MATH3200: APPLIED LINEAR ALGEBRA
SELF-STUDY AND PRACTICE MODULE 43: PROPERTIES OF THE LINEAR SPAN OF A SET OF VECTORS

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This module is based on Lecture 22. Recall from this lecture that the set of all linear combinations of vectors $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n$ is denoted by $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$ and called the *linear span* of these vectors. As in the lecture, let $\mathbb{R}^3$ denote here the set of all 1×3 row vectors.

In Lecture 22 we have seen one example of a linear span of the form $span(\vec{\mathbf{v}}_1)$ that is a line in $\mathbb{R}^3$ and one example of a linear span of the form $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2)$ that is a plane in $\mathbb{R}^3$.

Question 43.1: Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n, \vec{\mathbf{w}}$ be any vectors such that $\vec{\mathbf{w}}$ is in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$. Prove that then for any scalar λ the vector $\lambda\vec{\mathbf{w}}$ is also in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$.

Question 43.2: Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n, \vec{\mathbf{u}}, \vec{\mathbf{w}}$ be any vectors such that $\vec{\mathbf{u}}, \vec{\mathbf{w}}$ are in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$. Prove that then the vector $\vec{\mathbf{u}} + \vec{\mathbf{w}}$ is also in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$.

Question 43.3: Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n$ be any vectors of the same order. Prove that any linear combination of two vectors in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$ is also in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$.

Question 43.4: (Challenge) Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n$ be any vectors of the same order. Prove that any linear combination of any number of vectors in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$ is also in $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n)$.

Question 43.5: Is every line in $\mathbb{R}^3$ the linear span of some set of vectors?
Either give a short proof or find a counterexample.

Question 43.6: Let $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2$ be any vectors in $\mathbb{R}^3$. Is then the linear span $span(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2)$ always a plane in $\mathbb{R}^3$?
Either give a short proof or find a counterexample.