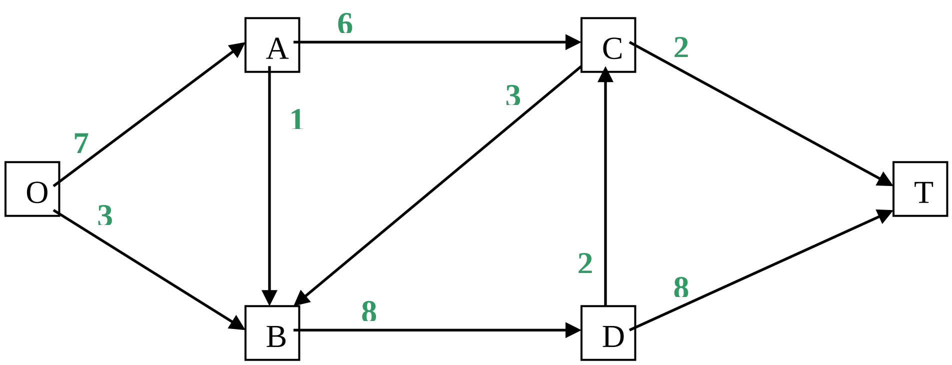
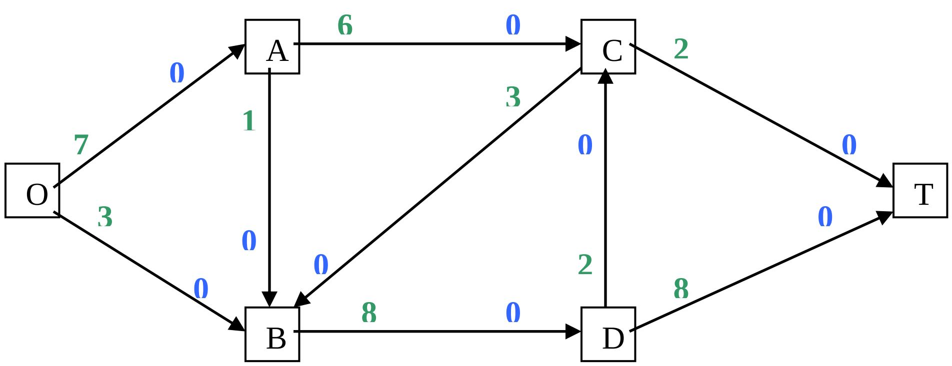
Solutions

Maximum Flow Problem.
You are given the following directed network with source O and sink T.

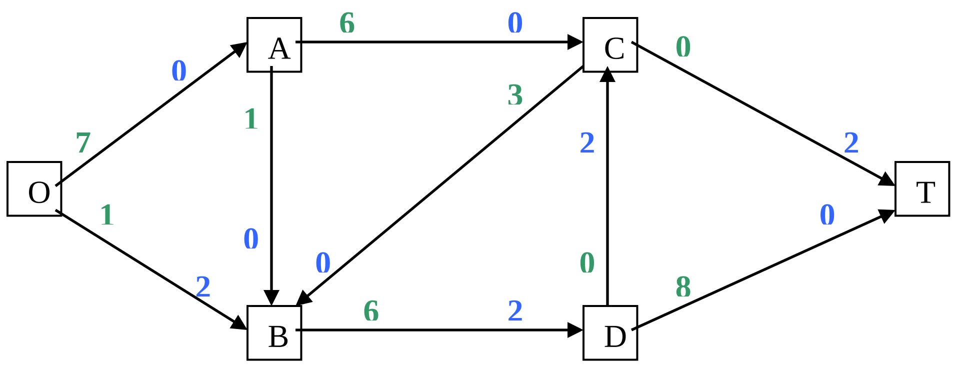


a) Find a maximum flow from O to T in the network (choose O→B→D→C→T as the first augmenting path).

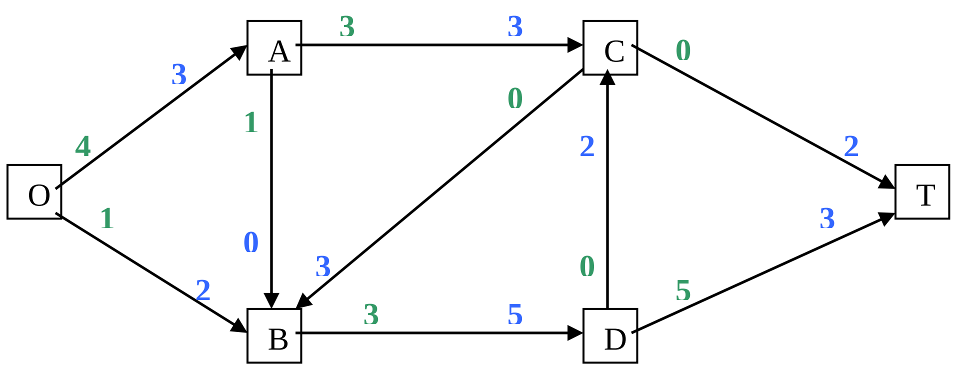
The first residual network:



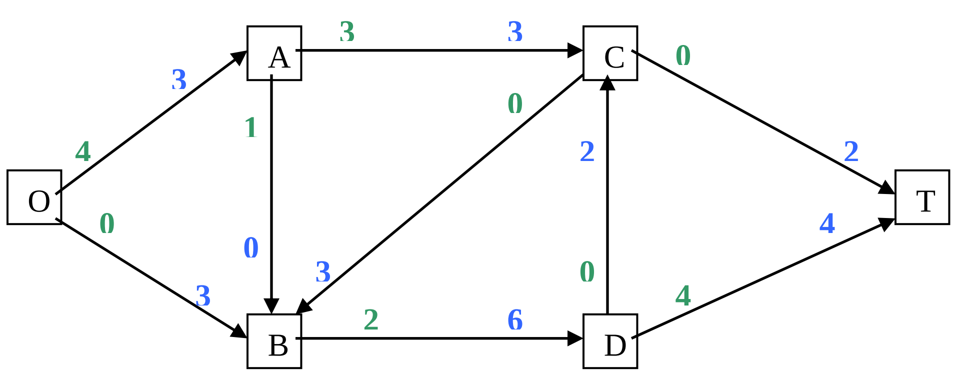
Iteration 1: As required, O→B→D→C→T is chosen as the first augmenting path. (The reason I asked you to start from this path is that it creates necessity for reversing some flow in later iterations.) Its residual capacity is min{3,8,2,2}=2. After sending 2 units of flow through the path, the resulting residual network is:



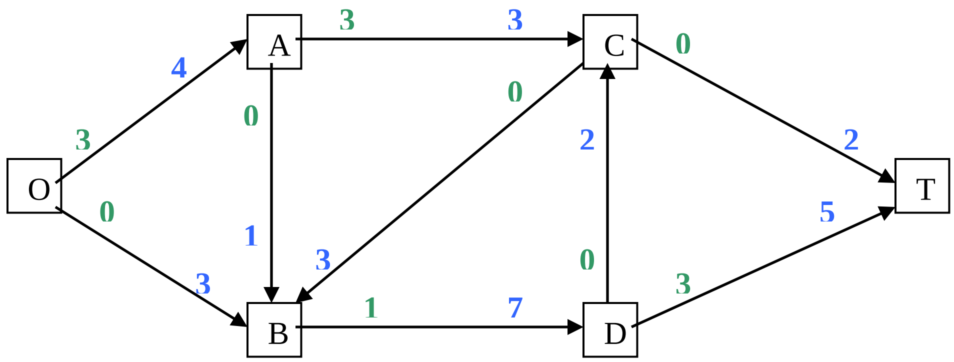
Iteration 2: Choose O→A→C→B→D→T as the next augmenting path. Its residual capacity is min{7,6,3,6,8}=3. After sending 3 units of flow through the path, the new residual network is:



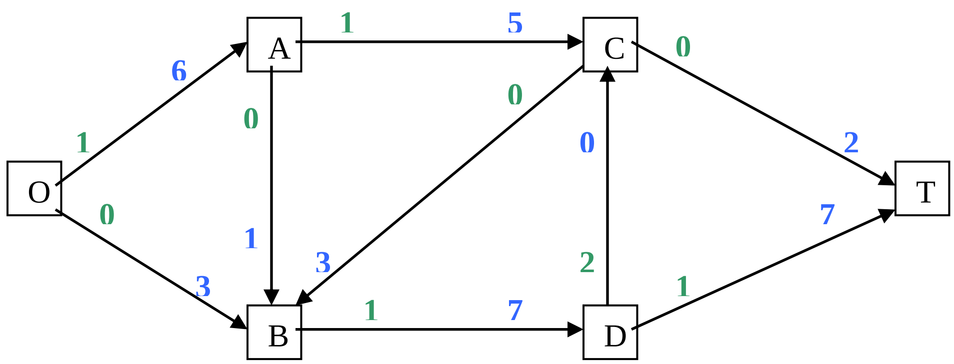
Iteration 3: Choose O→B→D→T as the next augmenting path. Its residual capacity is min{1,3,5}=1. After sending 1 units of flow through the path, the new residual network is:



Iteration 4: Choose O→A→B→D→T as the next augmenting path. Its residual capacity is min{4,1,2,4}=1. After sending 1 units of flow through the path, the new residual network is:



Iteration 5: Choose O→A→C→D→T as the next augmenting path. Its residual capacity is min{3,3,2,3}=2. (Note that we reverse the flow on arc D→C). After sending 2 units of flow through the path, the new residual network is:



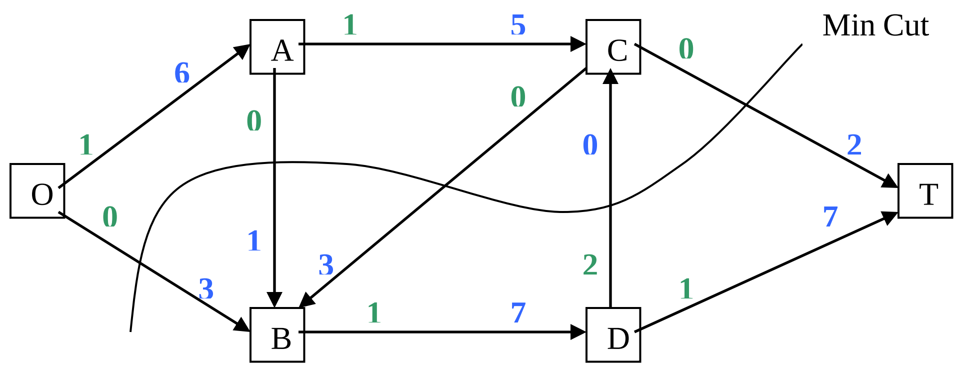
No more augmenting paths from O to T are left. Thus, the current residual network is optimal. The blue numbers on the arcs show the optimal flow values. Maximum flow value is 7+2 = 9 (the flow amount entering the sink).

b) Find a minimum cut. What is its capacity?

The nodes that can be reached from the source by augmenting paths are A and C. Thus, the O-side of the minimum cut is {O, A, C}. The minimum cut is

MinCut = {O→B, A→B, C→B, C→T}

(Note that arc D→C is not in the cut because it goes from T-side to O-side).
The capacity of the minimum cut is 3+1+3+2 = 9 which is equal to the maximum flow value.
The minimum cut is shown in the network below.



Min Cut
O
A
B
C
D
T
1
6
0
3
0
1
1
5
0
3
0
0
2
1
7
0
2
1
7