FWHM for ^{241}Am alphas, an active area of 50 mm^2, and a minimum depletion depth of 100 μm. The quoted resolution of an ORTEC detector is a measure of its quality. These resolutions can be measured only with a complete set of electronics, calibrated for standard conditions. The ORTEC guaranteed resolutions are measured with standard ORTEC electronics. A resolution of 20 keV or better is satisfactory for all parts of Experiment 4.

Since the shape of the detector is a circular disk, its active area is determined by the diameter of its face. At any given distance from the source, a larger area will subtend a larger angle, and thus intercept a greater portion of the total number of alpha particles that emanate from the source. A nominal area of 50 mm^2 is suggested for this experiment, but any area from 25 through 100 mm^2 will provide the information.

The depletion depth is synonymous with the sensitive should resemble that in Fig. 4.3. Determine the centroid channel number for the alpha peak. Call this channel C_0. In the example of Fig. 4.3, C_0 is channel 520; this represents the location in the spectrum for the ^{210}Po 5.305 MeV alpha events. The FWHM, measured in number of channels, is $\delta = 16$.

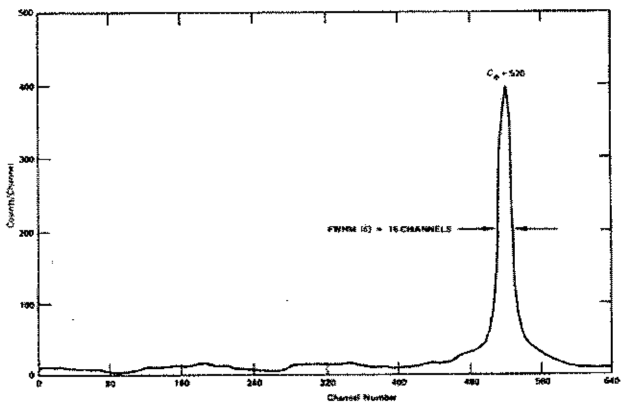


Fig. 4.3. Plot of a Typical ^{210}Po Alpha Spectrum.

5. Turn on the 480 Pulser and set its pulse-height dial at 531/1000. Adjust the attenuators and the calibration control until the pulse generator output is ~5 V in amplitude. The output pulses from the 575A Amplifier for the pulse generator input should now be approximately the same amplitude as the pulses from the ^{210}Po alphas. The pulses from both sources can be observed simultaneously on the oscilloscope.

6. Accumulate the pulser pulses in the MCA for ~20 seconds. Do they fall above or below channel C_0? Adjust the Calibrate control on the pulser as necessary to locate the peak exactly in channel C_0. The pulser is now calibrated to the system so that 5.31 MeV corresponds to 531/1000 on the pulse-height dial and, therefore, any setting of the pulse-height dial represents an identified energy level. For example, 600/1000 = 6 MeV, etc.

Table 4.1. Channel Numbers for Equivalent Energies

Accumulation Time (approx. s)	Pulse-Height Dial Setting	Equivalent Energy (MeV)	Channel Number of MCA Peak
20	100/1000	1.0	
20	200/1000	2.0	
20	300/1000	3.0	
20	400/1000	4.0	
40	500/1000	5.0	
20	600/1000	6.0	
20	700/1000	7.0	

7. Clear the MCA and accumulate the pulser pulses for ~20 seconds at each of the pulse-height values in Table 4.1. Determine their position with the cursor of the MCA.

EXERCISES

a. Fill in the column of data that is missing in Table 4.1. Make a plot on linear graph paper of energy (MeV) vs. channel number. Compare this plot with that in Fig. 4.4. For identification purposes, the 5 MeV point is accumulated for 40 seconds.

b. The slope of the curve in Fig. 4.4, $\Delta E/\Delta C$, is the energy per channel. For convenience this is usually expressed in keV/channel, and in Fig. 4.4 it is ~10 keV/channel. Determine the keV/channel for the plot you made in Exercise a.

c. The resolution in a spectrum is calculated as follows:

$$\text{resolution} = \frac{\Delta E}{\Delta C} \times \delta\,(\text{ch}) \qquad (1)$$

where δ (ch) = channels FWHM.

For example, in Fig. 4.3 the δ (ch) for FWHM is 16 channels for the energy range from channel 512 through 528. It is measured at the points in the spectrum where the number of counts per channel is half the number of counts at the peak. In the example the FWHM resolution is, then, 160 keV. Calculate the δ (ch) and the resolution of your alpha peak.

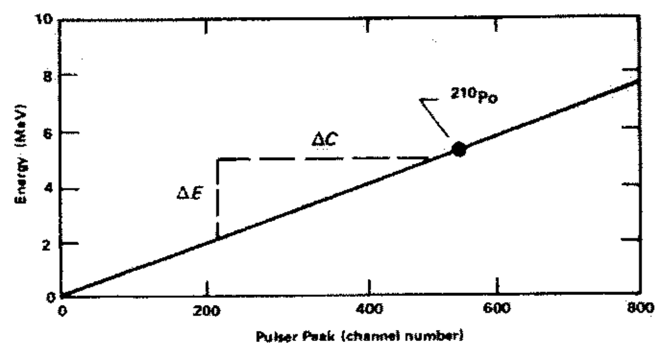


Fig. 4.4. Calibration Curve for Alpha Spectrometry.

EXERCISE

From the calibration curve, determine the keV/channel and the resolution as in Experiment 4.1. Generally speaking, the pulser method outlined in Experiment 4.1 is the better way to establish the calibration for the system.

EXPERIMENT 4.4

Absolute Activity of an Alpha Source

Purpose

The purpose of this experiment is to determine the absolute activity of an alpha source, which in this case is ^{210}Po.

As was mentioned earlier, ion-implanted and surface barrier detectors are essentially 100% efficient for their active area. It is, therefore, quite easy to determine an unknown source activity.

Procedure

1. Carefully place the ^{210}Po source in the 807 Vacuum Chamber exactly 4 cm from the face of the detector. Adjust the 575A Amplifier gain so that the 5.31 MeV alpha is about midscale in the MCA and accumulate a spectrum. Acquire the spectrum long enough for the sum under the peak ($\sum\alpha$) to be equal to ~2000 counts. Determine $\sum\alpha$.

2. Calculate the activity of the source from the following expression:

$$\text{activity (alpher per s)} = \left(\frac{\sum\alpha}{t}\right)\left(\frac{4\pi s^2}{\pi r^2}\right) \quad (2)$$

where

s = distance from source to detector (4 cm in our example),

r = radius of the detector (cm),

t = time in seconds,

$\sum\alpha$ = sum under the alpha peak.

Since 1 μCi = 3.7 x 10^4 disintegrations/second, the answer from Eq. (2) can easily be converted to μCi's and compared with the actual source activity. (If it is not written on the source, the laboratory instructor will supply the activity of the source). Remember, the half-life of ^{210}Po is 138 days. If the instructor gives the activity of the source when it was made, a correction will have to be made for its present activity.

EXPERIMENT 4.5

Decay Ratios for ^{241}Am

Procedure

1. Clear the MCA and accumulate the ^{241}Am spectrum long enough to see a spectrum similar to that in Fig. 4.7.

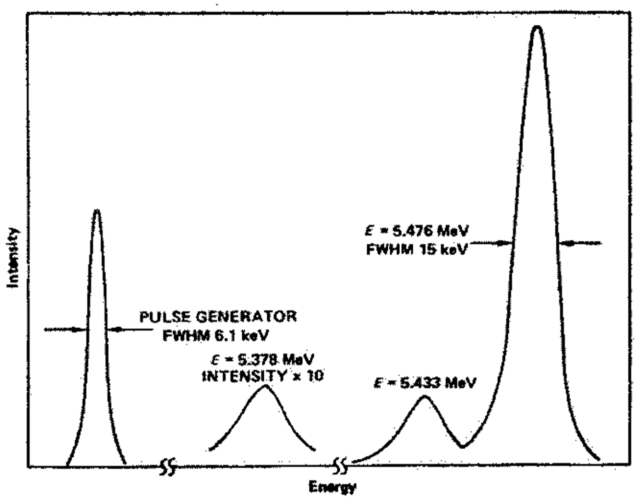


Fig. 4.7. ^{241}Am Alpha Spectrum.

2. From the MCA, determine the sum under the 5.476 MeV group. Call this the sum $\sum_2$ since it comes from alpha decay to the second excited state of ^{237}Np (Fig. 4.8). In the same manner, determine $\sum_4$ (5.433 MeV group) and $\sum_5$ (5.378 MeV).

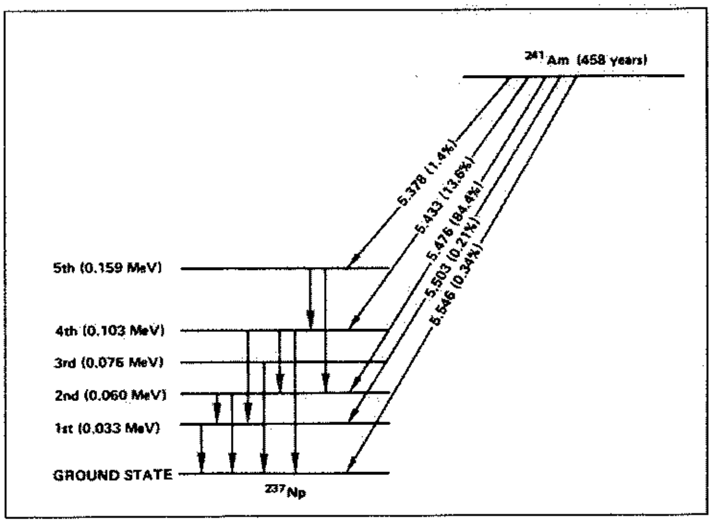


Fig. 4.8. Decay of ^{241}Am.

Rutherford Scattering of Alphas from Thin Gold Foil

EQUIPMENT NEEDED FROM EG&G ORTEC

142B Preamplifier	Oscilloscope
Bin and Power Supply	1 mCi ^{241}Am source
428 Detector Bias Supply	Source Kit SK-1A
480 Pulser	Target Kit M15
575A Amplifier	307 Rutherford Scattering Chamber
Surface Barrier Detector R-024-450-100	ORC-15 Cable Set

ACE-2K MCA System including suitable IBM PC (other EG&G ORTEC MCAs may be used)

Purpose

In this experiment the effect of gold foil for scattering alpha particles will be measured, and the results will be interpreted as experimental cross sections which will be compared with theoretical related expressions.

Introduction

No experiment in the history of nuclear physics has had a more profound impact than the Rutherford elastic scattering experiment. It was Rutherford's early calculations based on the elastic scattering measurements of Geiger and Marsden that gave us our first correct model of the atom. Prior to Rutherford's work, it was assumed that atoms were solid spherical volumes of protons and that electrons intermingled in a more or less random fashion. This model was proposed by Thomson and seemed to be better than most other atomic models at that time.

Geiger and Marsden made some early experimental measurements of alpha-particle scattering from very thin hammered-metal foils. They found that the number of alphas that scatter as a function of angle is peaked very strongly in the forward direction. However, these workers also found an appreciable number of scattering events occurring at angles $>90°$. Rutherford's surprise at this is this statement from one of his last lectures: "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a fifteen-inch shell at a piece of tissue paper and it came back and hit you."

Rutherford tried to analyze this angular dependence in terms of the atomic model that had been proposed by Thomson, but he observed that the Thomson model could not explain the relatively large back-angle cross section that had been found experimentally. Measurements by Geiger and Marsden revealed that 1 out of 8000 alpha particles incident on platinum foil experienced a deflection $>90°$. This was in conflict with calculations based on the Thomson model which predicted that only 1 alpha in 10^{14} would suffer such a deflection.

With intense effort, coupled with his unusual physical insight, Rutherford proposed the nuclear model of the atom. His calculations, based on Coulomb scattering from the proposed hard central core of charge, produced the required 10^{10} increase in cross section found by Geiger and Marsden. Of course, the cross section was very difficult to determine experimentally with the equipment available to these workers (an evacuated chamber with a movable microscope focused on a scintillating zinc sulfide screen). It was only through very careful and tedious measurements that the angular distribution was experimentally determined.

The term "cross section" mentioned above is a measure of the probability for the scattering reaction at a given angle. From a dimensional standpoint, cross section is expressed by units of area. This seems reasonable since the relative probability of an alpha striking a gold nucleus is proportional to the effective area of the nucleus. Cross sections are usually expressed in units called "barns," where one barn is 1×10^{-24} cm^2. This is a very small effective area but is not unreasonable when one considers the size of the nucleus in comparison to the size of the atom.

For a Rutherford scattering experiment it is most convenient to express the results in terms of cross section per solid angle. The solid angle referred to is the solid angle that the detector makes with respect to the target and is measured in steradians, (sr). The solid angle, $(\Delta\omega)$, in steradians is simply A/R^2, where A is the area of the detector and R is the distance of separation between the detector and the target. The measurement of cross section is expressed in barns/steradian or more conveniently millibarns/steradian. The cross section defined here is referred to as the differential cross section, and it represents the probability per unit solid angle that an alpha will be scattered at a given angle θ. The theoretical expression for the Rutherford elastic scattering cross section can be simplified to the following formula:

$$\frac{d\sigma}{d\Omega} = 1.296\left(\frac{\text{mb}}{\text{sr}}\right)\left(\frac{Z_1 Z_2}{E_\alpha}\right)^2 \left[\csc^4\left(\frac{\theta}{2}\right) - 2\left(\frac{M}{A}\right)^2\right] \quad (1)$$

where Z_1 and Z_2 are the atomic numbers of the projectile and target, E is the energy of the projectile in MeV, M is the mass

Calculate n_0, the number of gold target nuclei per cm^2 from the following formula:

$$n_0 = \frac{(\text{g/cm}^2 \text{ of the target})\ 6.023 \times 10^{23}}{197} \tag{4}$$

The value of the term n_0 will be used at a later time.

d. Calculate $\Delta\Omega$ from the formula

$$\Delta\Omega = \frac{\text{area of detector (cm}^2)}{R^2} \tag{5}$$

where R is the distance (in cm) from the detector to the gold foil.

---

5. Remove the gold foil and check the alignment of the apparatus by measuring the counting rates for the values in Table 15.2.

6. Plot the data in Table 15.2. If the instrument is properly aligned, the peak should be centered about zero degrees.

Table 15.2

Angle (deg)	Counts/m	Angle (deg)	Counts/m
0		0	
1		−1	
2		−2	
3		−3	
4		−4	
5		−5	
6		−6	
7		−7	

7. The number of alphas per unit time, (I_0), that impinge on the foil can be calculated from the following expression:

$$I_0 = \frac{(\text{activity of the source})\ (\text{area of the foil*})}{4\pi R_1^2} \tag{6}$$

(See Fig. 15.2) The activity of the source can be determined by the methods outlined in Experiment 4.

8. You are now ready to measure the cross section. Replace the gold foil. Set the detector at 10° and count for a period of time long enough to get good statistics in the peak. Calculate the counting rate, (I), at 10°. Repeat for all of the values listed in Table 15.1. It should be obvious from the theoretical cross section that the counting time will have to be increased as θ increases. You should try to get at least 15% statistics for all points.

*Area of the foil projected perpendicular to the source which is not shadowed by the collimator.

EXERCISE

e. Calculate the experimental cross section for each of the points in step **8** by using the following formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{I}{I_0 \Delta\Omega n_0}\right)\left(\frac{\text{cm}^2}{\text{sr}}\right). \tag{7}$$

Since 1 barn = 10^{-24} cm^2, the values calculated from Eq. (7) can be converted to millibarns per steradian and entered as $d\sigma/d\Omega$ (Experimental) in Table 15.1.

EXPERIMENT 15.2

The Z_2^2 Dependence of the Rutherford Cross Section

In this experiment alpha particles will be scattered from different foils to show the Z_2^2 dependence in Eq. (2). The foils used (which are included in the foil kit for this experiment) are aluminum ($Z_2 = 13$), nickel ($Z_2 = 28$), copper ($Z_2 = 29$), silver ($Z_2 = 47$), and gold ($Z_2 = 79$). The student will then plot this Z_2 dependence and show that it does agree with the theory.

Procedure

1. Repeat procedures 1 through 4 in Experiment 15.1 for each of the foils. For each foil calculate n_0 as in Experiment 15.1, Exercise **c**.

2. For each foil set $\theta = 45°$ and accumulate a pulse height spectrum for a period of time long enough to get at least 1000 counts in the scattered peak. Determine I (the number of scattered alphas per second) for each sample.

EXERCISE

Plot I as a function of $n_0 Z_2^2$ for each sample. The curve should be a straight line. The slope of the line can be determined by equating the two expressions for cross section Eqs. (2) and (7) and solving for the product $n_0 Z_2^2$.

Therefore

$$1.296\left(\frac{Z_1 Z_2}{E_\alpha}\right)^2 \csc^4\left(\frac{\theta}{2}\right) \times 10^{-27}\frac{\text{cm}^2}{\text{sr}} = \frac{I}{I_0 \Delta\Omega n_0}\ \text{cm}^2/\text{sr}, \tag{8}$$

and hence

$$I = \left[\frac{1.296\ Z_1^2 \csc^4\left(\frac{\theta}{2}\right) I_0 \Delta\Omega \times 10^{-27}}{E_\alpha^2}\right] n_0 Z_2^2. \tag{9}$$

Since every term inside the brackets is a constant for all foils:

$$I = K n_0 Z_2^2.$$

Therefore our experimental intensity should plot as a straight line in this exercise. The slope of the curve K is the value that is calculated from Eq. (9).
