Solutions: Three Practice Problems for ~Midterm Material for Practice for the Final

Physics 611

1. In a one-dimensional problem, consider a particle subject to potential energy $V(x) = -fx$, where f is a positive constant. For what physical problems might this potential be relevant?

(a) **Ehrenfest's theorem:** Determine the time derivatives of the expectation values of the position x and the momentum p of the particle.

(b) Integrate the equations you obtain part (a); compare with the classical motion.

(c) Show that $\langle(\Delta p^2)\rangle = \langle p^2\rangle - \langle p\rangle^2$ does not vary over time. Useful relation: $[AB, C] = A[B, C] + [A, C]B$.

Solution (partial):

a) use the Heisenberg EOM:

$<dx/dt> = d<x>/dt = < 1/i\hbar\ [x,H] > = < 1/i\hbar\ [x,p^2/2m] > = <p>/m$

$d<p>/dt = < 1/i\hbar\ [p,H] > = 1/i\hbar\ [p,V(x)] > = <(1/i\hbar)(-i\hbar\, dV(x)/dx) = f$ (V(x) = -fx)

b) $d<p> = f\, dt$ integrate $<p> = ft + C$; C = p(0). insert into $\int d<x> = \int <p>/m\, dt$

c) find $<p^2>(t)$ (expectation value as a function of time). Same way:

$d<p^2>/dt = <1/i\hbar\ [p^2,H] > = <1/i\hbar\ [p^2,V(x)] > = <1/i\hbar\ [pp,V] > = <1/ih\ (p[p,V] + [p,V]p) > = 2f<p>$

$d<\Delta p^2>/dt = d<p^2>/dt - d<p>^2/dt = 2f<p> - 2<p>d<p>/dt$ (chain rule) $= 2f<p> - 2f<p> = 0$ d/dt = 0 so no change

2.

Two Hermitian operators anticommute:

$$\{A, B\} = AB + BA = 0\,.$$

Is it possible to have a simultaneous (that is, common) eigenket of A and B? Prove or illustrate your assertion. Hint: Examining $\langle a''|\{A,B\}|a'\rangle$ will be helpful.

The most important part of this problem is setting up a familiar looking relation and evaluating it as we've done several times in class (following the hint!)

$<a|\{A,B\}|a'> = <a|AB+BA|a'> = (a+a')<a|B|a'> = 0$ Examine the last relation under various conditions. Note that if a,a' both > 0 it means $<a|B|a'>$ must be 0. Note that when proving commuting operators have common eigenkets, we said this proved that B must be diagonal too. Same here, which might lead us to think they do have common eigenkets, but when we examine the diagonal terms, we find that these can't be non-zero because if |a> = |a'> are eigenkets we get eigenval*<a|a> ≠ 0 unless eigenval is 0. So either all diagonal terms (eigenvals) are 0, in which case we have the trivial null operator B (which always commutes and anticommutes) , we can't have simultaneous. If either A,B eigenvalue is 0 for some subset of eigenkets, A,B can share these eigenkets.

3.

The observable A has eigenstates $|1\rangle$ and $|2\rangle$ and the hamiltonian operator is $H = C\,(|1\rangle\langle 2| \;+\; |2\rangle\langle 1|)$, where C is a constant.
(a) Derive the energy eigenstates and their eigenvalues.
(b) For a system in state $|1\rangle$ at $t = 0$, find the state vector (in Schrödinger picture) for $t > 0$ and the corresponding probability for it to be in state $|2\rangle$.
(c) What physical situation can this describe? What is then A, H and C ?

a) and c) Notice that matrix rep of H is $\begin{pmatrix} 0 & C \\ C & 0 \end{pmatrix}$ = $C\sigma_1$ -- like spin or sigma matrix, and 1,2, like +,-. Find eigenvectors traditional way (using matrix, generally this must be done for arbitrary operators) or remember Sx ∝ σ_1 eigenvectors ∝ |+> ± |-> → |1> ± |2> (normalized) w/ eigenvals = ± $\hbar$ /2

b) Use Shrodinger picture express |α, t=0> = |1> in Hamiltonian basis |1> ∝ |"Sx +"> + |"Sx –"> then evolve (operate) w/ U(t) = exp(iHt/$\hbar$) → meaning insert factor exp(-iE_nt/$\hbar$) factor for each state in the expansion→ |α ,t>

Probabilty for |2> will be $|c_2(t)|^2 = 1 - |c_1(t)|^2$ use previous expression (just take complex conjugate * answer |1>(t)))