MATH3200: APPLIED LINEAR ALGEBRA
COURSE DESCRIPTION

WINFRIED JUST, OHIO UNIVERSITY

1. What is applied linear algebra?

This is a course on linear algebra with an emphasis on applications and computations.

Linear algebra is the branch of mathematics that studies linear equations such as $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ and linear transformations such as the function that maps a vector $[x_1, \ldots, x_n]$ to the number $a_1x_1 + \ldots + a_nx_n$. These mathematical objects are studied in terms of matrices and matrix operations.

Together with calculus, differential equations, statistics, and probability theory, linear algebra provides the most important tools for modeling and computation in science and engineering.

2. What are you going to learn in this course?

A lot. The official course description mentions solutions to linear systems, matrices and matrix algebra, determinants, n-dimensional real vector spaces and subspaces, bases and dimension, eigenvalues and eigenvectors, diagonalization, norms, inner product spaces, orthogonality and least squares problems.

Do all of these words make sense to you? Probably not. More precisely: "Probably not yet." These words refer to important *concepts* that you will learn in this course. A large part of our work will consist of precisely defining and trying to understand these concepts. We will then have a common language for talking about the computations that make linear algebra such a powerful tool for applications.

Another large part of our work will consist of learning how to perform these computations step-by-step. In most of your professional life you will want to enlist a computer for doing the required computations. We also will practice this, to some extent, by letting the computer algebra system MatLab perform some of these calculations and discussing what to make of its output, especially if it isn't at all what we expect.

Computers are very good at doing calculations, but need to be told what, exactly, it is they are supposed to calculate. We will present in some depth applications of linear algebra to a few selected domains and discuss which of the computational procedures that you will learn are appropriate for which particular problems from these domains. Our knowledge of the relevant concepts will guide us in making these decisions.

Often it is not immediately clear whether a particular calculation will give the intended result. If you want to make sure the calculation will *always* work as expected, you often can perform it with symbols instead of numbers and check whether it works out in general. This is essentially what *mathematical proofs* do. We will practice writing this kind of simple mathematical proofs. Don't be afraid of them; think of them as calculations with symbols that can give you confidence in the correctness of your calculations with particular numbers.

Let us rephrase the above in a nice list of bullet points:

Learning objectives: Students in this course will

- become familiar with the meaning and use of the main concepts of linear algebra,
- become competent in performing standard computational procedures of linear algebra, both with pencil and paper and with the computer algebra system MatLab,
- become familiar with translations of real-world problems from selected domains into formal linear algebra problems, and
- become competent in reading and writing elementary proofs in linear algebra.

3. How are we going to learn all of this?

Why isn't the section titled: "How is this course going to teach you all of this?"
Because all a textbook, instructor, or tutor can do for you is to help you learn. Mathematics is not a spectator sport. You can only learn by doing it. By trying to do it, anyway. By usually getting it wrong at first. By asking questions, getting feedback, then trying again, still getting it partially wrong, getting more feedback, trying again, ... until all the pieces fall into place and beautifully fit together. You do have to go through this process. There is no shortcut.
This course is based on *active learning,* which in numerous educational studies has been shown to lead to better and deeper understanding than the passive kind. To see how and why active learning works, ask yourself the following question: Have you ever encountered a situation in a math course where you seemed to be able to follow all the instructor's explanations and work presented in class, but then your mind seemingly went blank on the test, and you just could not solve the problems? If you are among the lucky few who have never encountered this problem, relax and enjoy! You will do fine in this and other math courses regardless of the method of instruction. If you did encounter such situations, then it is likely that your instructor's explanations were clear enough to create the *illusion of understanding* without forcing you to work through all the potential sources of confusion. It was then only during the test that you were confronted with these sources of confusion. This happens a lot when the course is based on passive learning.
Active learning essentially requires you to engage with the material right when it is presented in such a way that under the guidance of your instructor you will work through these potential sources of confusion and sort things out. By the time the tests and final come around, you will have built up the understanding and confidence to succeed with the problems. In this way, active learning does ultimately improve student's performance and mastery of the material. However, by aiming at not giving you a false illusion of understanding, by aiming at sorting out potential confusion right away, active learning will at times put you into a state of confusion during class before everything can be clarified and sorted out. Thus active learning may not feel as good as passively absorbing neat explanations that are handed down by the instructor or the textbook. It does require hard work. And, unfortunately, during the process of learning new material via Top Hat you should expect to lose some points. Treat these as a small price you pay for getting well prepared for the tests and for the final. The grading scheme has enough built-in slack that allows you to make up for some points lost in the Top Hat component of your overall score.

Each topic will be covered in several parts:

- Course notes for each chapter. These will be posted online, but not covered in class. They give you, in fairly condensed form, the definitions, theorems, and descriptions of computational procedures covered in this course, together with some worked-out examples. Think of the course notes as your textbook. One key for success in this course is to read the relevant part of the course notes *before* the material is presented in class.
- Fairly traditional lectures on key concepts, computational procedures, and applications.
- Conversations.

- Modules with review exercises, graded homework exercises, and sometimes additional material for self-study.

The lectures contain some **Questions** to the audience that will be graded in part for participation and in part for correctness. It is important that you think about these questions and answer them before the lecture continues so as to stay engaged with the flow of the presentation.
The feature that makes this text unique are the conversations. Think of our protagonists Alice, Bob, Cindy, Denny, Frank, and Theo as students like yourself and your classmates who are trying to learn this material. They get it sometimes wrong, sometimes right, and give each other feedback. Almost like in a group work exercise. And you will be asked to join their discussions by providing answers to each **Question** that is embedded in the conversation before the dialogue continues.
The key to success is that you practice regularly what you learned in class. The modules give you close guidance, with some of their questions assigned for review, and some as graded homework. A number of these modules will teach you how to use the computer algebra system MatLab for linear algebra problems. Some of them contain material for self-study that will not directly be covered in class, but is also required knowledge for this course and may be used later in the classroom presentations.

4. Course outline

To make the structure of the material more transparent, it has been organized into chapters as outlined below. However, the order of presentation of the material may somewhat differ from this numbering and partition into chapters. Some parts that thematically belong to an earlier chapter will be presented at a later point in the course. The purpose of the latter is to create opportunities for reviewing earlier material and refreshing the memory of it.

- Chapter 1: Matrices and Matrix Operations
  - Covers the basic terminology for talking about matrices and vectors, and the definitions of sums, differences, and products of matrices. It also introduces several special kinds of matrices, such as symmetric, zero, identity, triangular, and diagonal matrices.
- Chapter 2: Systems of Linear Equations
  - Covers linear systems, the nature of their solution sets, their matrix representations, and introduces the inverse of a square matrix. You will learn methods for solving systems of linear equations, most notably, Gaussian elimination.
- Chapter 3: Key Concepts of Linear Algebra
  - Covers the conceptual framework of linear algebra: linear combinations, linear (in)dependence, linear span, vector spaces, rank and null space of a matrix, basis and dimension of a vector space, linear transformations. The emphasis will be on studying distinctions and connections between these concepts.
- Chapter 4: Important Tools of Linear Algebra
  - Covers determinants, eigenvectors, eigenvalues, and diagonalizations of square matrices.
- Chapter 5: The Geometry of Vectors
  - Covers vector norms, inner products, orthogonality, orthonormal basis, and least squares problems.