Unraveling complex systems: What do brains, the internet, and ant colonies have in common?

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Brains

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How can the firing patterns of the neurons *give rise to* **perceptions, feelings, thoughts, and actions?**

The World Wide Web

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How does Wikipedia, a reasonably reliable source of reference, *emerge* **out of the largely uncoordinated creativity of its authors? How does it happen that the accumulating mass of web pages reshapes the way we work, retrieve information, shop, socialize, and spend our leisure time?**

Ant colonies

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How can an ant colony build elaborate nests and even farm fungi, which amounts to creating a habitat for another species?

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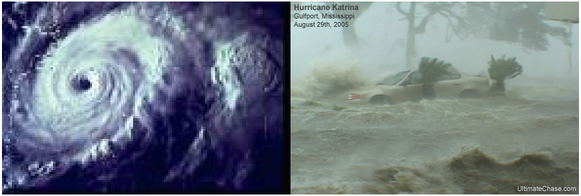
Hurricanes

Hurricane Katrina
Gulfport, Mississippi
August 29th, 2005

UltimateChase.com

Hurricanes



Hurricane Katrina
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How do the movements of individual air molecules that are driven by mechanical forces *self-organize* **into powerful winds that can devastate large coastal areas?**

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How do the individual decisions ***lead to*** **market equilibrium most of the time and market crashes some of the time? Which mechanisms can prevent systemic failure?**

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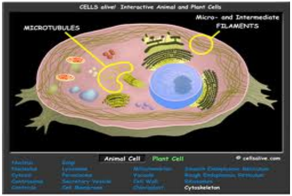
CELLS alive! Interactive Animal and Plant Cells
MICROTUBULES
Micro- and Intermediate FILAMENTS
Animal Cell
Plant Cell
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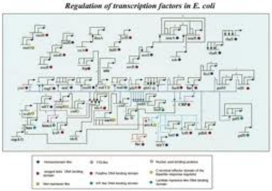
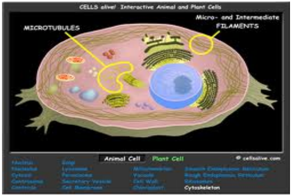
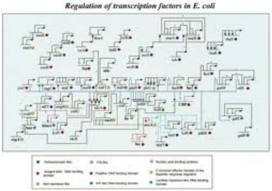
Nucleus
Nucleolus
Cytosol
Centrosome
Centriole
Golgi
Lysosome
Peroxisome
Secretory Vesicle
Cell Membrane
Mitochondrion
Vacuole
Cell Wall
Chloroplast
Smooth Endoplasmic Reticulum
Rough Endoplasmic Reticulum
Ribosomes
Cytoskeleton


Regulation of transcription factors in E. coli
CRP

Cells





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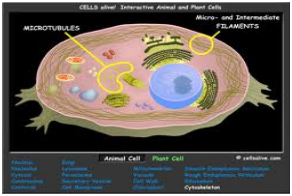

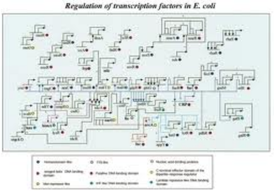

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How does this happen? Why does this work?

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How does this make institutions function, by and large, the way they are supposed to? When do things go wrong? Which mechanisms can prevent systemic failure?

A pioneer in the study of complex systems

The complex systems we listed are the subject matter of a variety of sciences. Nevertheless, they have a lot in common. The first scientist to clearly discern and spell out these communalities was **Herbert A. Simon (1916-2001)**. He studied administrative structures. And a lot of other things.

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He was not a mathematician, but used a lot of mathematics in his work. And he built on the work of others, including mathematicians Norbert Wiener (1894–1964) and Alfred J. Lotka (1880-1949).

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- Most complex systems have a **hierarchical** and **modular** structure. So there may be more than two levels.
- At the microscopic level agents may behave somewhat randomly and are prone to failure. Despite of this, the behavior of the system at the macroscopic level tends to be fairly robust.

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What makes a Boeing 787 different from our other examples?

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- All our other examples are complex adaptive systems.
- The distinction is not the same as between "natural" and "engineered" systems. Electrical power grids and transportation systems are complex and fairly adaptive. They are also a lot more robust to component failure than airplanes, but not as resilient as, say, ant colonies.

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Fortune cookie: Doing the impossible is kind of fun.

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Mathematics is uniquely suited to unravel the **common features** of complex adaptive systems that we have been marveling at.

Ohio University – Since 1804 Department of Mathematics

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- **Stochastic models** allow for some randomness in the trajectories, **deterministic models** assume that the current state uniquely determines the trajectory.
- **Continuous** (e.g. ODE, PDE) models assume that time can take any (nonnegative) real values; **discrete time** (e.g. difference equation, Boolean) models assume that time moves in discrete steps, that is, only takes integer values.

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- Try to predict (by simulations or deductions) the trajectories of the macroscopic variables, and, if applicable, the change of the equations for the microscopic variables.

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On the next slide we will give (sort of) a definition that will do for this talk.

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- Beware of anthropomorphisms! Systems really only behave **as if** they had "needs" or "tried to."
- Complex adaptive systems rarely find optimal solutions, only reasonably good ones. In Herbert Simon's terminology, they "satisfice."

Ideal gas. A counterexample?

An ideal gas consists of n moles of particles, which gives $\approx 6n\,10^{23}$ agents who can be characterized by $\approx 36n10^{23}$ microscopic variables and who interact according to the laws of (Newtonian) mechanics. This causes the macroscopic variables P, V, T to obey the law

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Personally, I prefer to look at this example as a limiting case of complex systems. We can learn a lot from this example.

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- Details (of the initial state, of the mechanics of the actual bumpings of particles into each other) **don't matter all that much** in the derivation of (1).
- (1) does not hold with certainty, only with probability very, very close to 1, and relies on the yet unproven **ergodic hypothesis.**

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While evolution is the primary mechanism by which biological systems adapt, emergence and self-organization also happen ins systems that do not evolve.

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Exhibit A: The ideal gas law.

Exhibit B: Hurricanes

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- The dynamics of complex systems may or may not be chaotic. Chaotic systems may or may not be complex. Chaos theory is something **very different** from a theory of complex systems.

Exhibit C: Yeast cells

Theorem

If cells respond to fairly general feedback mechanisms, then with substantial probability yeast cultures at certain cell densities in a well-stirred vat will spontaneously break up into several clusters that are near-synchronized for cell-cycle state and we will observe corresponding oscillations of oxygen levels.

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Results along these lines are being obtained in an ongoing NIH-funded research project led by Prof. T. R. Young that involves a number of OU graduate and undergraduate students as well as several internal and external collaborators.

Exhibit D: ODE vs. Boolean dynamics

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- A joint paper by W. Just, M. Korb, B. Elbert, and T. R. Young that will be submitted shortly proves another theorem of this kind.

Mathematics to learn

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Which areas of mathematics are important for the study of complex adaptive systems?

- Differential equations: ODEs, PDEs, differential delay equations
- Stochastic differential equations
- Stochastic processes
- More generally: dynamical systems (discrete, continuous, deterministic, stochastic)
- Theory of computation
- Finite automata
- Probability and statistics
- Statistical mechanics
- ... (you name it)

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- We will need some new tools (e.g., Exhibit D).

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  A candidate may be a branch of dynamical systems theory devoted to systems that adapt over time.

Picture sources

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