RINGS WHOSE CYCLIC MODULES HAVE CERTAIN
PROPERTIES AND THE DUALS

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The object of this article is to give a survey of the results for rings whose cyclic modules have certain properties, like, injectivity, quasi-injectivity, continuity, π-injectivity, rational completeness, projectivity and quasi-projectivity. The dual results for rings whose one-sided ideals are quasi-injective or quasi-projective are also given and open questions are stated. The contributors in this theory include Faith, Osofsky, Levy, Singh, Koehler, Mohamed, Ivanov, Hill, Courter, Klatt, Jain and others. We assume throughout that all rings have unity and all modules are right and unital.

In her dissertation Osofsky [36] proved the first important result in this direction for rings all of whose cyclic modules are injective.

<u>Theorem</u> (Osofsky [37]). If every cyclic R-module is injective then R is semisimple artinian.

Later she gave a simplified proof of the above theorem [38]. She claims that this result was also proved by Skornjakov [40] but that his proof had an error.

In his Ph.D. dissertation Cozzens [7] produced an example of a noetherian domain R in which every cyclic R-module is either free or

semisimple. Following the example of Cozzens, Boyle ([3], [4]) con-
sidered rings in which proper cyclic modules are injective. She called
these rings right PCI-rings and showed that noetherian right PCI-rings
are either semisimple artinian or simple hereditary domains. Faith
considered right PCI-rings without chain conditions and proved
Theorem (Faith [8]). A right PCI ring R is either semisimple, or a
right semihereditary simple domain.

How close right PCI-domains come to being hereditary rings is an
open question?

The example of Cozzens and the study of right PCI-rings led Goel,
Jain and Singh to consider rings R for which every cyclic R-module
is injective or projective. They proved the following theorem which,
in particular, provides a simple proof of Faith's result that regular
right PCI-ring is semisimple artinian.

Theorem (Goel-Jain-Singh [14]). If each cyclic R-module is injective or
projective then R = A $\oplus$ B where A is semisimple artinian and B is a
simple right semihereditary right Öre-domain whose every proper cyclic
right module is injective.

Faith's interest in the study of right PCI-rings stems from the
well-known cyclic decomposition of a module over a dedekind domain.
One way to obtain the cyclic decomposition is to observe that for a
dedekind domain R every proper cyclic module R/A is injective as
R/A-module. Thus Faith [8] proposed to study the class of rings R with
the condition:

(F)   Every cyclic module C $\neq$ R is injective modulo its annihilator ideal.

But instead he studied right PCI-rings R which do satisfy the
condition (F). Commutative rings with the condition (F) are precisely
the pre-self-injective rings characterised by Klatt and Levy. They

have shown

Theorem (Klatt-Levy [26]). A commutative ring R is pre-self-injective
if and only if it is one of the following.

(1)   An integral domain (necessarily a Prüfer domain) in which,
for every maximal ideal M, R_M is an almost maximal rank 1 valuation
domain; and every proper ideal is contained in only finitely many
maximal ideals.

(2)   The direct sum of a finite number of maximal rank 0 valua-
tion rings. (Here R is also self-injective).

(3)   An almost maximal rank 0 valuation ring.

(4)   A local ring whose maximal ideal M has composition
length 2 and satisfies $M^2 = 0$. (Here R is not self-injective.)

Right self-injective rings with the condition (F) have the
property that each cyclic R-module is quasi-injective. These rings
are studied by Ahsan [1] and are called qc-rings. Koehler has obtained
a complete characterisation of qc-rings in the following theorem.

Theorem (Koehler [29]). For a ring R the following are equivalent:

(i)   Each cyclic right R-module is quasi-injective.

(ii)   R = A ⊕ B where A is semisimple artinian and B is a
finite direct sum of rank 0 maximal valuation duo rings.

(iii)   Each cyclic left R-module is quasi-injective.

Thus, the above theorem proves

Theorem. Right self-injective rings with the condition (F) are pre-
cisely qc-rings.

Jain, Singh and Symonds call a ring R a right PCQI-ring if
each cyclic R-module C ≠ R is quasi-injective. Clearly rings with the

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(F)-condition are right PCQI-rings. The following are their main
results:

Theorem (Jain-Singh-Symonds [25]). A right PCQI-ring R is either
prime or semiperfect. Further, if R is non-prime, non-local, then R
is a right PCQI-ring iff either R is a qc-ring or R = $\begin{pmatrix} D & D \\ 0 & D \end{pmatrix}$ where
D is a division ring.

Theorem Jain-Singh-Symonds [25]). A local right PCQI-ring with maximal
ideal M is a right valuation ring or $M^2 = (0)$. In addition, if R
is prime then R is a valuation domain.

Theorem (Jain-Singh-Symonds [25]). A right PCQI-domain is a right
Öre-domain.

      The above result was proved by Faith [8] for PCI-domains. If
R is commutative then some of the results of Klatt and Levy [26]
follow as a special case of the above results for PCQI-rings.

      In an attempt to study rings R satisfying condition (F), Jain
and Singh have obtained the following partial solution.

Theorem (Jain-Singh [23]). Let R be a ring satisfying condition (F).
Then either R is a right Öre-domain or semiperfect. Further, a semi-
perfect ring R satisfies condition (F) iff R is a right PCQI-ring.

      So the above theorem leaves the question of characterising right
Öre-domains with condition (F) open.

      By assuming the ring R to be duo, Koehler has shown the
following:

Theorem (Koehler [30]). Let R be a duo domain. Then R satisfies
condition (F) (equivalently, R is a right PCQI-ring, or R is a right

pre-self-injective ring) iff

(1)   Every proper ideal is contained in only finitely many maximal ideals, and

(2)   For every maximal ideal M and every proper ideal A_M in the localization R_M, R_M/A_M is a maximal rank 0 duo valuation ring.

Analogous to continuous rings defined by Utumi [42], Mohamed and Bouhy [35] introduced the notion of a continuous module as a generalization of quasi-injective module. A module M is called continuous if it satisfies the following conditions:

(i)   Every submodule of M is essential in some direct summand of M.

(ii)   If a submodule A of M is isomorphic to a direct summand of M, then A itself is a direct summand of M.

A ring R is called a right cc-ring if every cyclic R-module is continuous. The following is a generalization of Koehler's theorem for qc-rings.

Theorem (Jain-Mohamed [21]). A semiperfect ring R is a right cc-ring iff R = A $\oplus$ B where A is semisimple artinian and B is a finite direct sum of right valuation right duo rings with nil radical.

Another generalization of Osofsky's theorem and Koehler's theorem is given by Goel and Jain. But first some terminology. Call an R-module M to be π-injective if for all R-modules A_1, A_2 of M with $A_1 \cap A_2 = (0)$, each projection $\pi_i: A_1 \oplus A_2 \to A_i$, i = 1,2 can be lifted to an endomorphism of M. Every quasi-injective module is π-injective but conversely π-injective modules need not be quasi-injective. For example, if R is a right Öre-domain which is not a division ring then R as a right R-module is π-injective but not quasi-injective. Some results

on π-injective modules are:

Theorem (Goel-Jain [15]). (i) M is π-injective iff M is (V,R)-submodule of $\hat{M}$, the injective hull of M, where V is the subring of the ring of endomorphisms of $\hat{M}$ generated by all the idempotents.

(ii) Let R be a right noetherian ring and M be a π-injective R-module. Then M is a finite direct sum of indecomposable π-injective modules.

(iii) Let M,N be R-modules such that M × N is π-injective. Then M is N-injective and N is M-injective.

(iv) A prime ring R with Z(R) = (0) is π-injective as an R-module iff R is self-injective or right Öre-domain.

The study of rings R over which each cyclic module is π-injective has been initiated by Goel and Jain [15]. The following theorems generalize the theorem of Osofsky [38] as well as that of Koehler [29]:

Theorem (Goel-Jain [15]). Let R be a right self-injective ring. Then each cyclic R-module is π-injective iff R = A $\oplus$ B where A is semisimple artinian and B is a finite direct sum of right self-injective right valuation rings.

Theorem (Goel-Jain [15]). Let R be a semiperfect ring. Then each cyclic R-module is π-injective iff R = A $\oplus$ B where A is semisimple artinian and B is a finite direct sum of right valuation rings.

Theorem (Goel-Jain [15]). Let R be a ring with right zero singular ideal such that each cyclic R-module is π-injective. Then R = A $\oplus$ B where A is semisimple artinian and B is a finite direct sum of right Öre-domains.

In this connection we ask: Is it true that a prime ring R
all of whose cyclic R-modules are π-injective must have zero singular
ideal? Since a right valuation ring, clearly, has the property that
each cyclic module is π-injective, an example of a prime right valua-
tion ring which is not a domain could settle this question in the
negative.

Findlay and Lambek ([10], [11]) calls an R-module M a rational
extension of N if N is a submodule of M and for each $x \in M$,
$0 \neq y \in M$ there exists $r \in R$ such that $xr \in N$ and $yr \neq 0$. It is known
that M is a rational extension of N iff for each submodule B of
M which contains N, each $f \in \hom(B,M)$ such that $f(N) = 0$ is the zero
mapping. A module M is rationally complete if M has no proper
rational extensions. They have shown

Theorem (Findlay-Lambek [11]). M is rationally complete if and only if
for each pair of R-modules A and B such that A is a submodule of
B and that for each $b \in B$ and for each $0 \neq m \in M$, there exists $r \in R$
such that $br \in A$ and $mr \neq 0$, each $f \in \hom(A,M)$ can be extended to a
$g \in \hom(B,M)$.

Thus, every injective module is rationally complete. Courter
has given the following characterisation for a ring R whose R-modules
are rationally complete.

Theorem (Courter [6]). Every R-module is rationally complete iff R
is a finite direct sum of rings R_i isomorphic to the $n_i \times n_i$ matrix ring
over a right and left perfect completely primary ring S_i.

Bland [2] has shown that every R-module is rationally complete
if and only if every R-module is quasi-rationally complete in the
following sense:

If N is a submodule of M such that (i) for each x ε M,
0 ≠ y ε M there exists r ε R such that xr ε N and yr ≠ 0 and (ii) M
is contained in the quasi-injective hull of N, then M is called a
quasi-rational extension of N. Further, a module M is called quasi-
rationally complete if M has no proper quasi-rational extensions.

It is an open problem to study the class of rings R each of
whose cyclic modules are rationally complete. Brown has proved some
results for rings over which every simple module is rationally complete.
Brown [5] has shown that the collection of rings over which every
simple module is rationally complete contains all finite direct sums of
matrix rings over duo rings.

Rings R for which a certain subclass of the cyclic R-modules,
simple R-modules, are injective or projective have been of interest to
various authors. For example, rings whose simple modules are injective,
due to Villamayor, are called V-rings and have been studied extensively.
We state below only two results. The reader is referred to Faith [9]
for results on V-rings.

Theorem (Villamayor). For a ring R the following are equivalent:

     (a) Each simple R-module is injective

     (b) Each right ideal is the intersection of maximal right
ideals

     (c) For all modules M, the radical of M = 0.

Theorem (Cozzens [7]). Let k be a universal differential field with
derivation D. The ring R = k[y,D] has the following properties:

     (a) R is a simple principal right and left ideal domain

     (b) R is a V-ring.

     (c) Up to isomorphism, R has a unique simple R-module.

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Rings whose simple module are injective or projective have been studied by Zaks [44], Fuller [12] and others.

Theorem (Fuller [12]). Let R be a right artinian ring with Jacobson radical N. Then the following are equivalent:

(a) Every submodule of a quasi-projective R-module is quasi-projective.

(b) Every factor ring of R is hereditary.

(c) Every factor module of a quasi-injective R-module is quasi-injective.

(d) R is hereditary and $N^2 = 0$.

(e) Every simple module is injective or projective.

It is remarked by Fuller that the conditions (a), (b), (d) and (e) are equivalent over a perfect ring R. It is not known whether (a) and (c) are equivalent over any larger class of rings.

Rings over which every cyclic module is projective are trivially semisimple artinian. The study of rings for which every cyclic module is quasi-projective was initiated by Koehler. She calls a ring R a right q*-ring if each cyclic R-module is quasi-projective. Some of the main results are:

Theorem (Koehler [27]). (i) A semiperfect ring R is a right q*-ring iff every right ideal in the Jacobson radical of R is an ideal.

(ii) Let R_n be the n × n matrix ring over a ring R with n > 1. Then R_n is a right q*-ring iff R is semisimple artinian.

(iii) Let R be a right self-injective semiperfect right q*-ring. Then R = A ⊕ B where A is semisimple artinian and $B = \oplus \sum_{i=1}^{n} e_i R$ is a finite direct sum of indecomposable right ideals $e_i R$ such that $e_i R \not\simeq e_j R$ if $i \neq j$.

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(iv) If R is a prime semiperfect q*-ring, then R is either simple artinian or R is local.

(v) Let R be a quasi-Frobenious ring. Then R is a right q*-ring iff R is a left q*-ring.

The study of rings with the dual property (rings in which every right ideal is quasi-injective) was begun by Jain, Mohamed and Singh. They call these rings with the above property right q-rings and have proved the following results.

<u>Theorem</u> (Jain-Mohamed-Singh [24]). (i) R is a right q-ring iff R is right self-injective and each essential right ideal is an ideal.

(ii) Let n > 1 be an integer. Then the n × n matrix ring R_n is a right q-ring iff R is semisimple artinian.

(iii) A prime ring R is a semisimple artinian iff R is a right q-ring.

(iv) A semi-prime ring R is a right q-ring iff R = A ⊕ B where A is semisimple artinian and B is a self-injective strongly regular ring.

Mohamed in his dissertation [32] studied various classes of q-rings. Some of his main results are:

<u>Theorem</u> (Mohamed [33]). (i) Let R be a semi local right q-ring. If eRe is not a division ring for any primitive idempotent e ε R, then R is a finite direct sum of local right self-injective duo rings.

(ii) Let R be a PF-ring (in the sense of Utumi [43]). If R is a q-ring then R = A ⊕ B where A is a quasi-Frobenious q-ring and B is a finite direct sum of local right self-injective duo rings with nonzero socle.

(iii) A right or left artinian right q-ring is also a left q-ring.

The next two theorems generalize Levy's [31] result for commutative noetherian rings whose homomorphic images are self-injective to the noncommutative case.

<u>Theorem</u> (Mohamed [34]). Let R be a nonprime right noetherian ring. Then every proper homomorphic image of R is a right q-ring if and only if

(a) $R = S \oplus T$ where S is semisimple artinian and T is a principal ideal duo ring with dcc, or

(b) If J denotes the Jacobson radical then R/J is artinian and J is a minimal ideal, or

(c) R is a local ring whose maximal right ideal M satisfies $M^2 = (0)$ and every proper homomorphic image of R contains at most one proper right (left) ideal.

<u>Theorem</u> (Mohamed [34]). Let R be a prime right noetherian ring with the property that every homomorphic image is a right q-ring. Then,

(i) Every ideal of R is a product of prime ideals, and

(ii) For every nonzero prime ideal of R, R/P is a division ring.

Hill [16] in his dissertation, has, among other results, characterized semiperfect q-rings.

<u>Theorem</u> (Hill [17]). Let R be semiperfect. If R is a right q-ring then R is a direct sum of a semisimple artinian ring and a right self-injective basic ring. Further, a right self-injective basic ring which is semiperfect need not be right q-ring.

Theorem (Hill [17]). Let R be an injective cogenerator as a right
R-module. Then R is a right q-ring iff R is a finite direct sum
of indecomposable rings R_i of the following three types.

   (a) R_i is simple artinian

   (b) R_i is local, and for each x $\in$ R_i, $xR_i = R_i x$.

   (c) R_i is basic, not local, and for each pair e,f of orthogonal
idempotents $eR_i f \subseteq$ right socle of R_i = left socle of R_i.

Corollary (Hill [17]). A right artinian ring is a right q-ring iff it
is a finite direct sum of indecomposable rings R_i of the above three
types (a), (b) or (c).

In his dissertation Ivanov [18] has determined the structure of
indecomposable non-local q-rings. He has shown that there are only two
types (both artinian) of indecomposable non-local q-rings and has
represented them as matrix rings, one of which is denoted by H(m,D,V)
and defined as follows: For any integer m $\geq$ 2, any division ring D,
and any null D-algebra V whose left and right D-dimensions are both
equal to one, H(m,D,V) denotes the ring of all m $\times$ m matrices whose only
non-zero entries are arbitrary elements of D along the diagonal, and
arbitrary elements of V at the places (2,1), ..., (m,m-1), (1,m).

Theorem (Ivanov [18]). Let R be an indecomposable non-local right
q-ring. Then R is artinian and is either simple artinian or is of
the form H(m,D,V). Conversely, every simple artinian ring or every
ring of the form H(m,D,V) is a q-ring.

Conjecture (Ivanov [19]). Every right q-ring is a direct sum of a
finite number of indecomposable non-local right q-rings and a right
q-ring all of whose idempotents are central.

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The conjecture is true iff every right ideal in a q-ring is contained in an ideal which is generated by a central idempotent and is minimal with respect to those properties iff the intersection of all right ideals which have (in the ring) a common homomorphic image disjoint from them is nonzero (c.f. [17]).

Some of the relationships between q-rings, q*-rings and qc-rings are given by Koehler.

Theorem (Koehler [29]). If R is a right qc-ring then R is a right q-ring and a right q*-ring.

Theorem (Koehler [27]). (i) If R is a semiperfect right q-ring then R is a left q*-ring.

(ii) If R is a left cogenerator and a left q-ring then R is a left q*-ring.

The next theorem proved by Koehler reproves a theorem of Mohamed that a right artinian right q-ring is a left q-ring.

Theorem (Koehler [27]). Let R be a quasi-Frobenious ring. Then the following statements are equivalent.

(i) R is a left q-ring.

(ii) R is a right q-ring.

(iii) R is a left q*-ring.

(iv) R is a right q*-ring.

We know that rings for which each right ideal is projective are hereditary rings, and left perfect rings are those for which each module has a projective cover. A class of commutative rings in which each ideal has a projective cover has been considered by Snider in the following theorem.

<u>Theorem</u> (Snider [41]). Let R be a commutative indecomposable ring.
Then each ideal of R has a projective cover iff R is either perfect,
dedekind or local noetherian.

It remains open to study other classes of rings with the property
that each right ideal has a projective cover.

Recall R is called a right qc-ring if each cyclic R-module is
quasi-injective. The class of rings with the dual property that each
right ideal is quasi-projective has been considered by Jain and Singh.
They have called a ring R a right qp-ring if each of its right ideals
is quasi-projective. Clearly, all right hereditary rings are right
qp-rings. However, the class of commutative principal ideal artinian
rings which are not direct sum of fields distinguishes qp-rings from
hereditary rings. The following results giving the structure of right
and left perfect right qp-rings R are obtained by Jain-Singh. N will
denote the Jacobson radical.

<u>Theorem</u> (Jain-Singh [22]). Let R be a local ring. Then R is a right
qp-ring iff N^2 = (0) or R is a principal right ideal ring with dcc
such that for each a ε R, ann a is an ideal in R.

<u>Theorem</u> (Jain-Singh [22]). If R is a perfect right qp-ring and T is
the sum of all those indecomposable right ideals of R which are not
projective then T is an ideal of R and N = T $\oplus$ L; L is a right
ideal of R such that every right subideal of L is projective, R/T
is hereditary, and R is hereditary iff T = (0).

<u>Theorem</u> (Jain-Singh [22]). If R is a perfect right qp-ring then
$R = \begin{pmatrix} S & M \\ 0 & T \end{pmatrix}$ where S is hereditary, T is a right qp-ring which is a
finite direct sum of local rings and M is (S,T)-bimodule such that

$_S M$ is projective. In particular, a perfect right qp-ring is semi-primary.

Question: Is M_T quasi-projective?

Theorem (Jain-Singh [22]). (i) Let R be an indecomposable ring such that it admits a faithful projective injective right module. Then R is a right qp-ring iff R is a local principal right ideal ring such that for each a ε R, ann a is an ideal or R is a right hereditary ring with dcc.

   (ii) Let R be an indecomposable quasi-Frobenious ring. Then R is a right qp-ring iff each homomorphic image of R is a q-ring.

Theorem (Jain-Singh [22]). If a right ideal A of a right qp-ring is not projective then the projective dimension of A is infinite. Thus gl. dim. R = 0, 1, or ∞.

   It has been shown by Goel-Jain [13] and Singh-Mohammad [39] independently that all the above results are true for right perfect right qp-rings. The study of right qp-rings other than for perfect rings remains open. Indeed this leads to another important problem, namely, what is the structure of quasi-projective modules over some other class of rings? In particular, quasi-projective modules over local rings have not been studied. Finitely generated quasi-projective modules over semiperfect rings have been characterized by Koehler [28].

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