HIGH SCHOOL PRE-CALCULUS QUIZ CONTEST 2024
SAMPLE QUESTIONS

(1) (a) The quadrant in which the point with polar coordinates \(-2, -\frac{3\pi}{4}\) lies is \underline{__________}. \textbf{Answer: First}

(b) The polar coordinates \((r, \theta)\) of the point with polar coordinates \((-1, \frac{\pi}{3})\) where \(r > 0\) and \(0 \leq \theta \leq 2\pi\) are \underline{__________}. \textbf{Answer: \(1, \frac{4\pi}{3}\)}

(2) (a) The center of the circle \((x + 1)^2 + (y - 2)^2 = 5\) lies in the \underline{quadrant}. \textbf{Answer: Second}

(b) The radius of the circle centered at \((1, -3)\) and touching the \(x\)-axis is \underline{__________}. \textbf{Answer: 3}

(3) If \(P(t)\) denotes the point on the unit circle with coordinates \(\left(\frac{5}{13}, \frac{-12}{13}\right)\), then what are the coordinates of the point on the unit circle that corresponds to \(P(\pi-t)\)? \textbf{Answer: \(\left(-\frac{5}{13}, \frac{12}{13}\right)\)}

(4) (a) What is the domain of the function \(f(x) = \frac{3}{\sqrt{5 - |x - 2|}}\)? \textbf{Answer: \((-3, 7)\)}

(b) What is the range of the quadratic function \(f(x) = -(x + 3)^2 - 2\)? \textbf{Answer: \((-\infty, -2]\)}

(5) (a) The number of points in which the graph of \(y = 3 - |x - 1|\) meets the \(x\)-axis is \underline{__________}. \textbf{Answer: 2}

(b) The number of points in which the graph of the function \(f(x) = \begin{cases} x^2 + x - 2, x \leq -2 \\ x - x^2, x > -2 \end{cases}\) meets the \(x\)-axis is \underline{__________}. \textbf{Answer: 3}

(c) The number of points in which the graph of the function \(f(x) = -3^x + 1\) meets the \(x\)-axis is \underline{__________}. \textbf{Answer: 1}
(d) The number of points in which the graph of the function \( f(x) = \sin 2x, \ 0 \leq x \leq 2\pi \) meets the x-axis is \( 5 \). (Answer: 5)

(6) (a) Every polynomial function of degree \( n \) has \( n \) real zeros, some of which may be repeated (True or False). (Answer: False)
(b) The equation \( \sin^2 x - \sin x - 6 = 0 \) has no solutions (True or False). (Answer: True)

(7) (a) If \( f(x) = \sqrt{x} \) and \( (f \circ g)(x) = \sqrt{x^2 + 1} \) then \( (g \circ f)(x) = \) \( x + 1 \). (Answer: \( x + 1 \))
(b) If \( g(x) = \sin x \) and \( (f \circ g)(x) = e^{\sin x} \) then \( (g \circ f)(x) = \) \( \sin e^x \). (Answer: \( \sin e^x \))

(8) What is the radius of the circle given by the equation \( x^2 + y^2 - 6x = 0 \)?

(Answer: 3)

(9) If \( 2^x = 1 \) then what is the value of \( 2^{3x-1} \)? (Answer: \( \frac{1}{2} \))

(10) If \( f \) is an odd function such that \( f(-2) = 1 \) and \( g \) is an even function such that \( g(1) = 2 \) then \( g(f(2)) = \) \( 2 \). (Answer: 2)

(11) What is the value of \( \sin(\arccos(-\frac{5}{13})) \)? (Answer: \( \frac{12}{13} \))

(12) (a) The function \( f(x) = |x - 3| \) is an even function (True or False). (Answer: False)
(b) The function \( f(x) = x \cos x + \sin x \) is an odd function (True or False). (Answer: True)

(13) (a) A polynomial with real coefficients has zeros \( i \) and \( 1 - i \), and 1. What is the smallest possible degree of the polynomial? (Answer: 5)
(b) A polynomial has zeros \( i \) and \( 1 - i \), and 1. What is the smallest possible degree of the polynomial? (Answer: 3)

(14) (a) If \( \frac{\log a}{\log b} = 4 \) then what is the value of \( \log_b a^3 \)? (Answer: 6)
(b) For any positive real number \( x \), \( \log_{x^2} \sqrt{x} = \) \( \frac{1}{4} \). (Answer: \( \frac{1}{4} \))

(15) If \( x - 1 \) is a factor of the polynomial \( x^3 + k^2 x^2 - kx - 3 \) find the values of \( k \)? (Answer: -1 and 2)

(16) If \( 2^{10} + 4^5 = 2^x \) then what is the value of \( x \)? (Answer: 11)
(17) What is the period of the function \( f(x) = \tan 5x \). (Answer: \( \frac{\pi}{5} \))

(18) (a) \( \tan 48^\circ \tan 138^\circ = \frac{-1}{\cos 18^\circ} \). (Answer: -1)
   (b) For any \( x \) such that \(-1 \leq x \leq 1\), the value of \( \cos(\sin^{-1} x + \cos^{-1} x) \) is \( \frac{1}{\sqrt{2}} \). (Answer: 0)

(19) If \( P(x) = (x^2 + x - 2)(x^2 - 4)(x + 2) \) what is the multiplicity of the zero \(-2\)? (Answer: 3)

(20) What is the maximum number of positive zeros of the polynomial \( x^7 - 13x^6 - 6x^5 - 7x^4 + 11x^3 + 3x^2 - 6x - 5 \). (Answer: 3)

(21) If \( x \neq 1 \) and \( x^3 = 1 \) then what is the value of \( x + x^2 \)? (Answer: -1)

(22) What is the value of \( \sin^2 \frac{3\pi}{8} + \sin^2 \frac{\pi}{8} \)? (Answer: 1)

(23) The value of \( e^{\ln 3 + 3\ln 2} \) is \( 24 \). (Answer: 24)

(24) (a) If one of \( \sin t \) and \( \cos t \) is positive and the other negative then the possible values of \( t \) satisfy
   (i) \( 0 < t < \frac{\pi}{2} \).
   (ii) \( \frac{\pi}{2} < t < \pi \).
   (iii) \( \pi < t < \frac{3\pi}{2} \).
   (iv) \( \frac{3\pi}{2} < t < 2\pi \).
   (v) \( \frac{\pi}{2} < t < \frac{2\pi}{3} \).
   (vi) \( \frac{2\pi}{3} < t < 2\pi \).
   (Answer: (iii) and (vi))

(b) If \( 0 \leq t \leq 2\pi \) and \( \tan t \) is negative then what is the sign of \( \sin 2t \)? (Answer: Negative)

(25) A quadratic equation has integer coefficients and leading coefficient in the equation is 1. If one of the roots of the quadratic equation is \( 2 + \sqrt{3} \) then the constant term in the equation is \( \frac{1}{4} \). (Answer: 1)