Conversation 11: Translating Word Problems into Linear Systems, Part II

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MATH 3200: Applied Linear Algebra

Back to Module 23

Bob: Let's do the next one from the same module, Question 23.2:

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: First we need to decide on our variables, right?

Denny: Yeah, but this one's a toughy. How many variables do we need, Frank?

Frank: A lot! Don't ask me. Let's skip 23.2 and do Question 23.3. That one looks easier.

Cindy: Let's give 23.2 a try. We may be able to figure it out. As Alice said: Step by step.

Frank and Denny: Then you do it, Cindy!

The first variable for this system

Bob: Go for it, Cindy! Here is the question:

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: Seems we need a variable for how much Cody spends on beverages. Can we call it c_b?

Theo: Yes. As I told you.

Cindy: Let c_b be the number of dollars that Cody spends on beverages.

Denny: Smart! So we wouldn't ever need to write $\$c_b$ later on and can remember what this variable stands for.

More variables for this system

Cindy: Let's keep the text handy for looking up the info:

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: Next let c_f be the number of dollars that Cody spends on food.

Frank: You forgot about Dan.

Cindy: Don't rush me please! We want to translate the info step by step, one piece at a time. Now let:

d_b be the number of dollars that Dan spends on beverages, and let

d_f be the number of dollars that Dan spends on food.

Denny: You got the variables down, Cindy, now do the equations!

Yet more variables for this system?

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: Wait! It say's "Overall" here. I don't know: Do I need some variables for "overall"?

Bob: Maybe. But maybe not. Difficult to tell at this point.

Question C11.5: Should Cindy introduce variables for the overall amounts of money spent?

Cindy: So to be on the safe side, let

c_o be the total number of dollars that Cody spends, and let

d_o be the total number of dollars that Dan spends.

Denny: But as Bob said, you may not even need these variables. Why go to the trouble of defining them?

Better safe than sorry

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: If it turns out that we don't need them, can't we then simply not use them? But if I do need them, I may get stuck somewhere. I don't like getting stuck.

Alice: Excellent strategy, Cindy! Better safe than sorry. First introduce all the variables you might need. This gives you a language into which you can translate the equations. Then translate the information into equations in the way that you find most convenient, and finally check whether you really needed all the variables, or whether you can get rid of some of them by simplifying your system of equations.

Translating info into equations

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: Now let's look at the info one piece at a time.
"Cody spends twice as much on beverages as Dan does."
That would be $c_b = 2d_b$.
Or as Theo wants us to put it: $c_b - 2d_b = 0$.

Theo: Precisely!

Cindy: " ... only one third as much as Dan on food."
That's what Cody spends. $c_f = \frac{1}{3}d_f$.
Or as Theo wants us to put it: $c_f - \frac{1}{3}d_f = 0$.

Denny: This is really easy with your nice variable names, Cindy!

Frank: Yeah. And so on. Let's skip the rest and do Question 23.3.

Translating info into equations, continued

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Denny: Don't rush her, Frank! I want to know how much Cody spent on those beverages. Please continue, Cindy!

Cindy: "Overall, Cody ends up spending 50% more money than Dan." That would be $c_o = 1.5d_o$.
Or as Theo wants us to put it: $c_o - 1.5d_o = 0$.

Denny: Now we are done!

Theo: Not yet. What does "overall" really mean?

Denny: Don't you understand plain English, Theo?

Theo: So what does "overall" mean here?

Denny: Don't play dumb, Theo! It means the total they spend on both food and beverages.

Translating all assumptions

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Cindy: I think Theo wants us to write two more equations:

$c_o = c_b + c_f$ and $d_o = d_b + d_f$. Or rather:

$c_b + c_f - c_o = 0,$

$d_b + d_f - d_o = 0.$

Theo: Neat! If Cody and Dan also bought some math textbooks, the meaning of "overall" would be different though.

Denny: Don't be so nitpicking, Theo!

Bob: Theo's point is that we always need to make sure we correctly interpret all the assumptions. We cannot read the mind of the author of the question.

Cindy: But here it says: *"Cody and Dan go shopping for food and beverages."* My equations translate this info too.

Translating all assumptions

"Cody and Dan go shopping for food and beverages. Cody spends twice as much on beverages as Dan does and only one third as much as Dan on food. Overall, Cody ends up spending 50% more money than Dan."

Alice: Excellent observation, Cindy! Always make sure to translate all pieces of information.

Question C11.6: Has Cindy now translated all the info?

Alice: Yes! Great job, Cindy! We obtained the following linear system:

$$\begin{array}{rcrcrcl} & & c_b & - & 2d_b & = & 0 \\ & & c_f & - & \frac{1}{3}d_f & = & 0 \\ & & c_o & - & 1.5d_o & = & 0 \\ c_b & + & c_f & - & c_o & = & 0 \\ d_b & + & d_f & - & d_o & = & 0 \end{array}$$

Denny: And this one is homogenous, even by Theo's standards!

Can we simplify this system?

We obtained the following system:

$$\begin{array}{rcrcrcl} & & c_b & - & 2d_b & = & 0 \\ & & c_f & - & \frac{1}{3}d_f & = & 0 \\ & & c_o & - & 1.5d_o & = & 0 \\ c_b & + & c_f & - & c_o & = & 0 \\ d_b & + & d_f & - & d_o & = & 0 \end{array}$$

Frank: This system is way too complicated!

Alice: We will discuss some other time how to simplify it.

Denny: But I want to know how much Cody spent on those beverages!

Alice: We will need the method of substitutions, which is best discussed in the context of solving systems.

Bob: This wasn't covered yet in the course.

Denny: Ok. Lets take a break and go party!

Take-home message

This conversation illustrates a number of useful strategies for translating word problems into systems of linear equations.

- Read the word problem closely. Keep a copy of the text handy and read it closely, several times, while working on the translation.
- Always start by setting up your variables. This is the key step.
  - Write down what each variable represents, including the units of the respective quantity.
  - It is helpful to use suggestive names for your variables.
  - Specify variables for all quantities that you might need.

Take-home message, completed

- Break up the text into small pieces of info and translate one such piece at a time.
- Don't rush! After each step, double-check whether your translation is correct. If in doubt, you can test on a numerical example whether it makes sense.
- You want to eventually write each of your equations in the form $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i$.

  But you don't need to do so right away. You can first express the info in a more convenient way, and then transform your equation into the standard format.
- You can translate the individual pieces of info in any order, but be sure to translate all of them.
- Be sure to also translate info that is implicit in common usage of English words or phrases and that relates your variables to each other.
- At the end, double-check whether you have correctly translated all of the given info.