MATH3200: APPLIED LINEAR ALGEBRA
PRACTICE MODULE 82A: INNER PRODUCTS AND ORTHOGONALITY

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This module is based on Lecture 42. Recall the following definitions and facts from this lecture:

The (standard) *inner product* $\langle \vec{\mathbf{x}}, \vec{\mathbf{y}} \rangle$ of vectors $\vec{\mathbf{x}} = [x_1, x_2, \dots, x_n], \vec{\mathbf{y}} = [y_1, y_2, \dots, y_n]$ is defines as $\langle \vec{\mathbf{x}}, \vec{\mathbf{y}} \rangle = x_1y_1 + x_2y_2 + \cdots + x_ny_n = \sum_{i=1}^n x_iy_i$.

$\sqrt{\langle \vec{\mathbf{x}}, \vec{\mathbf{x}} \rangle} = \|\vec{\mathbf{x}}\|$ is the Euclidean norm of $\vec{\mathbf{x}}$. Thus $\langle \vec{\mathbf{x}}, \vec{\mathbf{x}} \rangle = \|\vec{\mathbf{x}}\|^2$.

Two vectors $\vec{\mathbf{x}}, \vec{\mathbf{y}}$ are *orthogonal* if, and only if, $\langle \vec{\mathbf{x}}, \vec{\mathbf{y}} \rangle = 0$.

The angle Θ in $[0, \pi]$ between two nonzero vectors in $\mathbb{R}^n$ can be computed from the Law of Cosines $\cos\Theta = \frac{\langle \vec{\mathbf{x}}, \vec{\mathbf{y}} \rangle}{\|\vec{\mathbf{x}}\| \, \|\vec{\mathbf{y}}\|}$. The angle Θ then becomes $\Theta = \arccos \frac{\langle \vec{\mathbf{x}}, \vec{\mathbf{y}} \rangle}{\|\vec{\mathbf{x}}\| \, \|\vec{\mathbf{y}}\|}$.

Question 82.1: Consider the following vectors in $\mathbb{R}^3$:
$\vec{\mathbf{u}} = [1, 2, 3]$, $\vec{\mathbf{v}} = [-1, -1, 1]$, $\vec{\mathbf{w}} = [1, 0, 1]$, $\vec{\mathbf{0}} = [0, 0, 0]$.
Compute $\langle \vec{\mathbf{x}}, \vec{\mathbf{y}} \rangle$ for all six pairs of these vectors.

Question 82.2: Which of the six pairs of vectors of Question 82.2 are orthogonal?

Question 82.3: Is there a vector $\vec{\mathbf{x}}$ in $\mathbb{R}^3$ that is orthogonal to itself?

Question 82.4: Let $\vec{\mathbf{x}} = [\sqrt{3}, 1]$, $\vec{\mathbf{y}} = [3, 0]$, $\vec{\mathbf{z}} = [-1, 1]$.

(a) Find the angle between $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$.
(b) Find the angle between $\vec{\mathbf{y}}$ and $\vec{\mathbf{z}}$.