MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 41: INTRODUCTION TO LINEAR COMBINATIONS

WINFRIED JUST, OHIO UNIVERSITY

This module is based on Lecture 21.

Recall from Lecture 21 that a vector $\vec{\mathbf{w}}$ is a *linear combination* of vectors $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n$ if there exist scalars $d_1, d_2, \dots, d_n$, called *coefficients* of the linear combination such that
$\vec{\mathbf{w}} = d_1\vec{\mathbf{v}}_1 + d_2\vec{\mathbf{v}}_2 + \cdots + d_n\vec{\mathbf{v}}_n$.
The vectors $\vec{\mathbf{w}}, \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n$ are assumed to be all of the same order. The zero vector $\vec{\mathbf{0}}$ of the same order as $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \dots, \vec{\mathbf{v}}_n$ is always a linear combination of these vectors.

Question 41.1: Would the last sentence remain correct if the phrase "of the same order" were replaced by "of the same dimension"?

Now consider the vectors $\vec{\mathbf{v}}_1 = [1, 2, 3], \vec{\mathbf{v}}_2 = [-3, 8, 5], \vec{\mathbf{v}}_3 = [4, 4, 4]$. Let $\vec{\mathbf{w}} = [1, 4, 7]$.
Then $\vec{\mathbf{w}} = 3\vec{\mathbf{v}}_1 - 0.5\vec{\mathbf{v}}_3$. Thus $\vec{\mathbf{w}}$ is a linear combination of $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_3$, since there are scalars $d_1 = 3$ and $d_3 = -0.5$ such that $\vec{\mathbf{w}} = d_1\vec{\mathbf{v}}_1 + d_3\vec{\mathbf{v}}_3$.

Question 41.2: Would it also be correct to say that $\vec{\mathbf{w}}$ is a linear combination of the vectors $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2$, and $\vec{\mathbf{v}}_3$?