MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 63B: PROOFS OF SOME PROPERTIES OF DETERMINANTS

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This module is based on Lecture 33.

In the first two questions you will be asked to construct a proof of the fact that a square matrix $\mathbf{A}$ is singular if, and only if, it does not have full rank for the case of 2×2 matrices.

Question 63.9: Let $\mathbf{A}$ be a matrix of order 2×2. Prove that if one row of $\mathbf{A}$ is a scalar multiple of the other row, then $\det(\mathbf{A}) = 0$.

When $\mathbf{A}$ is a 2×2 matrix with $\det(\mathbf{A}) = 0$, then the two rows form a linearly independent set so that one of them is a scalar multiple of the other. The next question asks you to prove this fact under an additional assumption on $\mathbf{A}$.

Question 63.10: Let $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a matrix of order 2×2 such that $a \neq 0$.

Prove that if $\det(\mathbf{A}) = 0$, then the two rows of $\mathbf{A}$ form a linearly dependent set of vectors.

Recall the following theorem from Lecture 33:

Theorem 1. *Let* $\mathbf{A}, \mathbf{B}$ *be two square matrices. Then*
(i) $\det(\mathbf{A}^T) = \det(\mathbf{A})$.
(ii) $\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$.

Question 63.11: Prove part (i) of Theorem 1 for the case when $\mathbf{A}$ has order 2×2.

Question 63.12: Prove part (ii) of Theorem 1 for the case when $\mathbf{A}, \mathbf{B}$ have order 2×2.