MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 61: INTRODUCTION TO DETERMINANTS

WINFRIED JUST, OHIO UNIVERSITY

This module is based on Lecture 31.

1. Determinants of 2×2 matrices

Recall that the *determinant* of a 2×2 matrix is defined as $\det(\mathbf{A}) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

Question 61.1: Find the determinants of the following matrices:

$$\mathbf{A} = \begin{bmatrix} 6 & 7 \\ -3 & 3.5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 6 & 7 \\ -3 & -3.5 \end{bmatrix}$$

Recall that in Lecture 31 we proved the following results for matrices of order 2×2:

Theorem 1. *Let* $\mathbf{A}$ *be a square matrix.*

(i) *If* $\mathbf{B}$ *is obtained from* $\mathbf{A}$ *by switching two rows of* $\mathbf{A}$*, then* $\det(\mathbf{B}) = -\det(\mathbf{A})$.
(ii) *If* $\mathbf{B}$ *is obtained from* $\mathbf{A}$ *by multiplying one row of* $\mathbf{A}$ *by a scalar* λ*,* *then* $\det(\mathbf{B}) = \lambda \det(\mathbf{A})$.
(iii) *If* $\mathbf{B}$ *is obtained from* $\mathbf{A}$ *by adding a scalar multiple of one row of* $\mathbf{A}$ *to another row of* $\mathbf{A}$*, then* $\det(\mathbf{B}) = \det(\mathbf{A})$.

This theorem has an exact analogue for column operations:

Theorem 2. *Let* $\mathbf{A}$ *be a square matrix.*

(i) *If* $\mathbf{B}$ *is obtained from* $\mathbf{A}$ *by switching two columns of* $\mathbf{A}$*, then* $\det(\mathbf{B}) = -\det(\mathbf{A})$.
(ii) *If* $\mathbf{B}$ *is obtained from* $\mathbf{A}$ *by multiplying columns row of* $\mathbf{A}$ *by a scalar* λ*,* *then* $\det(\mathbf{B}) = \lambda \det(\mathbf{A})$.
(iii) *If* $\mathbf{B}$ *is obtained from* $\mathbf{A}$ *by adding a scalar multiple of one column of* $\mathbf{A}$ *to another column of* $\mathbf{A}$*, then* $\det(\mathbf{B}) = \det(\mathbf{A})$.

Question 61.2: Prove Theorem 2 for the case when $\mathbf{A}$ is of order 2×2.

2. Determinants of 3×3 matrices

Recall the following formula for the determinant of a 3×3 matrix $\mathbf{A}$ from Lecture 31:

$$\det(\mathbf{A}) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}.$$

While you are not required to memorize this formula, you are expected to be able to use it if it is given to you.

Question 61.3: Use the above formula to find $\det(\mathbf{A}) = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

Question 61.4: Use the above formula to prove that if $\mathbf{B}$ is obtained by multiplying the third row of a 3×3 matrix $\mathbf{A}$ by a scalar λ, then $\det(\mathbf{B}) = \lambda \det(\mathbf{A})$.

Question 61.5: Use the above formula to prove that if $\mathbf{B}$ is obtained by switching the first two rows of a 3×3 matrix $\mathbf{A}$, then $\det(\mathbf{B}) = -\det(\mathbf{A})$.