MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 10: SOME SPECIAL MATRICES

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We will use the terminology and notation of Lecture 7 and of Module 9.

Recall from Module 9 that $\mathbf{N} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent matrix with $\mathbf{N}^3 = \mathbf{O}_{3\times 3}$.

Notice that this matrix is upper-triangular and has only zeros on the (main) diagonal. The next questions asks you to generalize this observation.

Question 10.1: Prove that if $\mathbf{U}$ is an upper-triangular matrix of order 3×3 that has only zeros on the main diagonal, then $\mathbf{U}^3$ is the zero matrix of order 3×3.

In Lecture 7 we illustrated the formula for products of diagonal matrices with an example. Now let us prove it for matrices of order 2×2.

Question 10.2: Prove that for all 2×2 diagonal matrices the following equality holds:

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix} = \begin{bmatrix} \lambda_1\kappa_1 & 0 \\ 0 & \lambda_2\kappa_2 \end{bmatrix}$$

Question 10.3: Let $\mathbf{U}_1, \mathbf{U}_2$ be two upper-triangular matrices of order 2×2. Prove that their product $\mathbf{U}_1\mathbf{U}_2$ is an upper-triangular matrix.

Question 10.4: Prove that for every square matrix $\mathbf{A}$, if the sum $\mathbf{A} + \mathbf{A}^T$ is defined, then it is a symmetric matrix.