MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 3: VECTORS

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We will use here the notation and terminology of Lecture 2.

Consider the vectors $\vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$ and $\vec{\mathbf{w}} = [2,\ -4,\ 0.1]$.

Question 3.1: Do $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ have the same order?

Question 3.2: Do $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ have the same dimension?

For Questions 3.3 and 3.4 below, consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 6 & 0.2 \end{bmatrix} \tag{1}$$

Question 3.3: Find $\vec{\mathbf{a}}_{*1}, \vec{\mathbf{a}}_{*2}$, and $\mathbf{A}_{cols}$.

In analogy with the constructions of column vectors $\vec{\mathbf{a}}_{*j}$ and matrix of column vectors $\mathbf{A}_{cols}$, we can also construct row vectors and a matrix of row vectors as follows:

Consider an $m \times n$ matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

The rows of $\mathbf{A}$ are $1 \times n$ row vectors

$\vec{\mathbf{a}}_{1*} = [a_{11}, a_{12},\ \dots\ , a_{1n}]$,

$\vec{\mathbf{a}}_{2*} = [a_{21}, a_{22},\ \dots\ , a_{2n}]$,

$\dots$

$\vec{\mathbf{a}}_{m*} = [a_{m1}, a_{m2},\ \dots\ , a_{mn}]$.

We can now define an $m \times 1$ matrix

$$\mathbf{A}_{rows} = \begin{bmatrix} \vec{\mathbf{a}}_{1*} \\ \vec{\mathbf{a}}_{2*} \\ \vdots \\ \vec{\mathbf{a}}_{m*} \end{bmatrix}$$

where $\vec{\mathbf{a}}_{1*}, \vec{\mathbf{a}}_{2*},\ \dots\ , \vec{\mathbf{a}}_{m*}$ are as above

Then the matrices $\mathbf{A}$ and $\mathbf{A}_{rows}$ are *not equal,* but are *equivalent* in the sense that they are two different ways of representing the same information.

Question 3.4: Find $\vec{\mathbf{a}}_{1*}, \vec{\mathbf{a}}_{2*}, \vec{\mathbf{a}}_{3*}$, and $\mathbf{A}_{rows}$ for the matrix $\mathbf{A}$ of Equation (1).

Let's return to the example of the instructor's spreadsheet

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

Recall that here we assume that m is the number of students in the class, n is the number of gradable items, and the entry a_{ij} represents the score of student i on gradable item number j.

Question 3.5: What does $\vec{\mathbf{a}}_{*j}$ represent?

Question 3.6: What does $\vec{\mathbf{a}}_{i*}$ represent?