

MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 50: CONSISTENCY OF SYSTEMS OF LINEAR
EQUATIONS AND THE RANKS OF THEIR COEFFICIENT AND
EXTENDED MATRICES

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This module is based on Lecture 27 and uses results from Module 49.

Recall from Lecture 27 that:

- A linear system $\mathbf{A}\vec{x} = \vec{b}$ is consistent if, and only if, $r(\mathbf{A}) = r([\mathbf{A}, \vec{b}])$.
- When $r(\mathbf{A}) = m$ is equal to the number of rows of \mathbf{A} , then *every* system of the form $\mathbf{A}\vec{x} = \vec{b}$ is consistent.

Question 50.1: Consider the system of linear equations

$$\begin{array}{rrcr} -x_1 & + & 3x_2 & = & 41 \\ -2x_1 & + & 6x_2 & = & 84 \end{array}$$

- (a) Does the answer to Question 49.3 of Module 49 allow us to conclude that this system is consistent?
- (b) Find the rank $r([\mathbf{A}, \vec{b}])$ of the extended matrix for this system.
- (c) Is the above system consistent?

Question 50.2: Consider the system of linear equations

$$\begin{array}{rrrrcr} 3x_1 & + & 2x_2 & + & x_3 & = & 71 \\ x_1 & + & x_2 & + & x_3 & = & 13 \\ 6x_1 & + & 5x_2 & + & 6x_3 & = & 84 \end{array}$$

- (a) Does the answer to Question 49.4 of Module 49 allow us to conclude that this system is consistent?
- (b) Find the rank $r([\mathbf{A}, \vec{b}])$ of the extended matrix for this system.
- (c) Is the above system consistent?