Lecture 6: Submatrices

Winfried Just
Department of Mathematics, Ohio University

MATH3200: Applied Linear Algebra

Submatrices

A *submatrix* **B** of a given matrix **A** is any matrix that can be obtained by removing some rows and/or columns from **A**. The rows and columns that remain do *not* need to be adjacent in **A**.

$$\text{Consider} \qquad \mathbf{A} = \begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ 21 & 22 & 23 & 24 & 25 \\ 31 & 32 & 33 & 34 & 35 \\ 41 & 42 & 43 & 44 & 45 \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

Question L6.1: Is $\mathbf{B}_1$ below a submatrix of **A**?

$$\mathbf{B}_1 = \begin{bmatrix} 11 & 13 & 14 \\ 21 & 23 & 24 \\ 31 & 33 & 34 \\ 41 & 43 & 44 \\ 51 & 53 & 54 \end{bmatrix}$$

Yes. Remove from **A**:

$$\begin{bmatrix} 11 & \blacksquare & 13 & 14 & \blacksquare \\ 21 & \blacksquare & 23 & 24 & \blacksquare \\ 31 & \blacksquare & 33 & 34 & \blacksquare \\ 41 & \blacksquare & 43 & 44 & \blacksquare \\ 51 & \blacksquare & 53 & 54 & \blacksquare \end{bmatrix}$$

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$$\text{Consider} \qquad \mathbf{A} = \begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ 21 & 22 & 23 & 24 & 25 \\ 31 & 32 & 33 & 34 & 35 \\ 41 & 42 & 43 & 44 & 45 \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

Question L6.2: Is $\mathbf{B}_2$ below a submatrix of **A**?

$$\mathbf{B}_2 = \begin{bmatrix} 21 & 22 & 25 \\ 31 & 32 & 35 \\ 41 & 42 & 45 \\ 51 & 52 & 55 \end{bmatrix}$$

Yes. Remove from **A**:

$$\begin{bmatrix} \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ 21 & 22 & \blacksquare & \blacksquare & 25 \\ 31 & 32 & \blacksquare & \blacksquare & 35 \\ 41 & 42 & \blacksquare & \blacksquare & 45 \\ 51 & 52 & \blacksquare & \blacksquare & 55 \end{bmatrix}$$

Submatrices

A *submatrix* **B** of a given matrix **A** is any matrix that can be obtained by removing some rows and/or columns from **A**. The rows and columns that remain do *not* need to be adjacent in **A**.

$$\text{Consider} \qquad \mathbf{A} = \begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ 21 & 22 & 23 & 24 & 25 \\ 31 & 32 & 33 & 34 & 35 \\ 41 & 42 & 43 & 44 & 45 \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

Question L6.3: Is $\mathbf{B}_3$ below a submatrix of **A**?

$$\mathbf{B}_3 = \begin{bmatrix} 21 & 22 \\ 31 & 35 \\ 42 & 45 \\ 51 & 55 \end{bmatrix} \qquad \text{No. Remove from } \mathbf{A}\text{:} \qquad \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ 21 & 22 & \blacksquare & \blacksquare & \blacksquare \\ 31 & \blacksquare & \blacksquare & \blacksquare & 35 \\ \blacksquare & 42 & \blacksquare & \blacksquare & 45 \\ 51 & \blacksquare & \blacksquare & \blacksquare & 55 \end{bmatrix}$$

We cannot obtain $\mathbf{B}_3$ by obtaining entire rows and columns.

Submatrices

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$$\text{Consider} \qquad \mathbf{A} = \begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ 21 & 22 & 23 & 24 & 25 \\ 31 & 32 & 33 & 34 & 35 \\ 41 & 42 & 43 & 44 & 45 \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

Question L6.4: Is $\mathbf{B}_4$ below a submatrix of **A**?

$$\mathbf{B}_4 = \begin{bmatrix} 11 & 14 & 13 \\ 21 & 24 & 23 \\ 31 & 34 & 33 \\ 41 & 44 & 43 \\ 51 & 54 & 53 \end{bmatrix} \qquad \text{No. Remove from } \mathbf{A}\text{:} \qquad \begin{bmatrix} 11 & \blacksquare & 13 & 14 & \blacksquare \\ 21 & \blacksquare & 23 & 24 & \blacksquare \\ 31 & \blacksquare & 33 & 34 & \blacksquare \\ 41 & \blacksquare & 43 & 44 & \blacksquare \\ 51 & \blacksquare & 53 & 54 & \blacksquare \end{bmatrix}$$

The order of the columns in $\mathbf{B}_4$ is different from what we get by removal.