MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 7: MATRIX PRODUCTS

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We will use here the notation and terminology of Lecture 5.

For Questions 7.1 through 7.5, consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 10 & -2 \\ 2 & 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & -4 \\ -5 & 9 & 0 \end{bmatrix}$$

Question 7.1: Find the matrix product $\mathbf{AC}$ or state that it is undefined.

Question 7.2: Find the matrix product $\mathbf{BA}$ or state that it is undefined.

Question 7.3: Find the matrix product $\mathbf{AB}$ or state that it is undefined.

Question 7.4: Find the matrix product $\mathbf{AC}^T$ or state that it is undefined.

Question 7.5: Find the matrix product $\mathbf{C}^T\mathbf{A}$ or state that it is undefined.

We have seen that when $\vec{\mathbf{x}}$ is a $1 \times n$ row vector and $\vec{\mathbf{y}}$ is an $m \times 1$ column vector, then the inner product $\vec{\mathbf{x}}\vec{\mathbf{y}}$ may not exist. But the so-called *outer product* $\vec{\mathbf{y}}\vec{\mathbf{x}}$ always exists. It is a matrix $\mathbf{C} = [c_{ij}]_{m\times n}$ of order $m \times n$ of the following form:

$$\vec{\mathbf{y}}\vec{\mathbf{x}} = \begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_m \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_j & \dots & x_n \end{bmatrix} = \begin{bmatrix} y_1x_1 & \dots & y_1x_j & \dots & y_1x_n \\ \vdots & & \vdots & & \vdots \\ y_ix_1 & \dots & y_ix_j & \dots & y_ix_n \\ \vdots & & \vdots & & \vdots \\ y_mx_1 & \dots & y_mx_j & \dots & y_mx_n \end{bmatrix}$$

Consider $\vec{\mathbf{x}} = \begin{bmatrix} 2 & 4 & -1 \end{bmatrix}$ and $\vec{\mathbf{y}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ for which the inner product $\vec{\mathbf{x}}\vec{\mathbf{y}}$ is undefined.

Question 7.6: Find the outer product of the above vectors $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$.

For every angle α, let us define a matrix

$$\mathbf{R}_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Question 7.7: Derive a formula for the product $\mathbf{R}_\alpha\mathbf{R}_\beta$ that is as simple as possible.

Question 7.8: Let $\vec{\mathbf{v}}$ be the $1 \times m$ row vector all of whose elements are 1, let $\vec{\mathbf{w}}$ be the $n \times 1$ column vector all of whose elements are 1, and let $\mathbf{A}$ be an $m \times n$ matrix. Give verbal descriptions of $\vec{\mathbf{v}}\mathbf{A}$ and of $\mathbf{A}\vec{\mathbf{w}}$.

For Questions 7.9 through 7.12, consider an arbitrary matrix $\mathbf{A}$ of order $m \times n$, where m and n are positive integers.

Question 7.9: True or false? "For any matrix $\mathbf{A}$, the product $\mathbf{A}\mathbf{A}^T$ is always defined."

Question 7.10: True or false? "The product $\mathbf{AA}$ of $\mathbf{A}$ with itself is always defined."

Question 7.11: True or false? "If $m = n$, the product $\mathbf{AA}$ has the same order as $\mathbf{A}$."

Question 7.12: True or false? "$\mathbf{A}\mathbf{A}^T$ and $\mathbf{A}^T\mathbf{A}$ always have the same order."