Lecture 1: Matrices

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MATH3200: Applied Linear Algebra

The general form of a matrix

Definition

A *matrix* is a rectangular array of *elements* arranged in horizontal *rows* and vertical *columns.*

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

Question L1.1: According to this definition, do the symbols a_{ij} always stand for numbers?

Not necessarily. While some textbooks allow only numbers as *elements* or *entries* of a matrix, our the definition allows arbitrary elements.
In particular, the entries can be matrices themselves.

Most of the time we will work with matrices whose elements are numbers.
Such a restriction on the entries will usually be **implied by the context.**

The order of a matrix

Consider the following examples of matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & \pi & 0 \\ -6 & 13.8 & 84 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ -6 & -8 \\ 3-2i & 3+2i \end{bmatrix}$$

The *order* aka *size* aka *shape* of the matrix **A** shown here is $m \times n$, where m is the number of *rows* of **A** and n is the number of *columns* of **A**.

Question L1.2: Is the order of the matrices **A** and **B** equal to 6?

No. The order is not a number, but an expression involving two positive integers and the symbol $\times$.

The order of the matrix **A** is 2×3,
while the order of the matrix **B** is 3×2.

The numbers m and n are sometimes called the *dimensions* of the matrix.

Example: Spreadsheets are matrices

During this semester, your instructor will record the grades of all students in this class in a matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

Here m is the number of students in the class.

The entries $a_{11}, a_{21}, \dots, a_{m1}$ in the first column will normally be the names of the students, but let us assume here that your instructor remembers each student by his or her number rather than name and does not record the names.

Then n will be the number of gradable items and the entry a_{ij} represents the score of student i on gradable item number j.

Calculating grades

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

Assume m is the number of students in the class, n is the number of gradable items, and the entry a_{ij} represents the score of student i on gradable item number j.

Question L1.3: How does one calculate the score of a given student from this matrix?

The score of student i is the sum of entries in row number i.

Calculating grades, continued

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

Assume m is the number of students in the class, n is the number of gradable items, and the entry a_{ij} represents the score of student i on gradable item number j.

Question L1.4: How does one calculate the mean score of a given gradable item?

That depends on what, exactly, we want the phrase "mean score gradable item j" to signify.

It could be either the sum of entries in column number j, divided by the total number m of students, or the sum of entries in column number j, divided by the number of students who actually took this test or quiz.

Summary

- A *matrix* is a rectangular array of *elements* arranged in horizontal *rows* and vertical *columns.*
- The elements aka *entries* of a matrix are usually numbers, but we also allow them to be other objects. It will usually be implied by the context whether or not the elements of a matrix are assumed to be numbers.
- The *order* aka *size* aka *shape* of a matrix $\mathbf{A}$ is an expression of the form $m \times n$, where m is the number of *rows* of $\mathbf{A}$ and n is the number of *columns* of $\mathbf{A}$. The numbers m and n are sometimes called the *dimensions* of the matrix.
- Spreadsheets are examples of matrices.