

MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 44: APPLICATIONS OF LINEAR COMBINATIONS
AND OF THE LINEAR SPAN TO MOVEMENT IN THREE DIMENSIONS

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This module is based on Conversation 22.

Conversation 22 illustrates that a point \vec{w} in three-dimensional space can be reached by traveling (forward or backward) from the origin first in the direction of one vector \vec{v}_1 and then in the direction of another vector \vec{v}_2 if, and only if, \vec{w} is in the linear span $\text{span}(\vec{v}_1, \vec{v}_2)$ of these vectors.

Question 44.1: You may have heard some version of the following story: “Paul and Paula are beautiful but lonely birds of paradise. Paul sits perched on a tree at position $[10, 30, 5]$ and sings his song that will make the heart of any she-bird of paradise melt in an instant. Paula, sitting on the ground at position $[0, 0, 0]$ hears the song, her heart does melt, and she flies first for some time in the direction of vector \vec{v}_1 , and then for some time in the direction of vector \vec{v}_2 in search for the singer. When she reaches Paul’s location, the two birds look each other deep into the eyes. Stunned by the beauty of each other’s feathers, Paul and Paula engage in a trance-like dance at the end of which they join their fates and live together happily ever after.”

Your instructor heard in fact three versions of it, but did not find all of them believable:

Version One: $\vec{v}_1 = [1, 2, 0]$, $\vec{v}_2 = [2, 1, 0]$.

Version Two: $\vec{v}_1 = [2, 7, 3]$, $\vec{v}_2 = [1, 1, -3.5]$.

Version Three: $\vec{v}_1 = [1, 2, 3]$, $\vec{v}_2 = [2, -1, 2.5]$.

Which version(s) is/are somewhat believable?

Question 44.2: Eliza is a proud eagle who travels only (forward or backward) in the directions of the vectors $\vec{v}_1 = [1, 2, 3]$ and $\vec{v}_2 = [-1, -3, -4]$ and shuns all other directions. She roosts at a place with x -coordinate 3 and y -coordinate 4. Can you tell the z -coordinate (height) of her roost if you know she can reach it from the origin? If so, what is this number? (Assume that all coordinates are measured in suitable units, like meters.)

Question 44.3: Baldwin is a hungry buzzard who spots a mouse on the ground when he is at coordinates $[7, 20, 30]$ and is not moving. The x -coordinate of the mouse is $x = 5$ and its y -coordinate is $y = -3$. Baldwin decides to try catching the mouse by first flying t_1 seconds horizontally in the direction of $\vec{v}_1 = [-1, -2, 0]$ and then taking a dive in the direction of $\vec{v}_2 = [-1, -4, -10]$. Does Baldwin have a chance to catch the mouse? If so, how long will it take him to reach the mouse? *Hint:* Note that in this question neither Baldwin nor the mouse are at the origin.